LEARNING TO FIND PROOFS AND THEOREMS
BY LEARNING TO REFINE SEARCH STRATEGIES

THE CASE OF LOOP INVARIANT SYNTHESIS

NEURAL INFORMATION PROCESSING SYSTEMS

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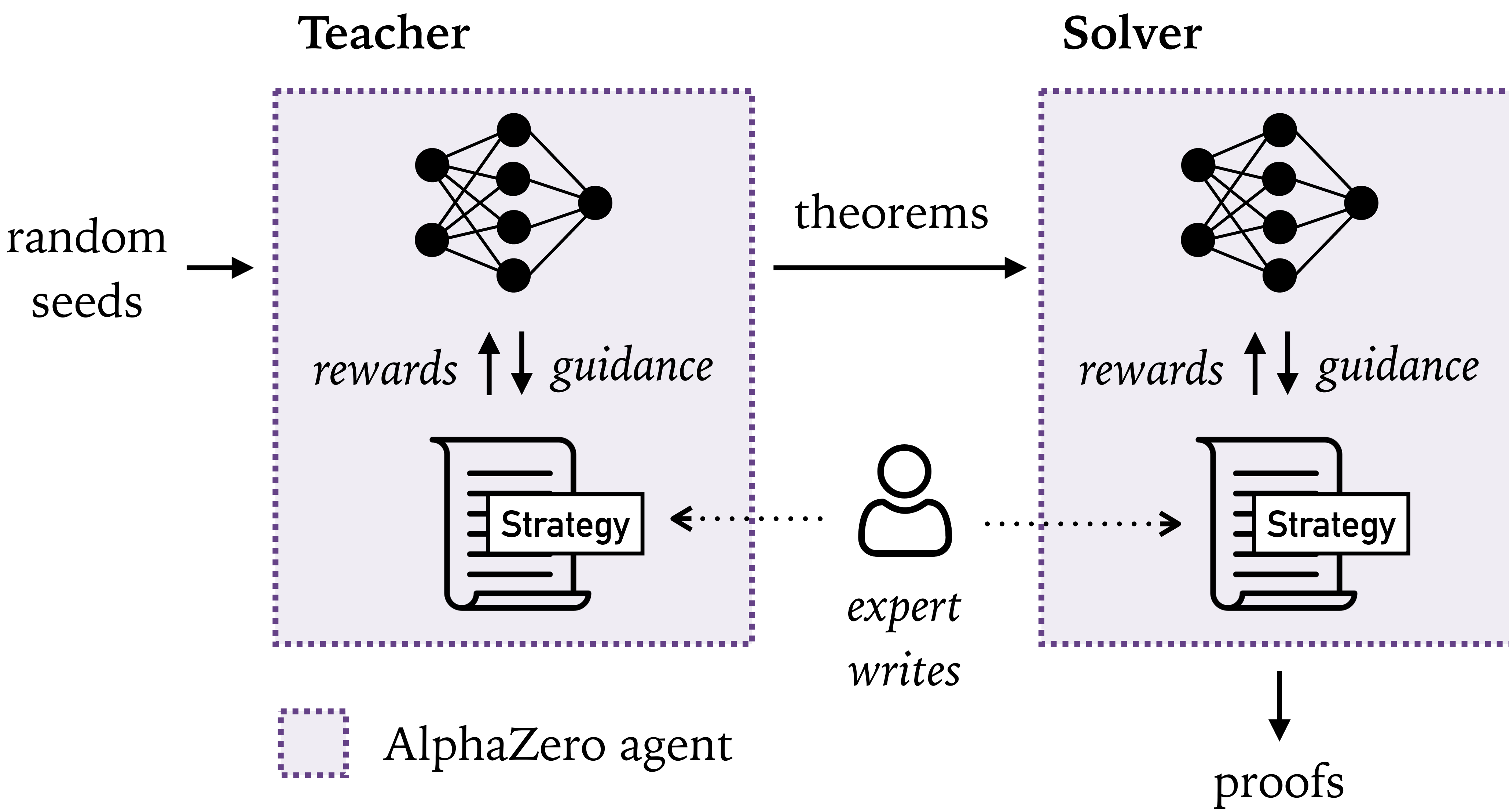
Can theorem proving be learned without a single example of a proof or theorem?

- **Imitation learning** is limited by the scarcity of human proofs
- **Reinforcement learning** presents challenges:
  - Infinite action spaces are hardly amenable to exploration
  - Theorems are still needed as training tasks

PROPOSED APPROACH



LOOP INVARIANT SYNTHESIS

- Training data unavailable and hard to generate!
- No pre-existing deep-learning agent capable of generalizing across instances.

```
assume x >= 1
y = 0
while y < 1000 {
   x = x + y
   y = y + 1
}
assert x >= y
```

To prove the final assertion, one must find a <u>loop invariant</u> that:

1. is true before the loop
2. is preserved by the loop body (when the loop guard holds)
3. implies the final assertion (when the loop guard does not hold)

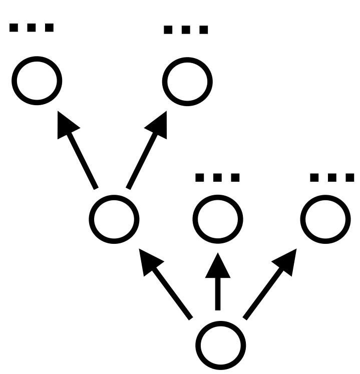
Invariant: $x \geq y \land x \geq 1 \land y \geq 0$

A LANGUAGE FOR EXPRESSING STRATEGIES

We define a strategy language based on **choose** and **event** operators.

Expert strategy

compiled into



MDP amenable to RL and neural-guided search

```
def solver(
    init: Formula, guard: Formula,
    body: Program, post: Formula) -> Formula:
    def prove_inv(inv: Formula) -> List[Formula]:
        assert valid(Implies(init, inv))
        ind = Implies(And(guard, inv), wlp(body, inv))
        event(PROVE_INV_EVENT)
        match abduct(ind):
        case Valid:
            return [inv]
        case [*suggestions]:
            aux = choose(suggestions)
            return [inv] + prove_inv(aux)
inv_cand = choose(abduct(Implies(Not(guard), post)))
inv_conjuncts = prove_inv(inv_cand)
return And(*inv_conjuncts)
```

▲ A **solver strategy** for invariant synthesis

GENERATING TRAINING PROBLEMS

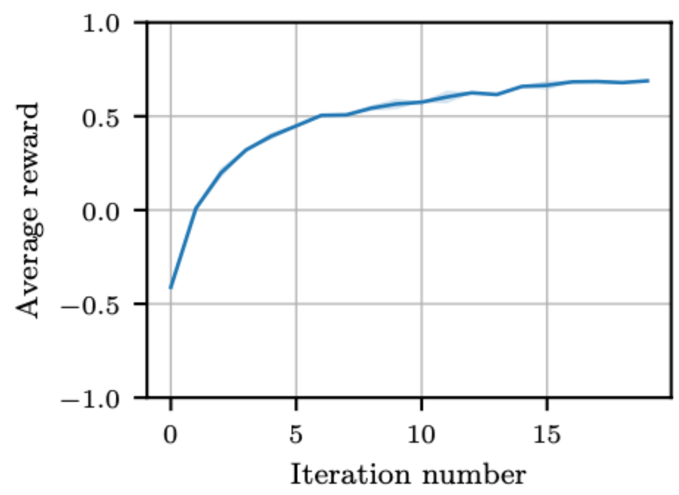
- Generating interesting theorems is harder than proving those!
- **Our approach:** refining **conditional generative strategies** using RL.

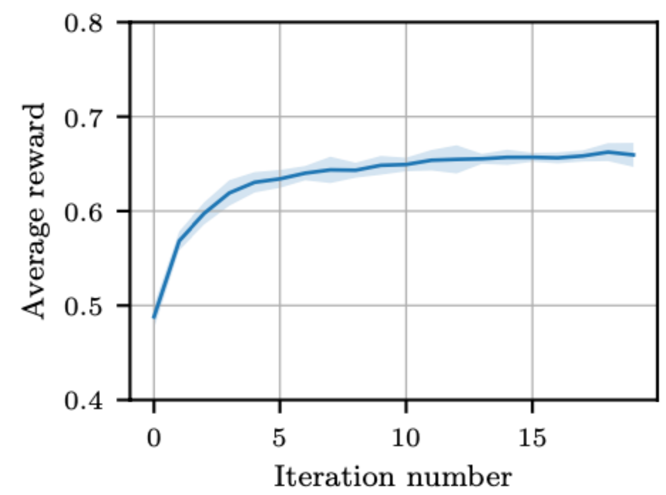
```
def teacher(rng: RandGen) -> Prog:
    cs = sample_constrs(rng)
    p = generate_prog(cs)
    p = transform(p, rng)
    p = hide_invariants(p)
    return p

def generate_prog(cs: Constrs):
    p = Prog("
      assume init;
      while (guard) {
        invariant inv_lin;
        invariant inv_aux;
        invariant inv_main;
        body; }
      assert post;")
    p = refine_guard(p, cs)
    p = refine_inv(p, cs)
    p = refine_body(p, cs)
    assert valid(inv_preserved(p))
    p = refine_post(p, cs)
    assert valid(inv_post(p))
    p = refine_init(p, cs)
    assert valid(inv_init(p))
    penalize_violations(p, cs)
    return p

def transform(p: Prog, rng: RandGen):
    p = shuffle_formulas(p, rng)
    p = add_useless_init(p, rng)
   ...
    return p
```

▲ Outline of a **teacher strategy** for invariant synthesis

RESULTS ON INVARIANT SYNTHESIS

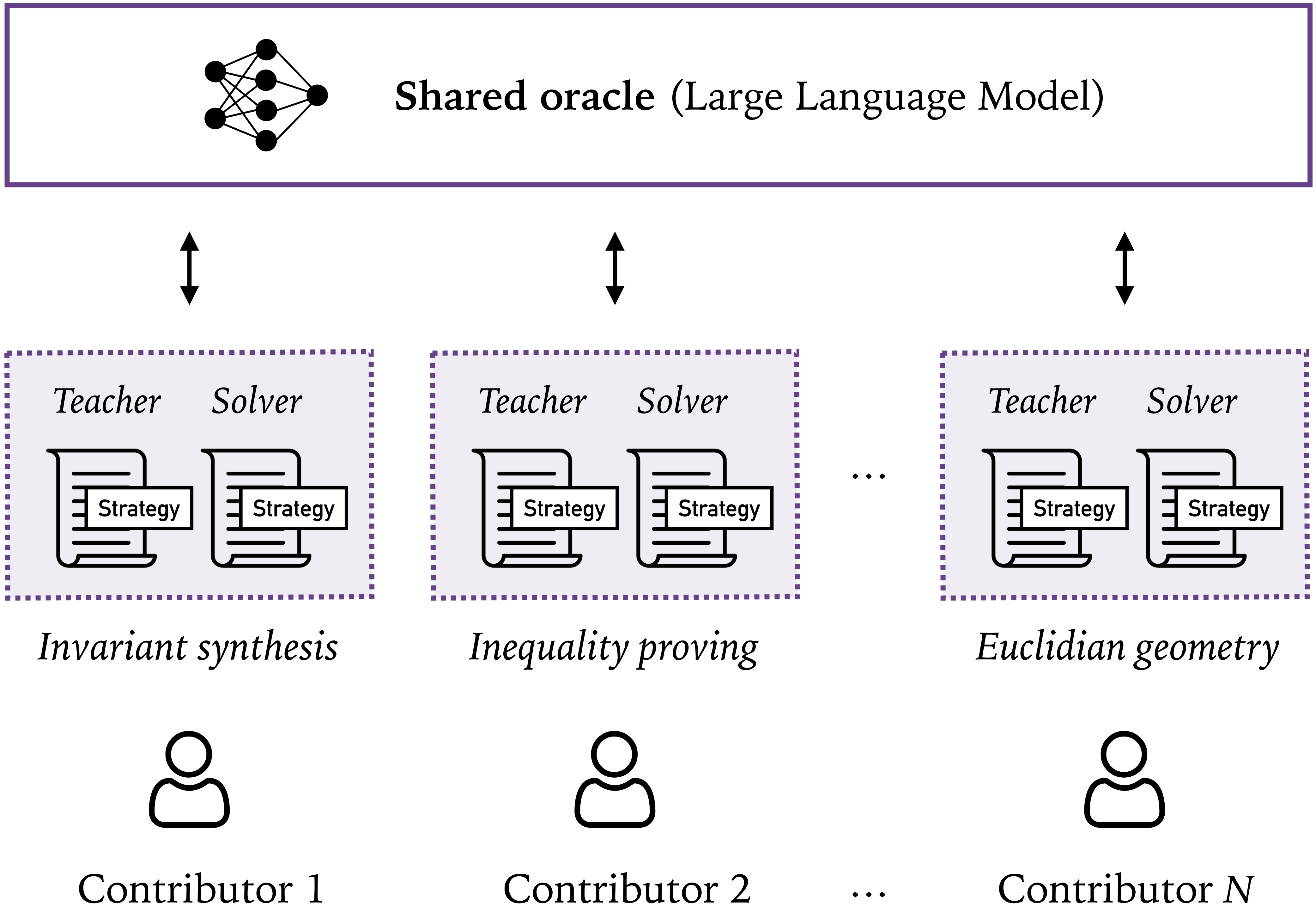
- Training curves for the teacher and the solver (respectively):





- Experimental results on Code2Inv (no backtracking search):

Policy	% Problems solved
Random	18.4 ± 0.0
Network (untrained teacher)	39.7 ± 1.6
Network (trained teacher)	$\mathbf{61.5} \pm \mathbf{0.4}$



Shared oracle (Large Language Model)
Teacher
Solver
Strategy
Strategy
Invariant synthesis
Teacher
Solver
Strategy
Strategy
Inequality proving
...
Teacher
Solver
Strategy
Strategy
Euclidian geometry
Contributor 1
Contributor 2
...
Contributor N