LEARNING TO FIND PROOFS AND THEOREMS
BY LEARNING TO REFINE SEARCH STRATEGIES

THE CASE OF LOOP INVARIANT SYNTHESIS

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Can theorem proving be learned without a single example of a proof or theorem?

- **Imitation learning** is limited by the scarcity of human proofs
- **Reinforcement learning** presents challenges:
  - Infinite action spaces are hardly amenable to exploration
  - Theorems are still needed as training tasks
PROPOSED APPROACH

Teacher

random seeds →

rewards ↑↓ guidance

Solver

theorems →

rewards ↑↓ guidance

expert writes

AlphaZero agent

proofs
LOOP INVARIANT SYNTHESIS

• Training data unavailable and hard to generate!
• No pre-existing deep-learning agent capable of generalizing across instances.

To prove the final assertion, one must find a loop invariant that:

1. is true before the loop
2. is preserved by the loop body (when the loop guard holds)
3. implies the final assertion (when the loop guard does not hold)

**Invariant:** \( x \geq y \land x \geq 1 \land y \geq 0 \)
A LANGUAGE FOR EXPRESSING STRATEGIES

We define a strategy language based on `choose` and `event` operators.

```
def solver(
    init: Formula, guard: Formula,
    body: Program, post: Formula) -> Formula:

def prove_inv(inv: Formula) -> List[Formula]:
    assert valid(Implies(init, inv))
    ind = Implies(And(guard, inv), wlp(body, inv))
    event(PROVE_INV_EVENT)
    match abduct(ind):
        case Valid:
            return [inv]
        case [*suggestions]:
            aux = choose(suggestions)
            return [inv] + prove_inv(aux)
        inv_cand = choose(abduct(Implies(Not(guard), post)))
        inv_conjuncts = prove_inv(inv_cand)
        return And(*inv_conjuncts)
```

▲ A solver strategy for invariant synthesis
GENERATING TRAINING PROBLEMS

• Generating interesting theorems is harder than proving those!

• Our approach: refining conditional generative strategies using RL.

▲ Outline of a teacher strategy for invariant synthesis

```python
def teacher(rng: RandGen) -> Prog:
    cs = sample_constrs(rng)
    p = generate_prog(cs)
    p = transform(p, rng)
    p = hide_invariants(p)
    return p

def generate_prog(cs: Constrs):
    p = Prog("assume init;
               while (guard) {
                   invariant inv_lin;
                   invariant inv_aux;
                   invariant inv_main;
                   body; }
               assert post;")

    p = refine_guard(p, cs)
    p = refine_inv(p, cs)
    p = refine_body(p, cs)
    assert valid(inv_preserved(p))
    p = refine_post(p, cs)
    assert valid(inv_post(p))
    p = refine_init(p, cs)
    assert valid(inv_init(p))
    penalize_violations(p, cs)
    return p

def transform(p: Prog, rng: RandGen):
    p = shuffle_formulas(p, rng)
    p = add_useless_init(p, rng)
    ...
    return p
```
RESULTS ON INVARIANT SYNTHESIS

• Training curves for the teacher and the solver (respectively):

![Training curves](image)

• Experimental results on Code2Inv (no backtracking search):

<table>
<thead>
<tr>
<th>Policy</th>
<th>% Problems solved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>18.4 ± 0.0</td>
</tr>
<tr>
<td>Network (untrained teacher)</td>
<td>39.7 ± 1.6</td>
</tr>
<tr>
<td>Network (trained teacher)</td>
<td>61.5 ± 0.4</td>
</tr>
</tbody>
</table>
Shared oracle (Large Language Model)

Invariant synthesis

Inequality proving

Euclidian geometry

Contributor 1

Contributor 2

... (omitted)

Contributor N