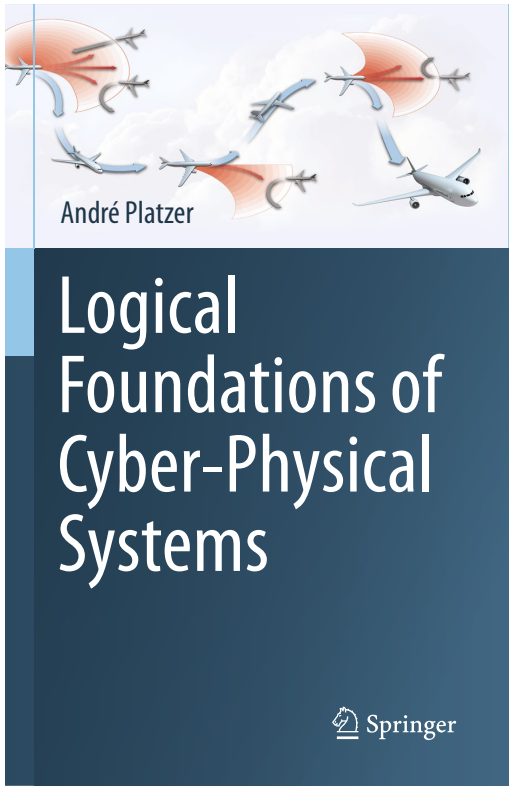
19: Verified Models & Verified Runtime Validation

Logical Foundations of Cyber-Physical Systems



André Platzer

Carnegie Mellon University
Computer Science Department

Outline

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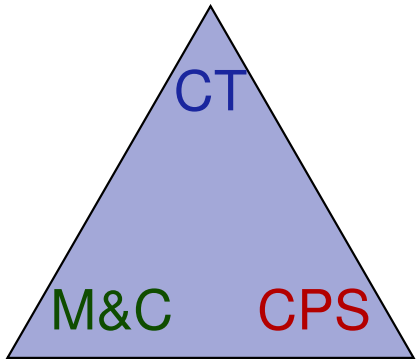
Learning Objectives

proof in a model vs. truth in reality
tracing assumptions
turning provers upside down
correct-by-construction
dynamic contracts
proofs for CPS implementations



models vs. reality
inevitable differences
model compliance
architectural design

tame CPS complexity
runtime validation
online monitor
prediction vs. run

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What Else Could Possibly Go Wrong?

Proposition (System Proved Safe)

$$A \rightarrow [(ctrl; plant)^*]S$$

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only apply if CPS fits to the model

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$\rightsquigarrow$ Verifiably correct runtime model validation

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Runtime Monitor for Runtime Validation of Model

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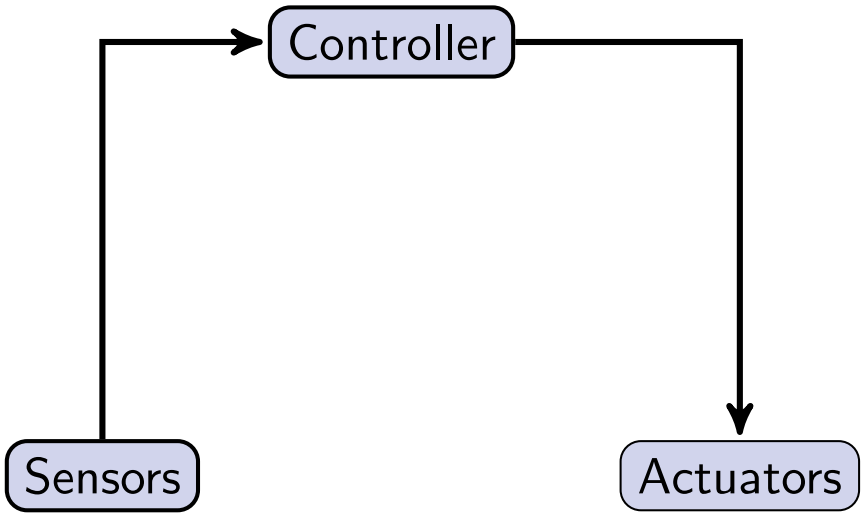

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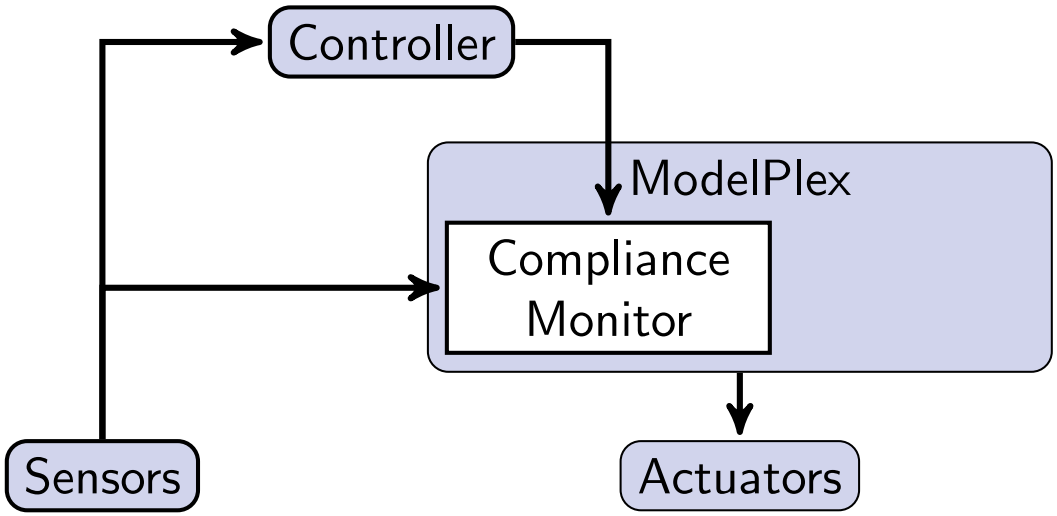

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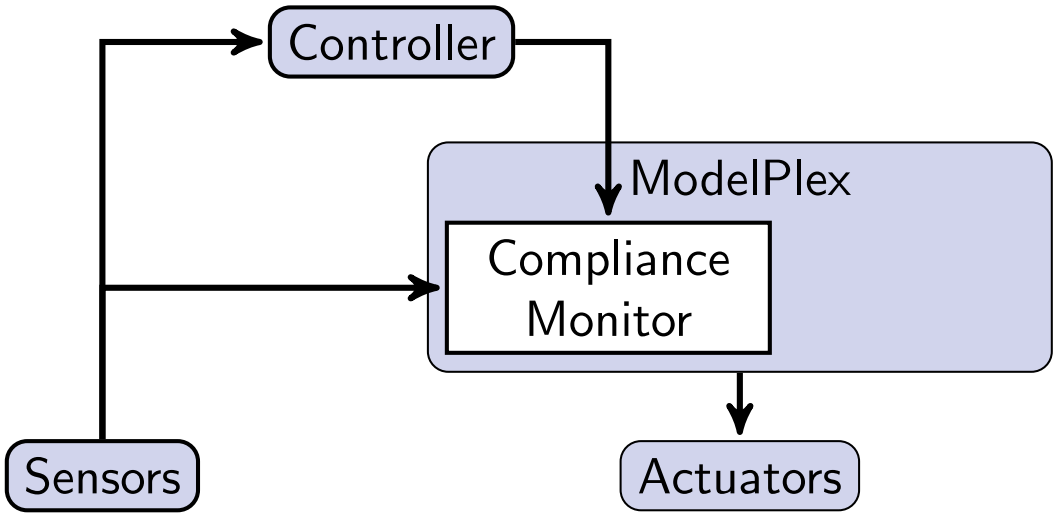

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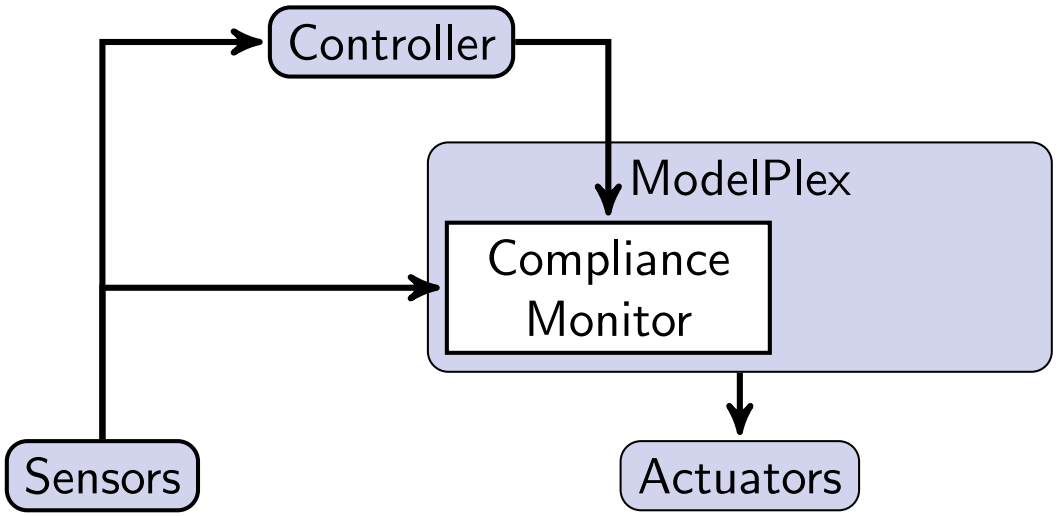

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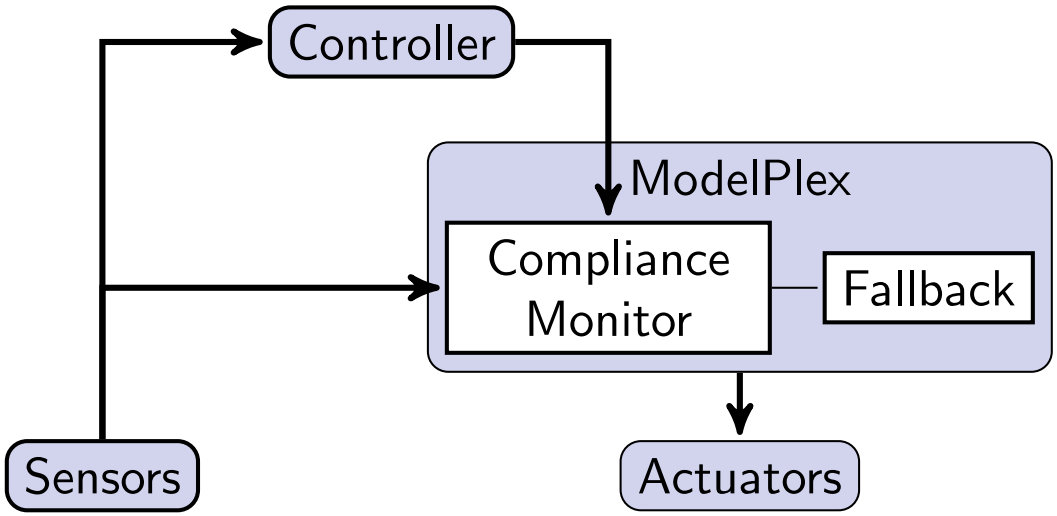

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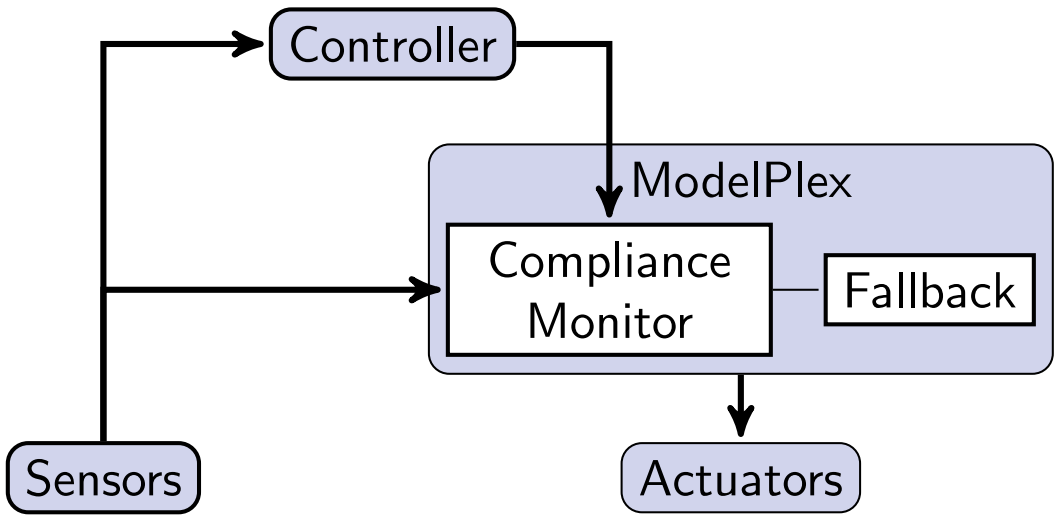

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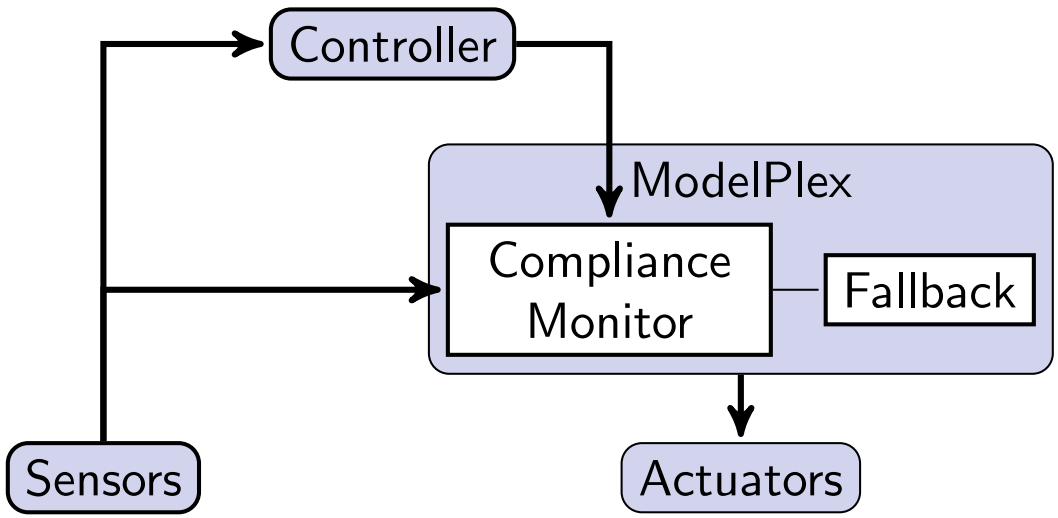
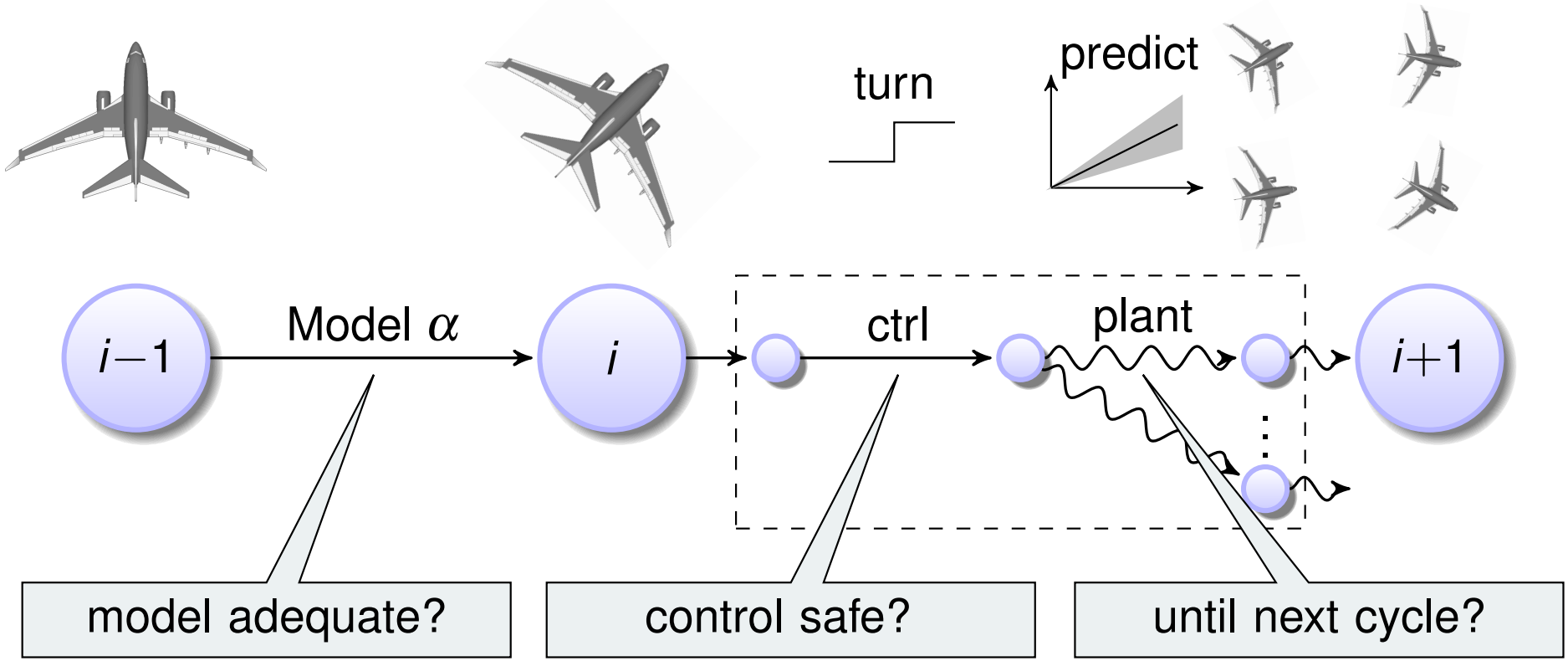


ModelPlex **ensures that verification results** about models
apply to CPS implementations



ModelPlex: Verified Runtime Validation of Models

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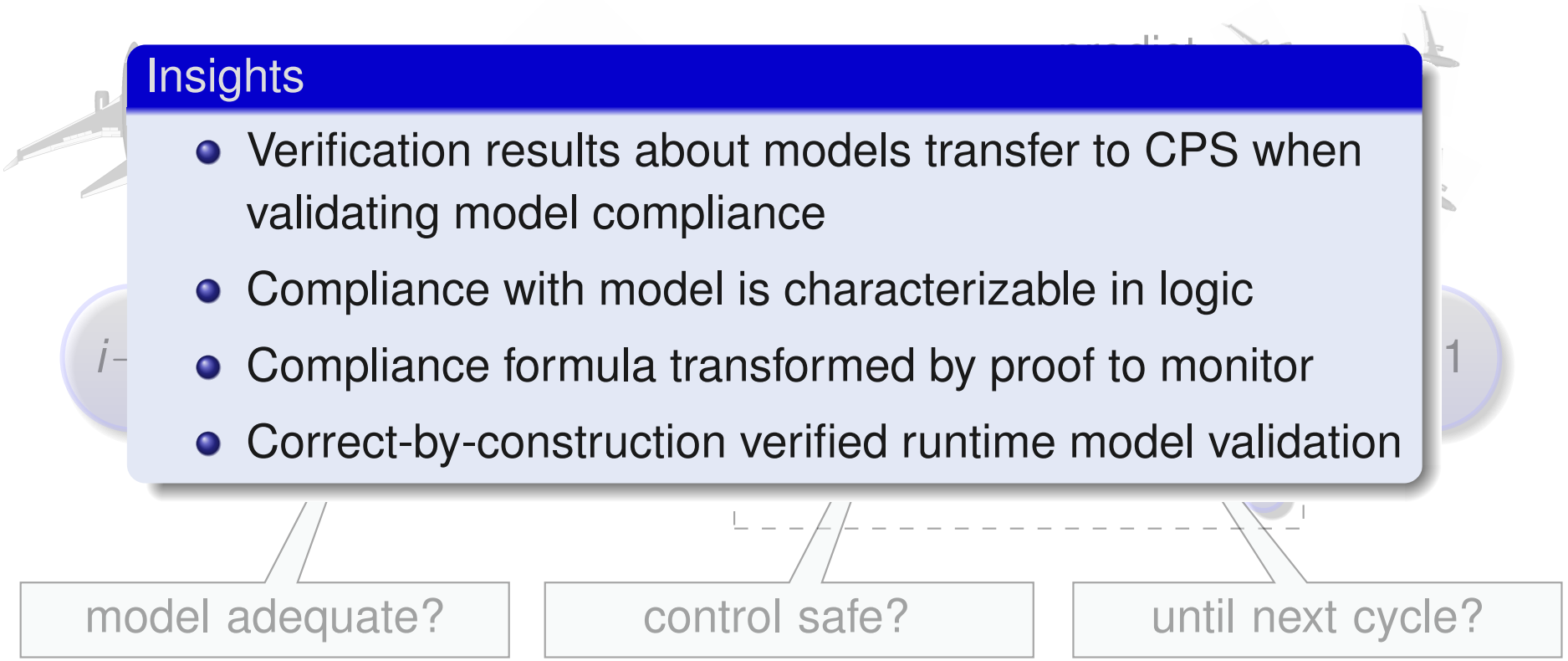

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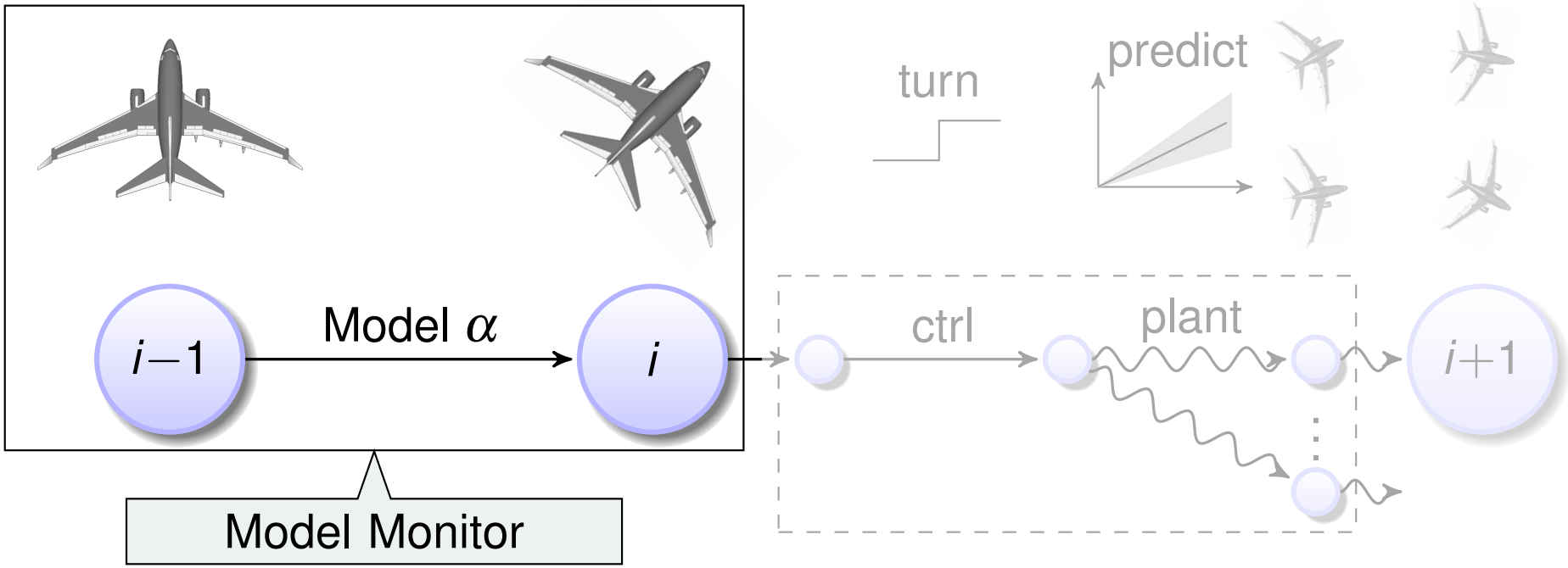
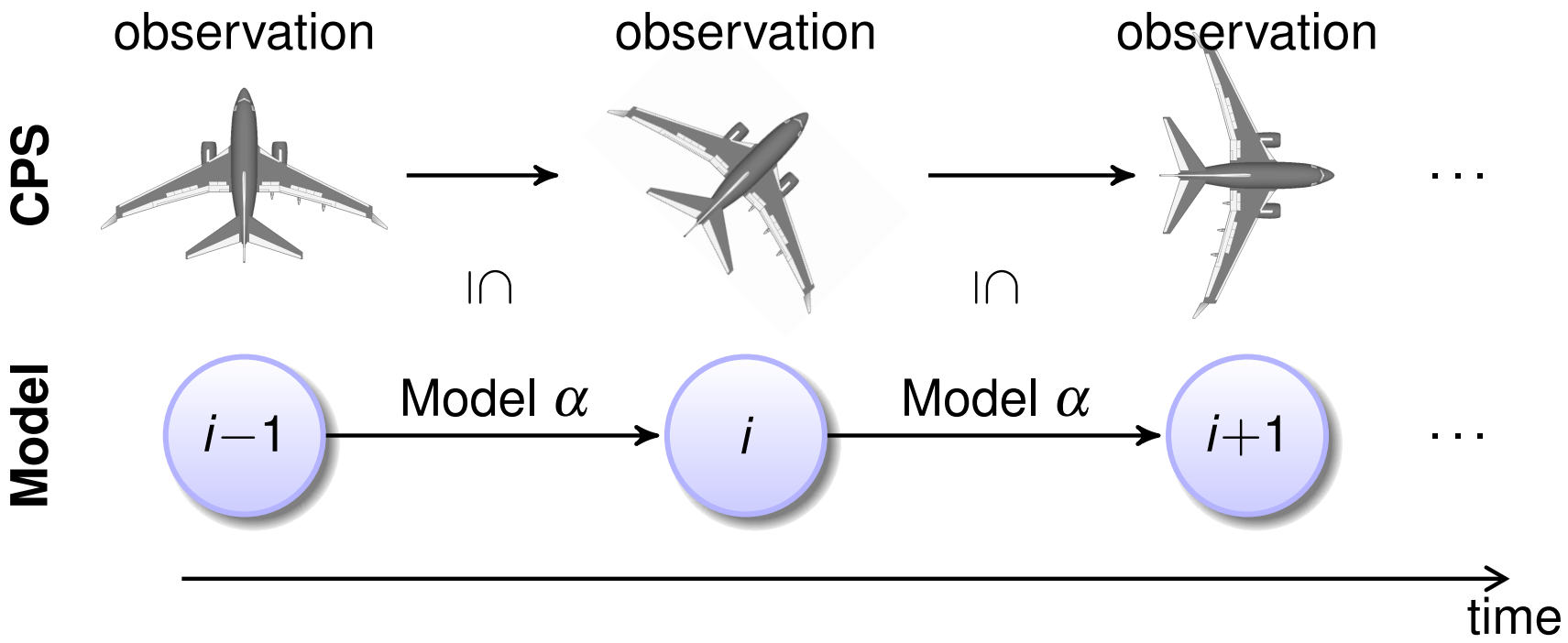


Model Compliance

Is present CPS behavior included in the behavior of the model?

- CPS observed through sensors
- Model describes all possible behavior of CPS between states



Detect non-compliance ASAP to initiate fallback actions while still safe

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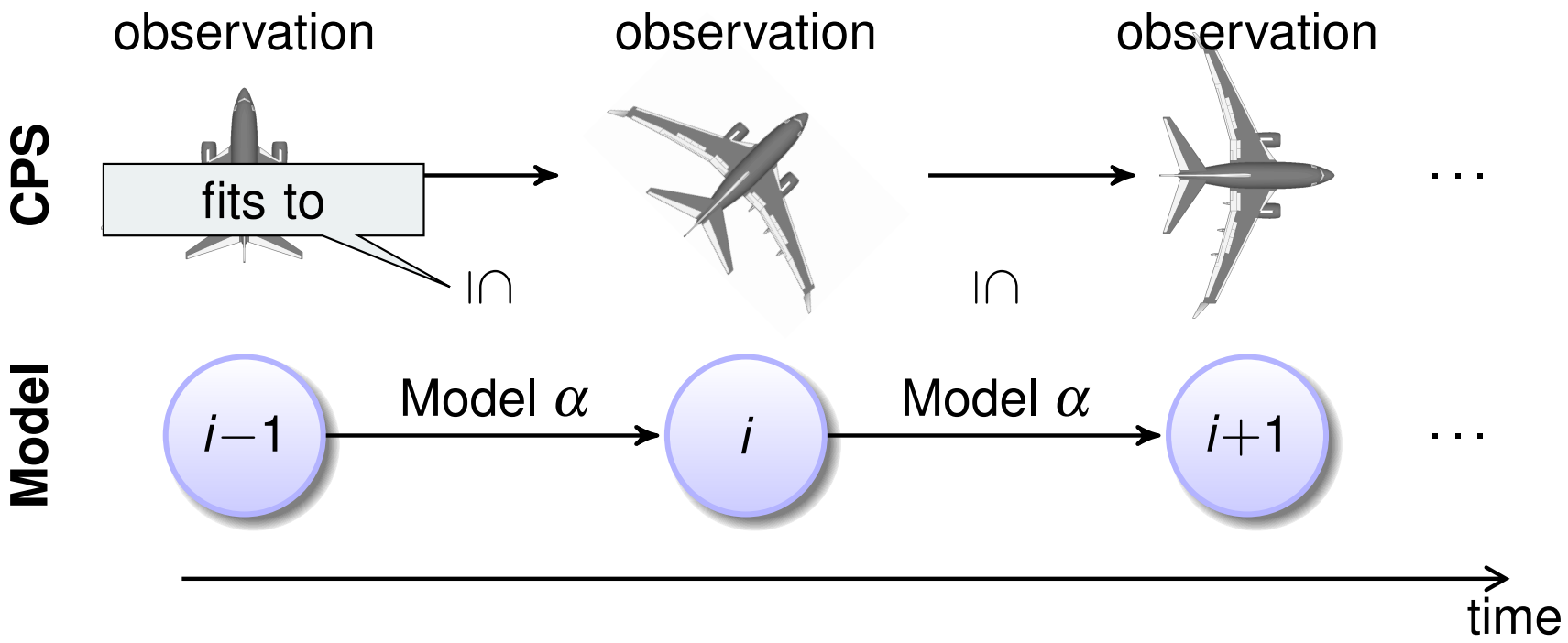

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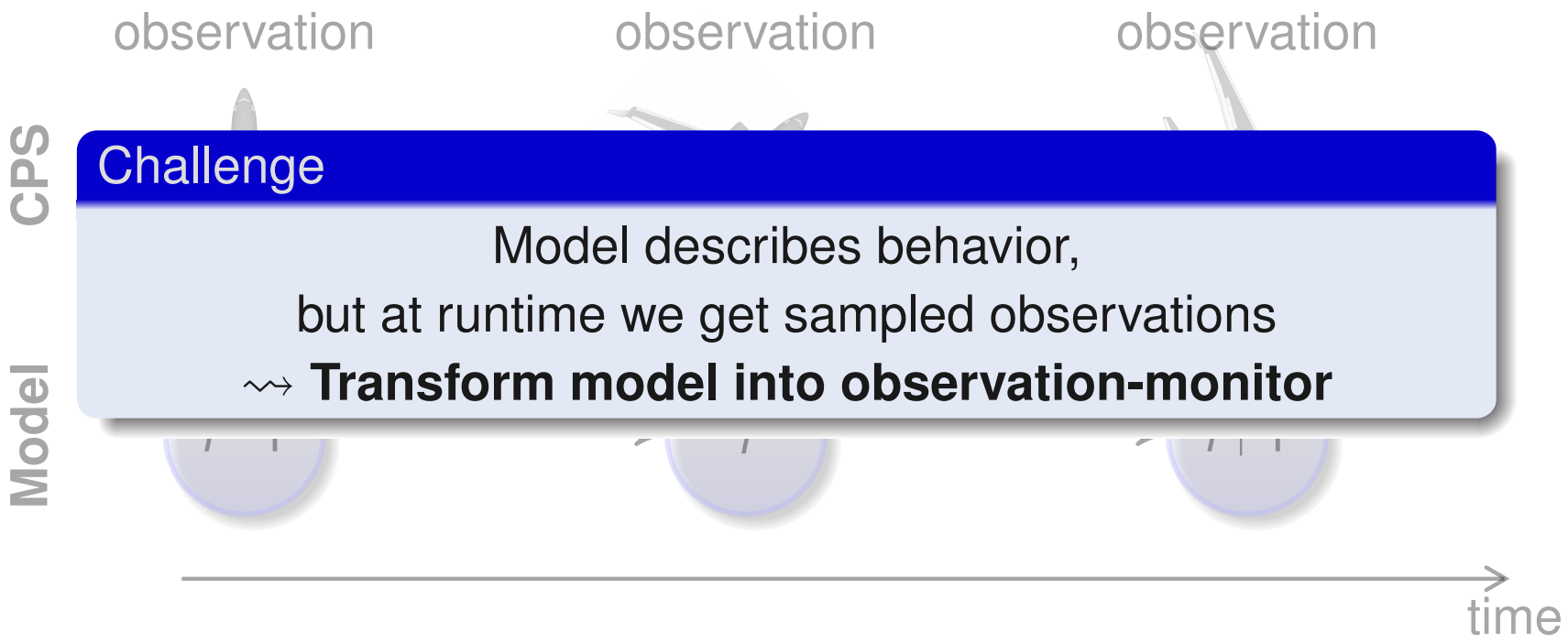

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Quantum's Bouncing Ball Monitors

Proposition (Quantum can bounce around safely)

$$0 \leq x \wedge x = H \wedge v = 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \rightarrow$$
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Example (Controller Monitor)

control changes (x, v) to (x^+, v^+)

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test+domain

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$$(v^+ = v - gt \wedge x^+ = x + vt - \tfrac{g}{2}t^2)$$

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from invariant — $2gx = 2gH - v^2$

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directionality: always falling

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Example (Controller Monitor)

$(x = 0$

Takeaway

Monitors are subtle, in desperate need of correctness proof.
What proof implies a safe system if the monitors pass?

Exam

$2g(x^+ -$

Example (Model Monitor)

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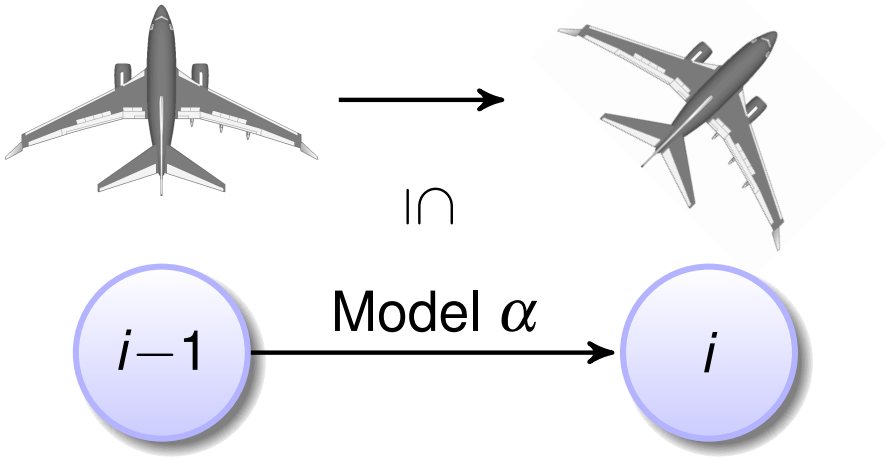
Characterizing State Relations in Logic

When are two states linked through a run of model α?



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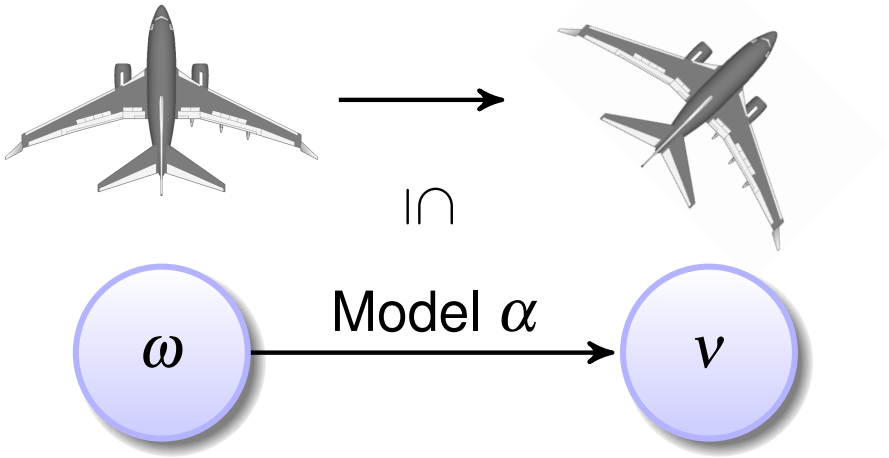

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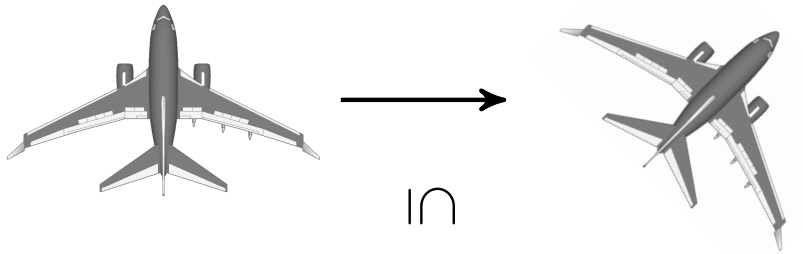

a prior state characterized by x

a posterior state characterized by x^+

ω — Model α → ν

Semantical: $(\omega, \nu) \in [\![\alpha]\!]$ ◁ reachability relation of α

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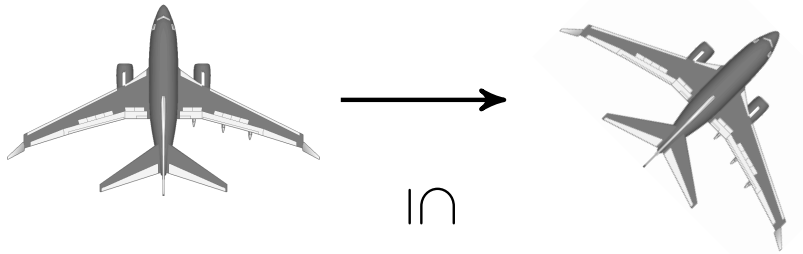

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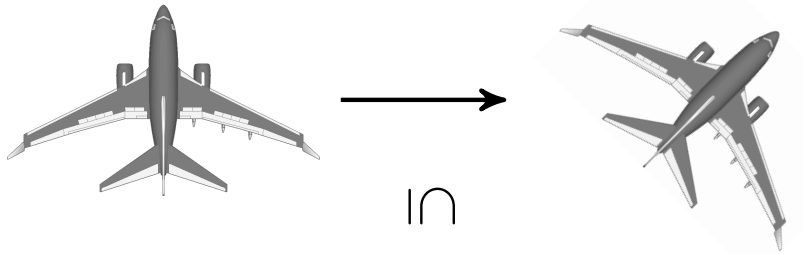

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check at runtime (efficient)

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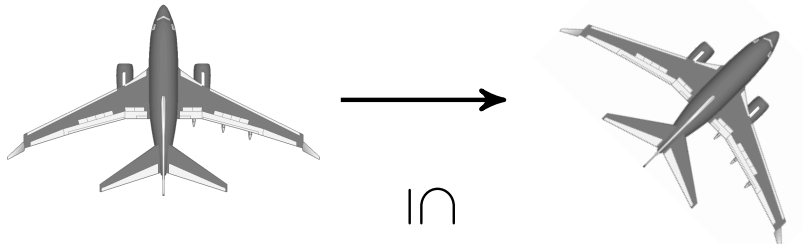

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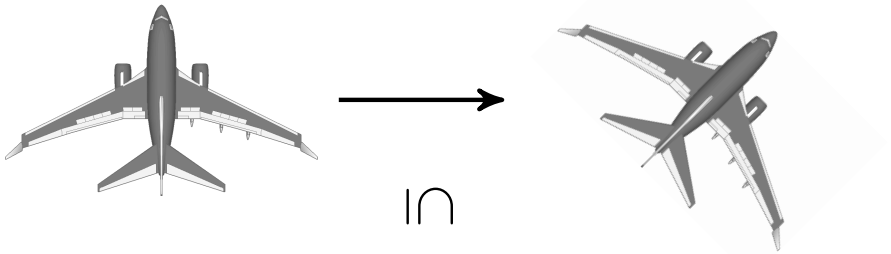
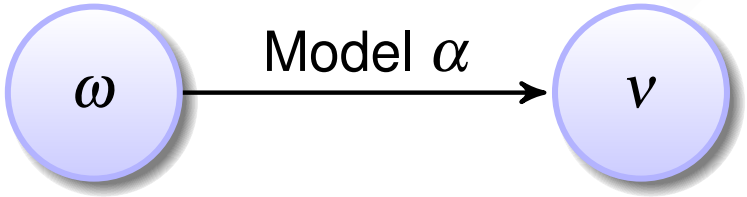
Logical Reductions for Model Safety Transfer

Logic reduces CPS safety to runtime monitor with offline proof



dL proof $A \to [\alpha]S$



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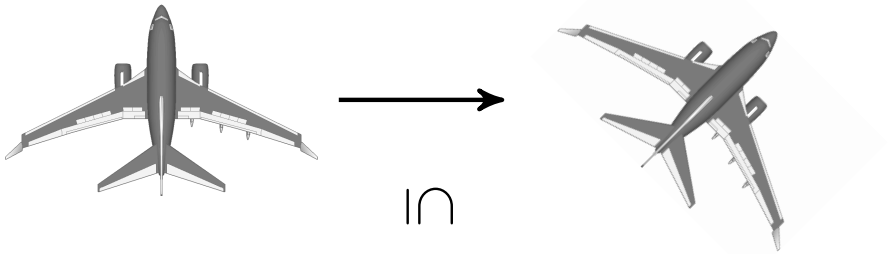

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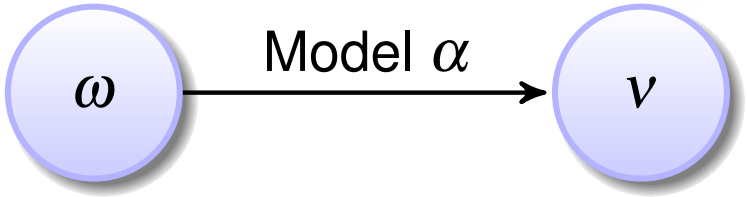

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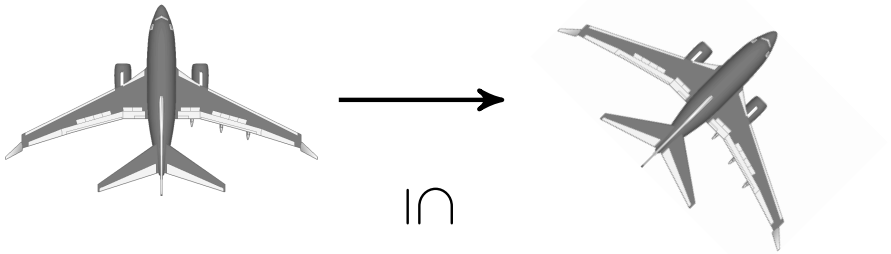

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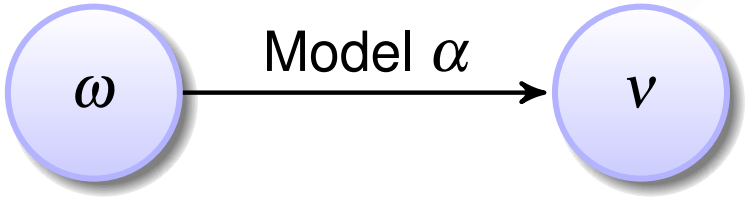

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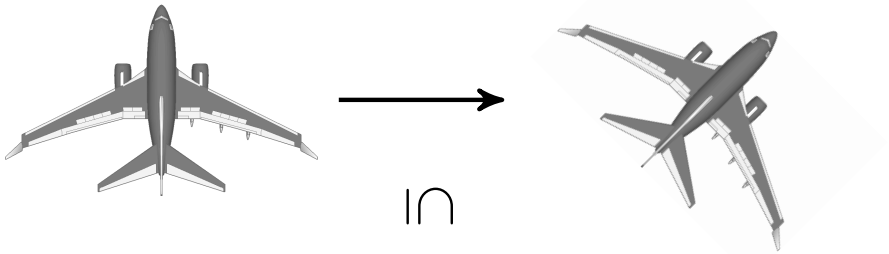

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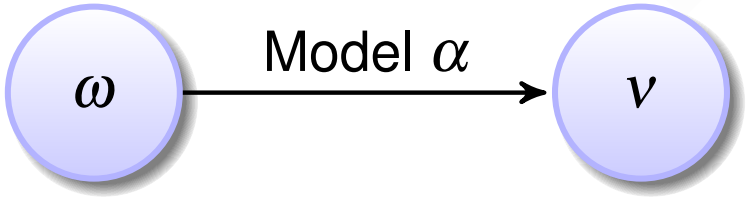

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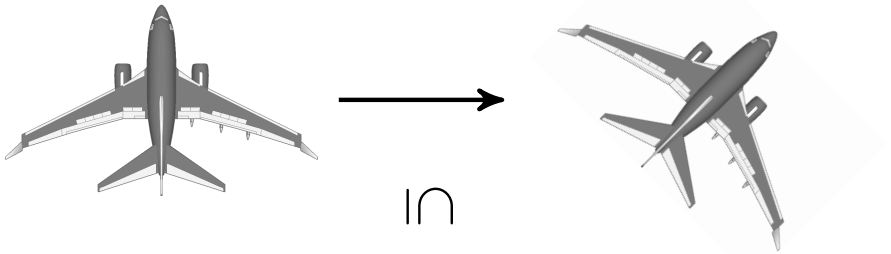

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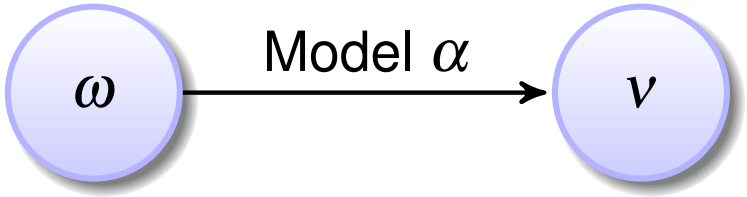

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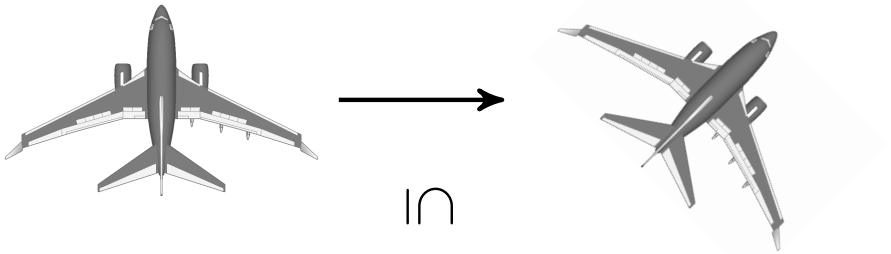

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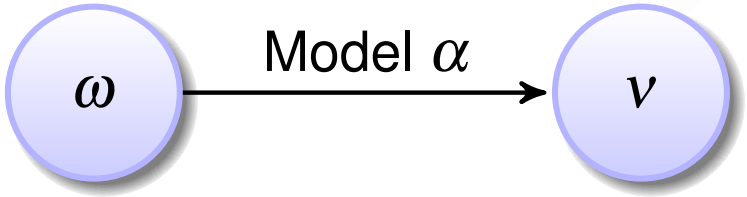

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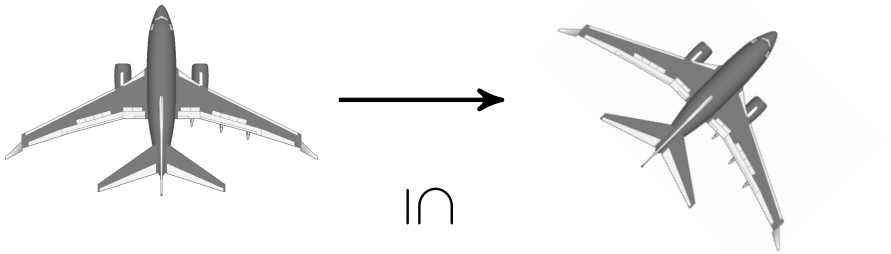

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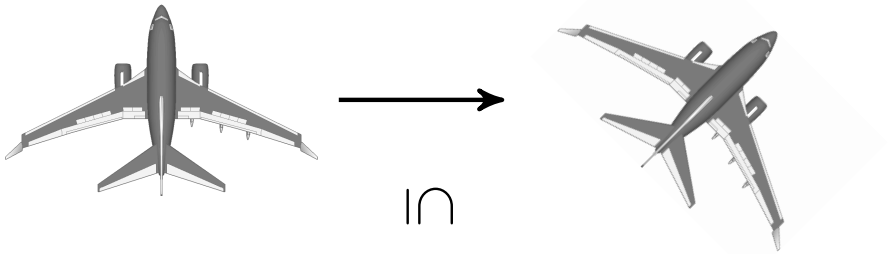

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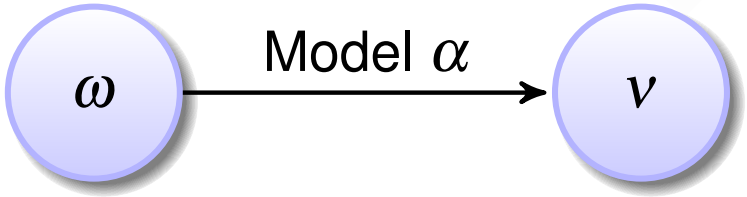

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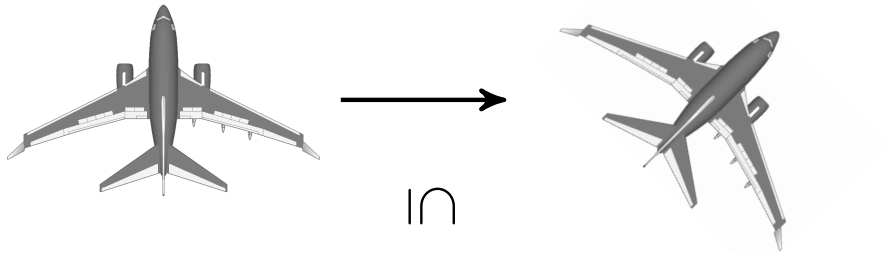

Not initial state.
Model repeats...

dL proof $A \to [\alpha]S$

ω — Model α → ν

Offline

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Semantical: $(\omega, \nu) \in [\![\alpha]\!]$

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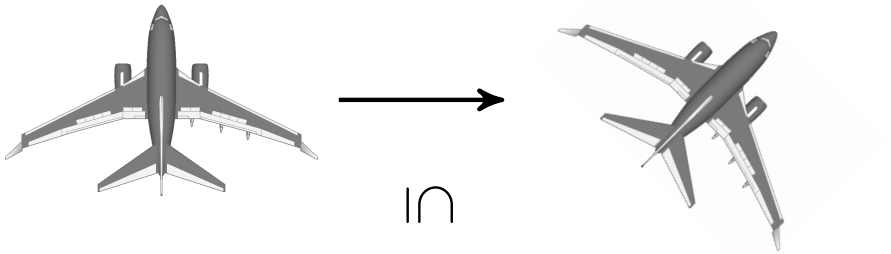
$\Uparrow$ dL proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

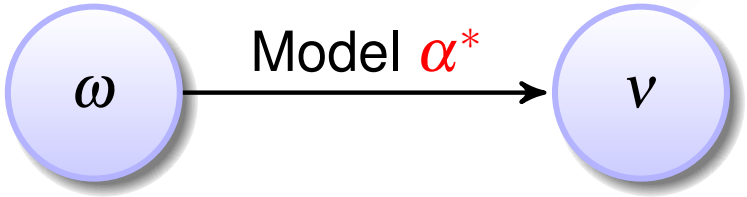
check at runtime (efficient)

Logical Reductions for α^* Model Safety Transfer

Logic reduces CPS safety to runtime monitor with offline proof



dL proof $A \rightarrow [\alpha^*]S$



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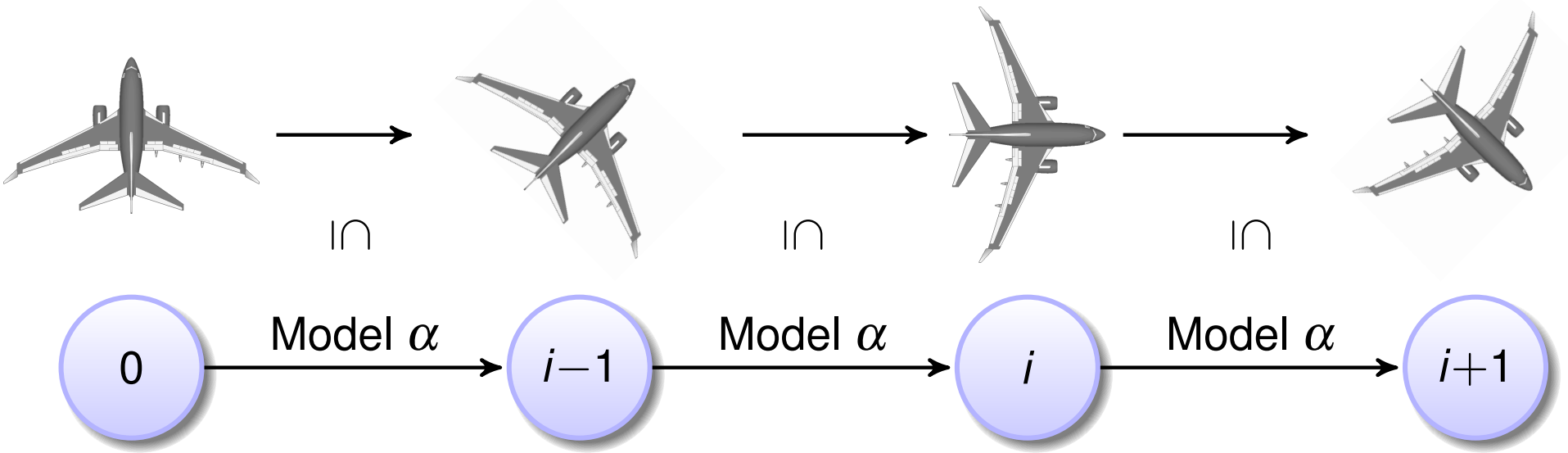
ModelPlex Model Monitor Correctness



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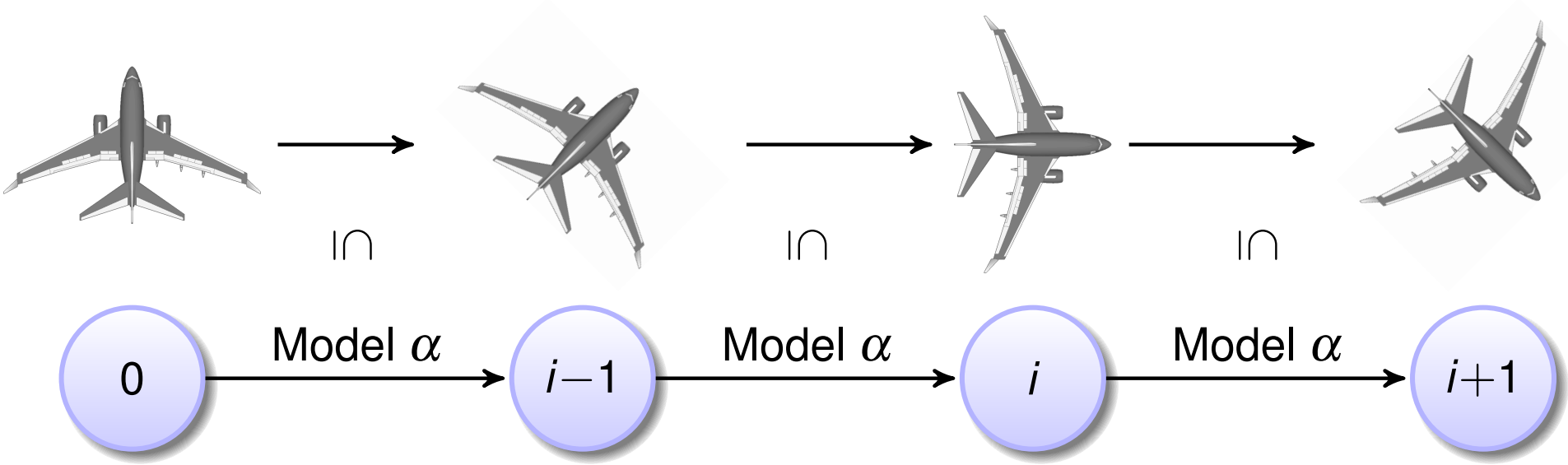

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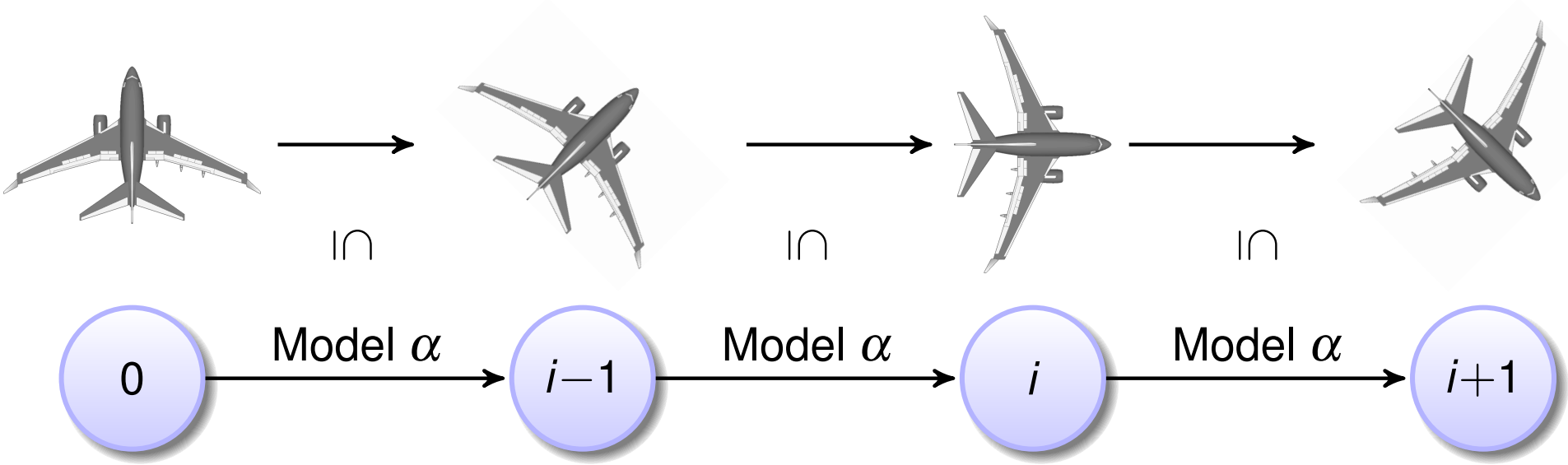

Init $0 \models A$

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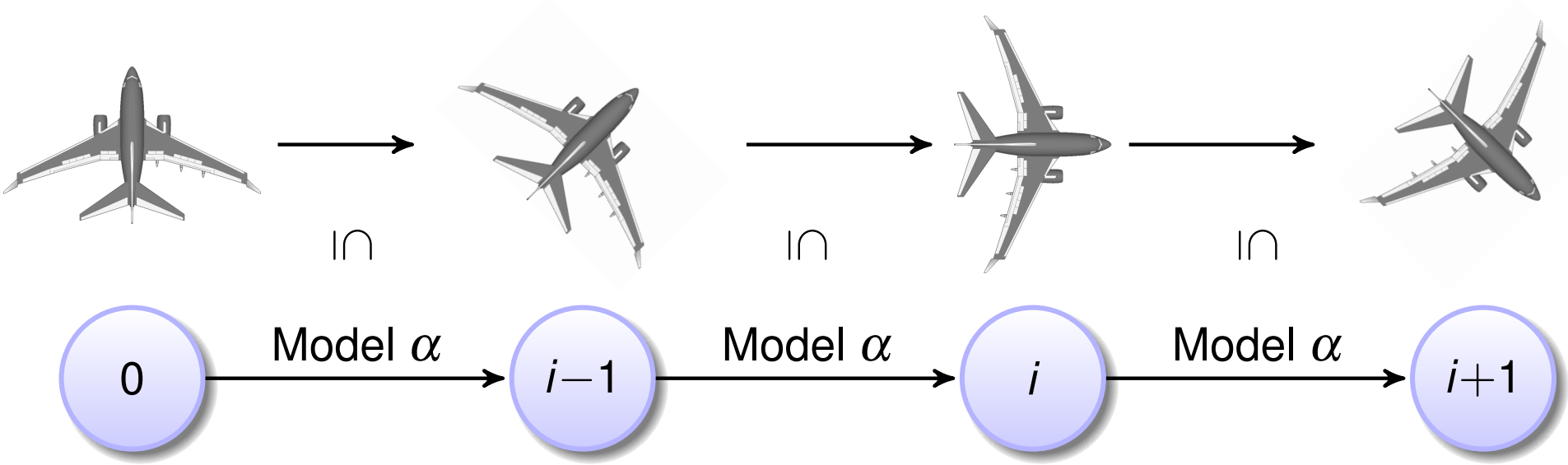

Init $0 \models A$

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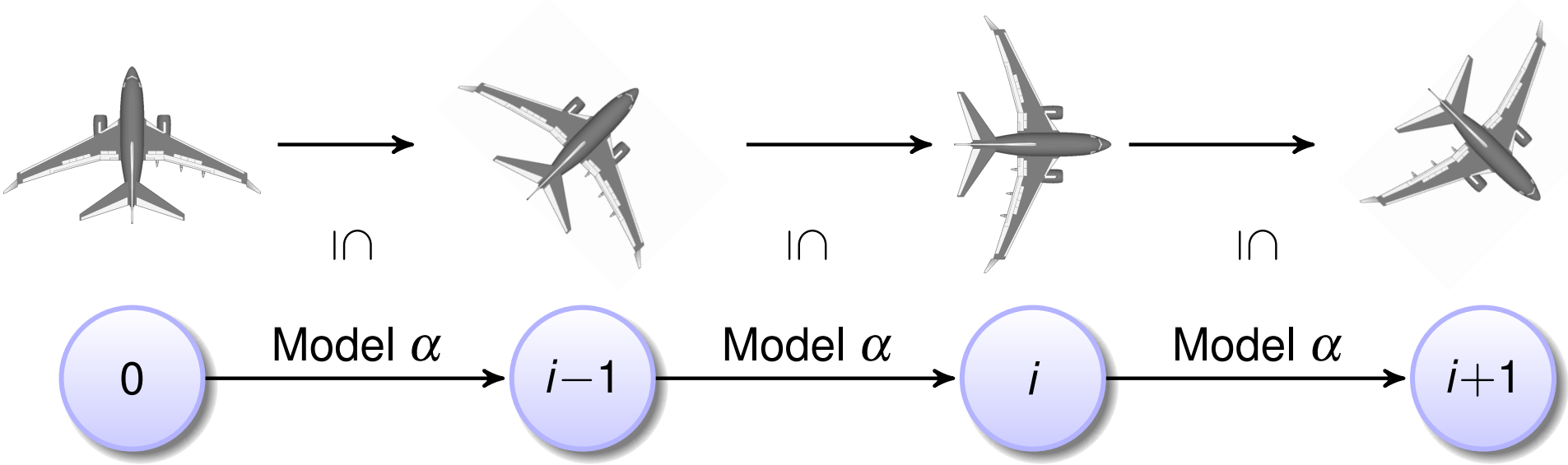

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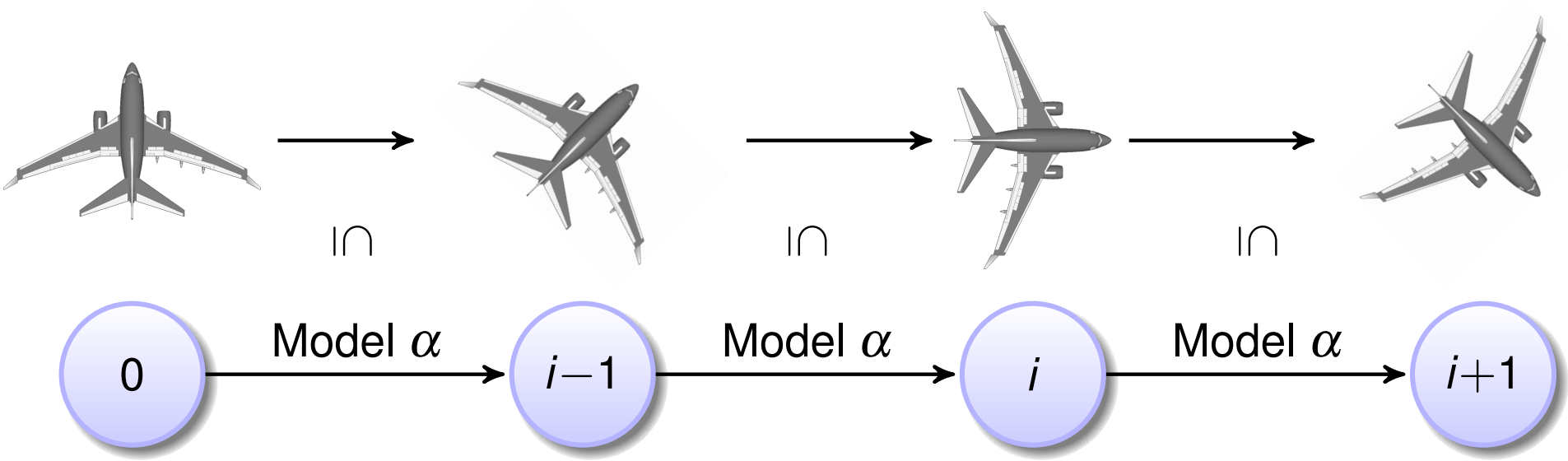

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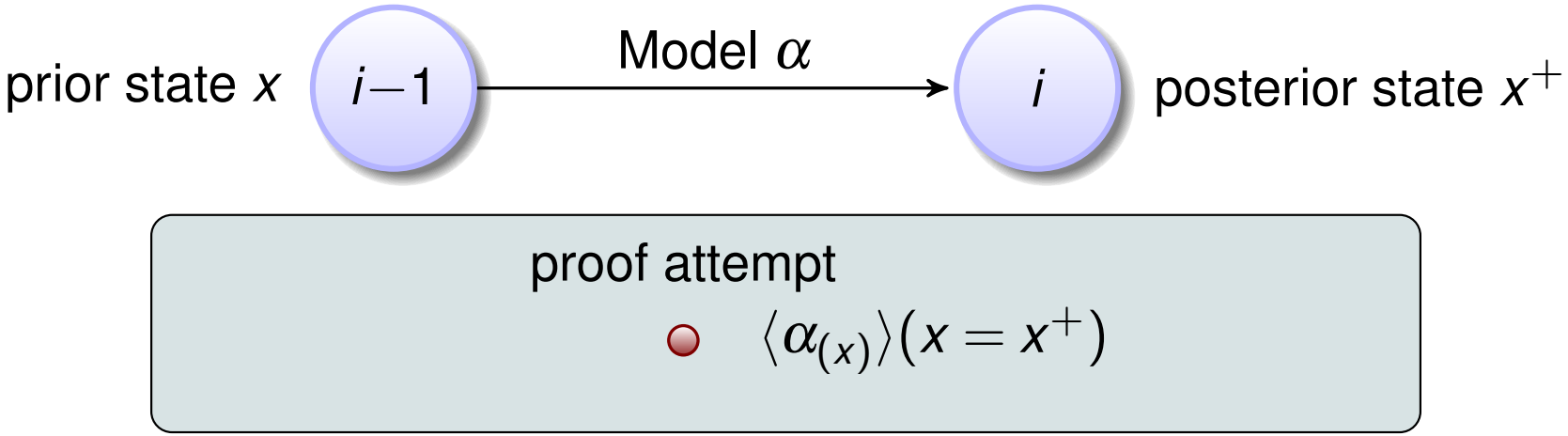
Theorem (Model Monitor Correctness) (FMSD'16)

System safe as long as monitor satisfied.

Correct-by-Construction Synthesis

- dL proof calculus executes models symbolically



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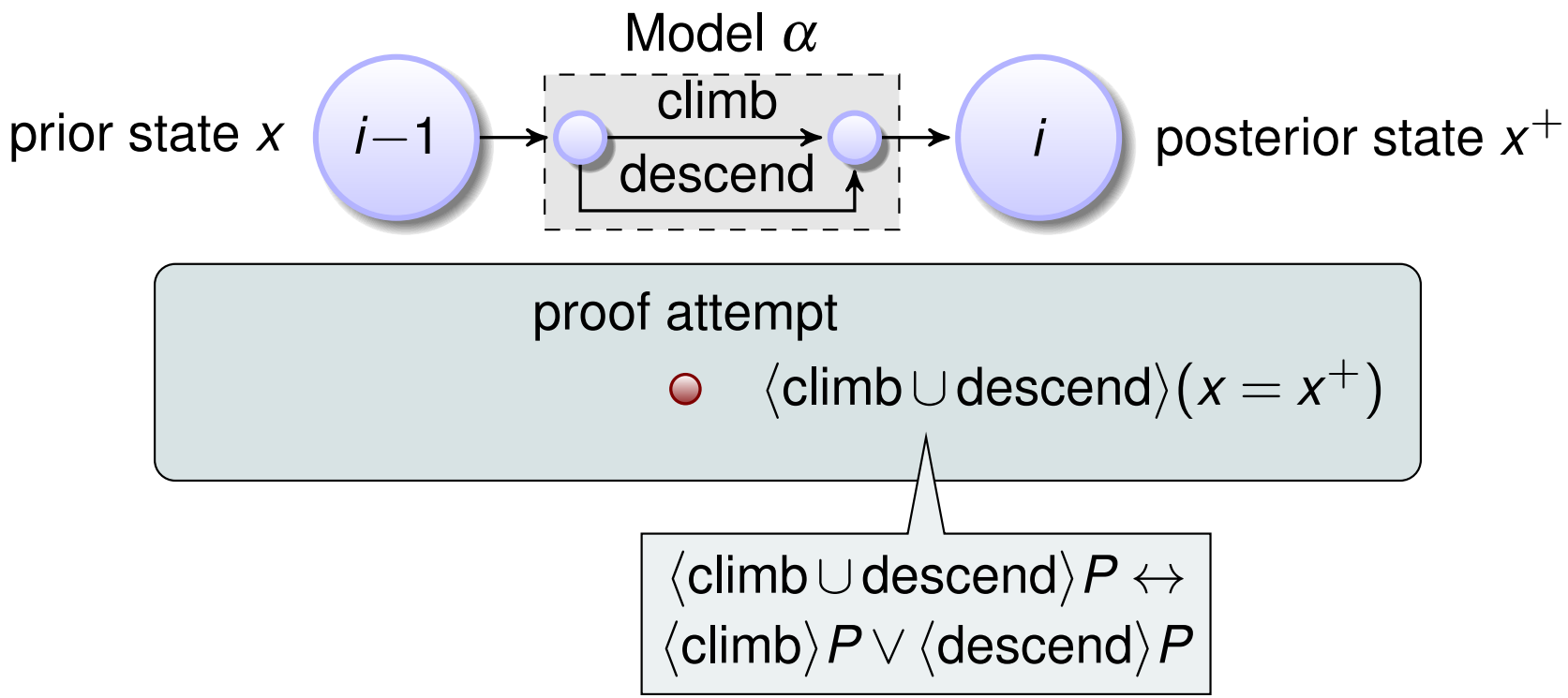

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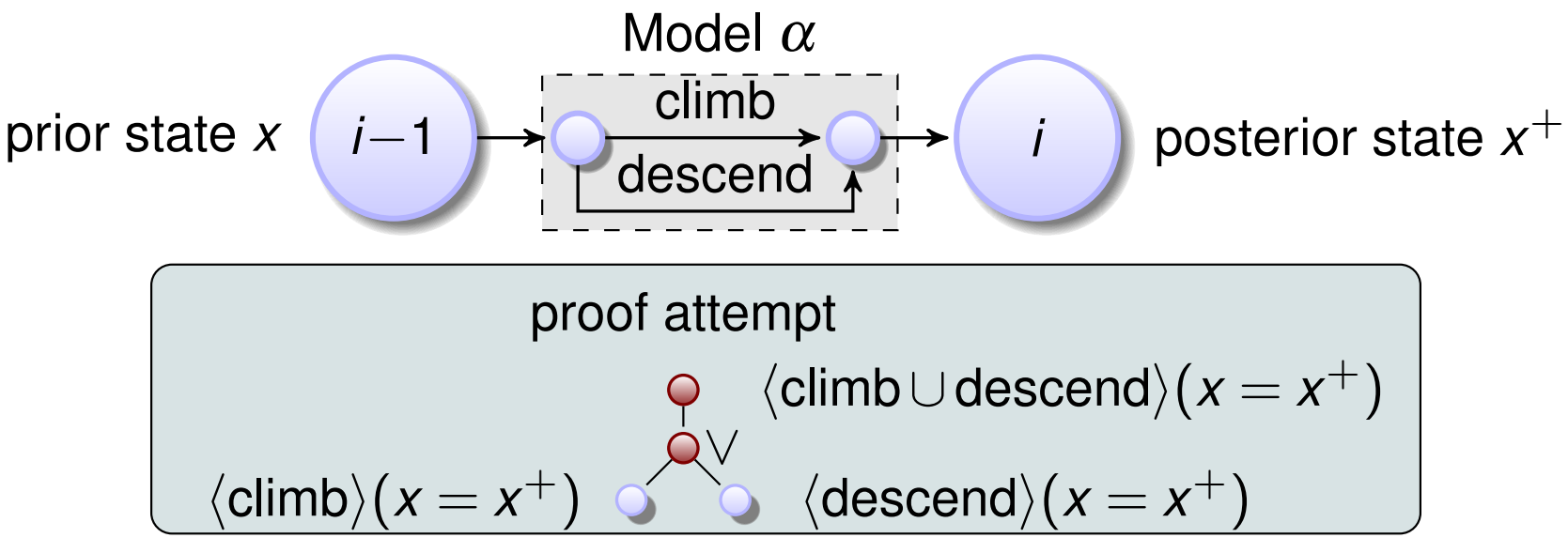

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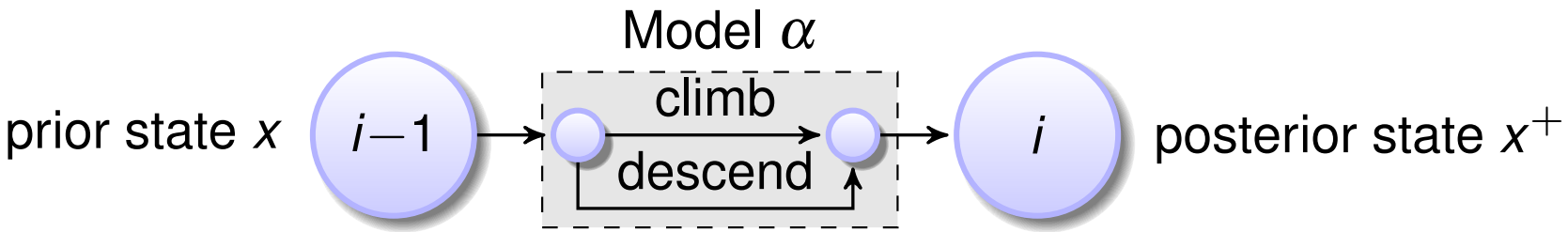

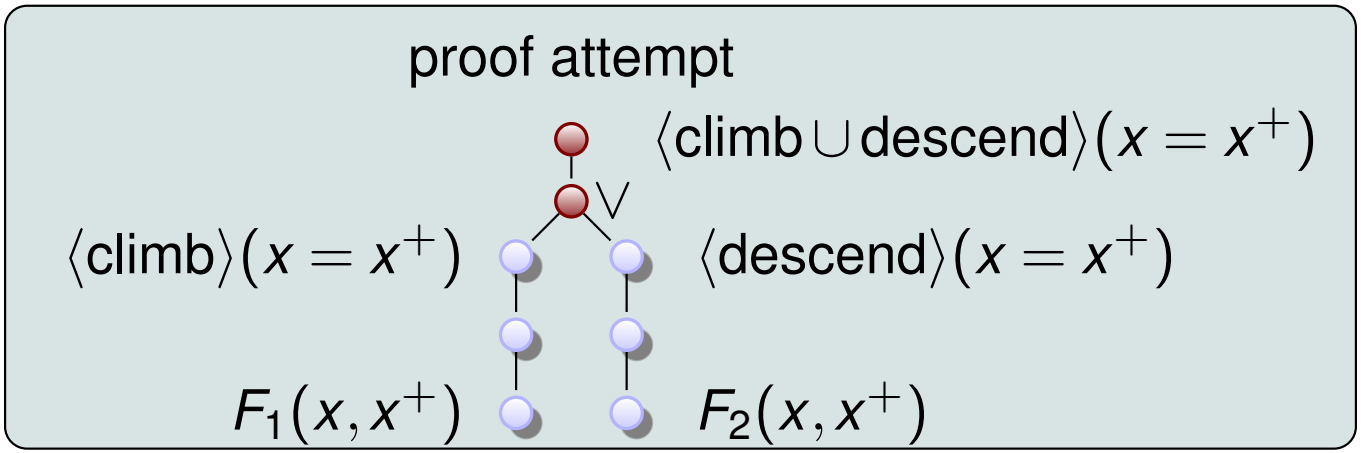

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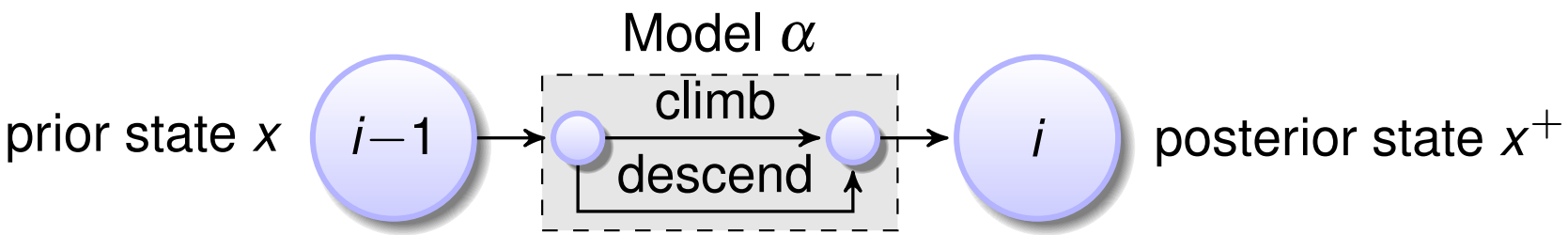

proof attempt

$\langle \text{climb} \cup \text{descend} \rangle (x = x^+)$

$\vee$

$\langle \text{climb} \rangle (x = x^+)$ $\quad$ $\langle \text{descend} \rangle (x = x^+)$

$F_1(x, x^+)$ $\quad$ $F_2(x, x^+)$

Monitor: $\overbrace{F_1(x, x^+) \vee F_2(x, x^+)}$

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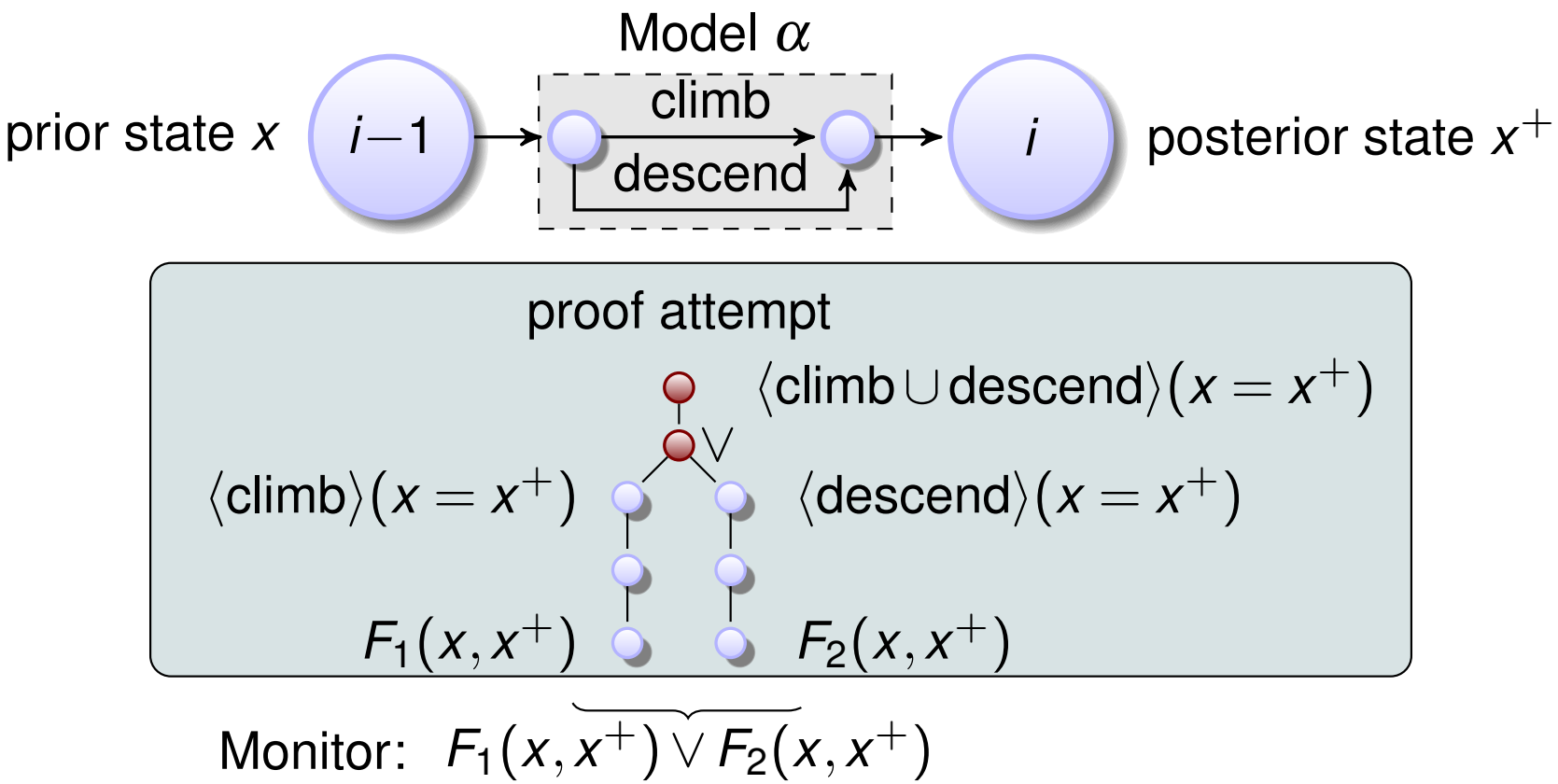

- The subgoals that cannot be proved express all the conditions on the relations of variables imposed by the model $\rightsquigarrow$ prove at runtime

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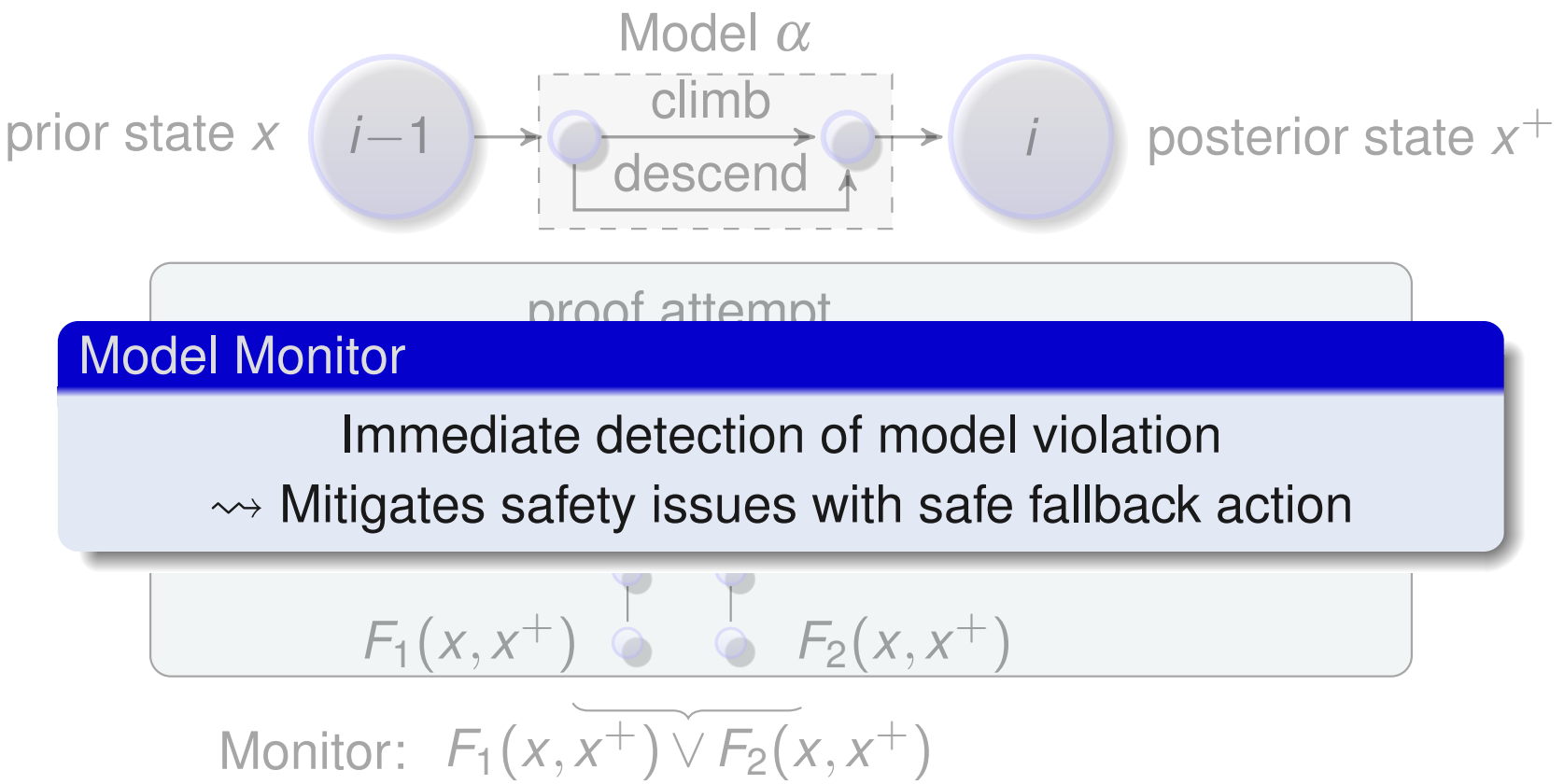

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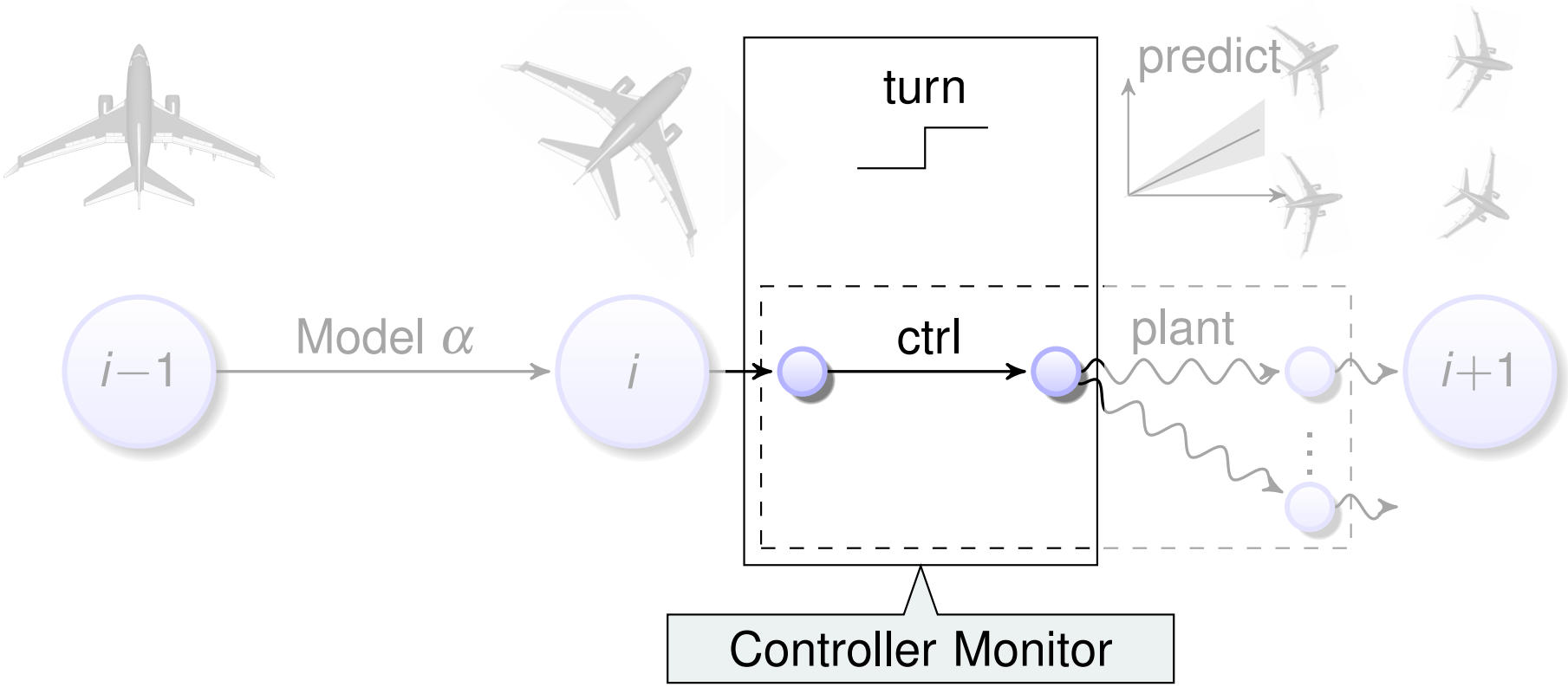
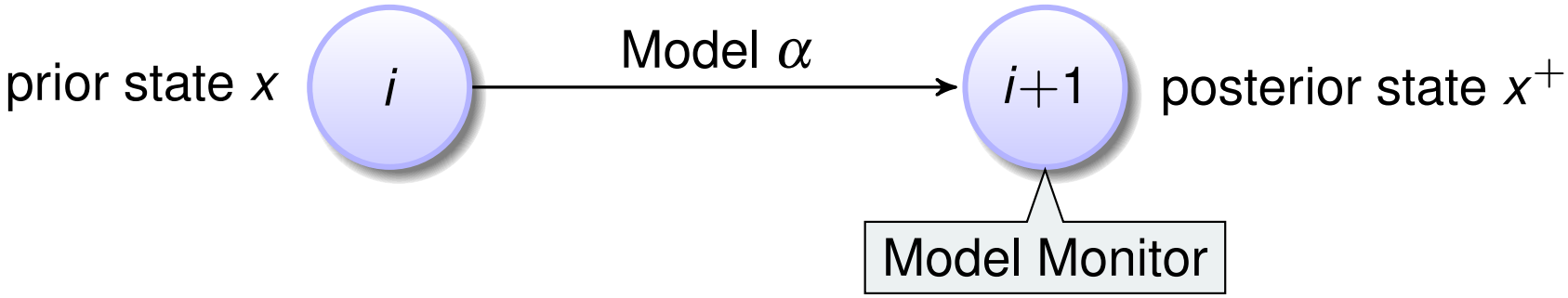
Typical $(\text{ctrl}; \text{plant})^*$ models can check earlier



Controller Monitor: Veto Early If Noncompliant



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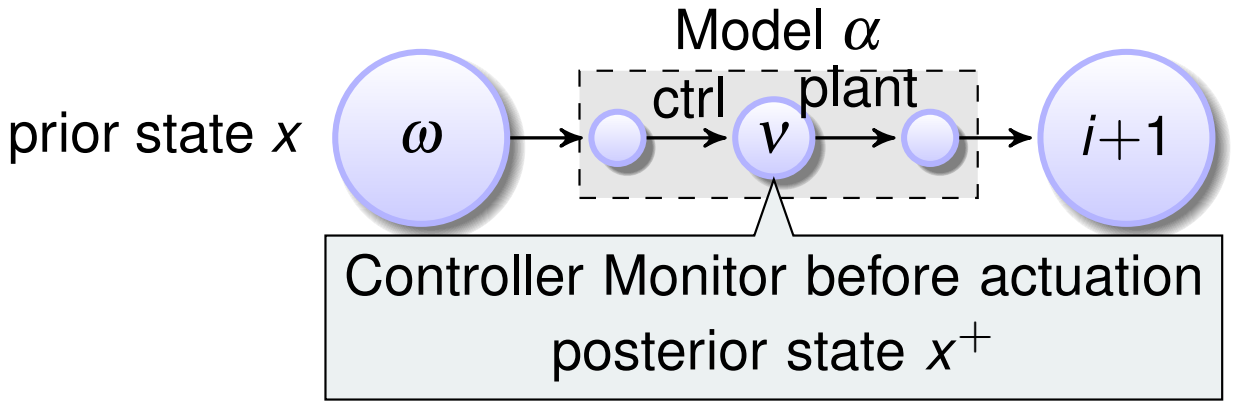
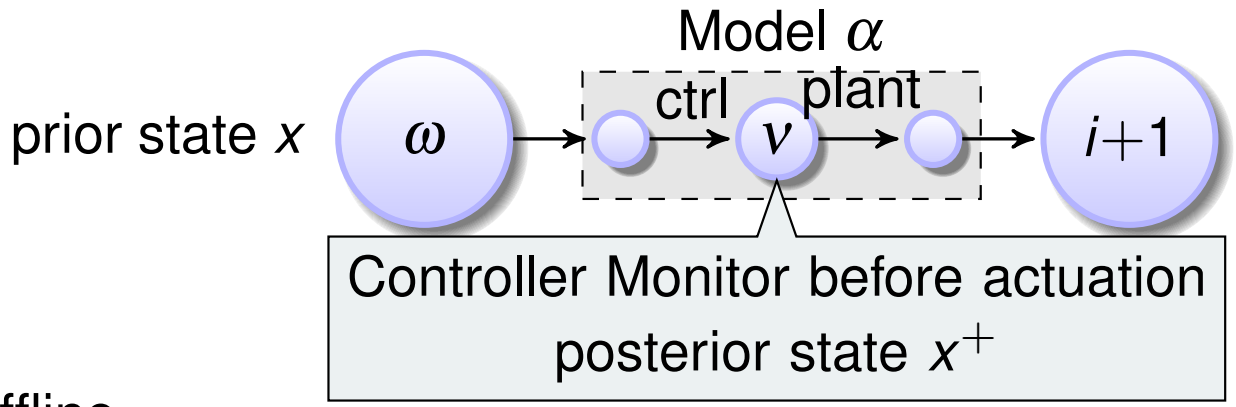


Semantical: $(\omega, \nu) \in [\![\text{ctrl}]\!]$ reachability relation of ctrl

Controller Monitor: Veto Early If Noncompliant



Offline

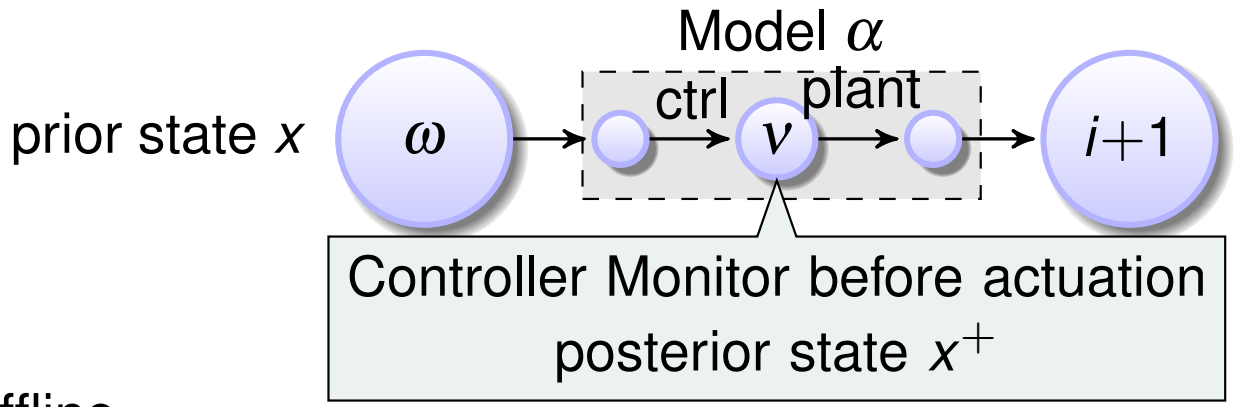
Semantical: $(\omega, \nu) \in [\![\text{ctrl}]\!]$

$\Updownarrow$ Theorem

Logical dL: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+)$

exists a run of ctrl to a state where $x = x^+$

Controller Monitor: Veto Early If Noncompliant



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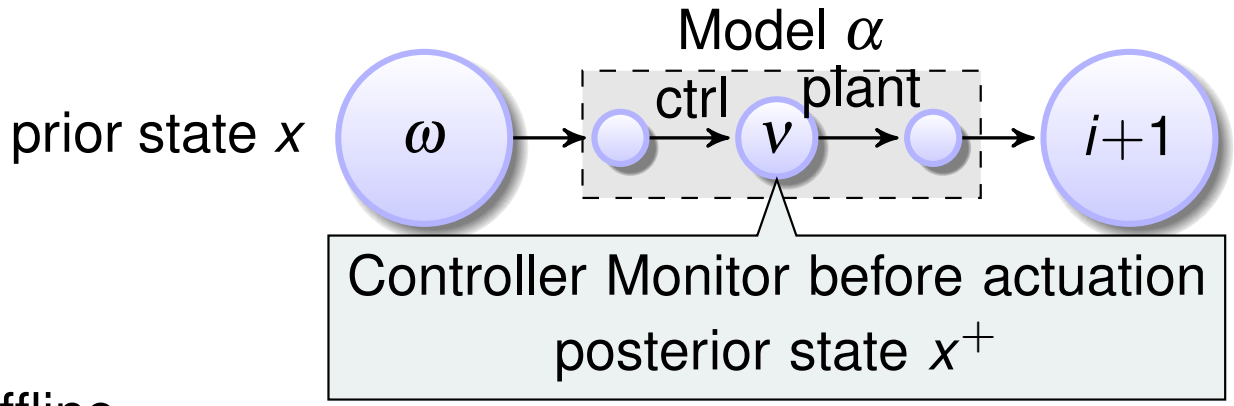
$\Uparrow$ dL proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)

Controller Monitor: Veto Early If Noncompliant



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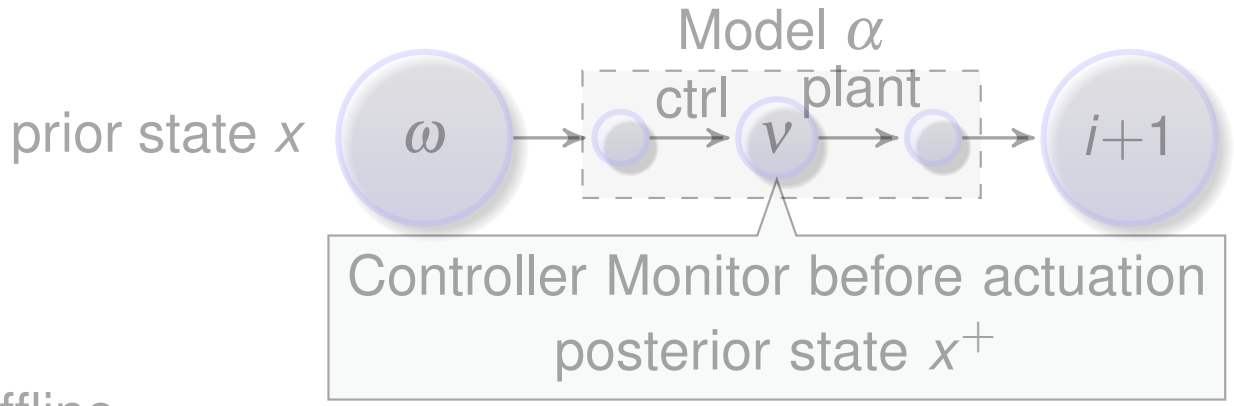
Theorem (Controller Monitor Correctness) (FMSD'16)

Controller safe and in plant bounds as long as monitor satisfied.

Controller Monitor: Veto Early If Noncompliant



Offline

Controller Monitor

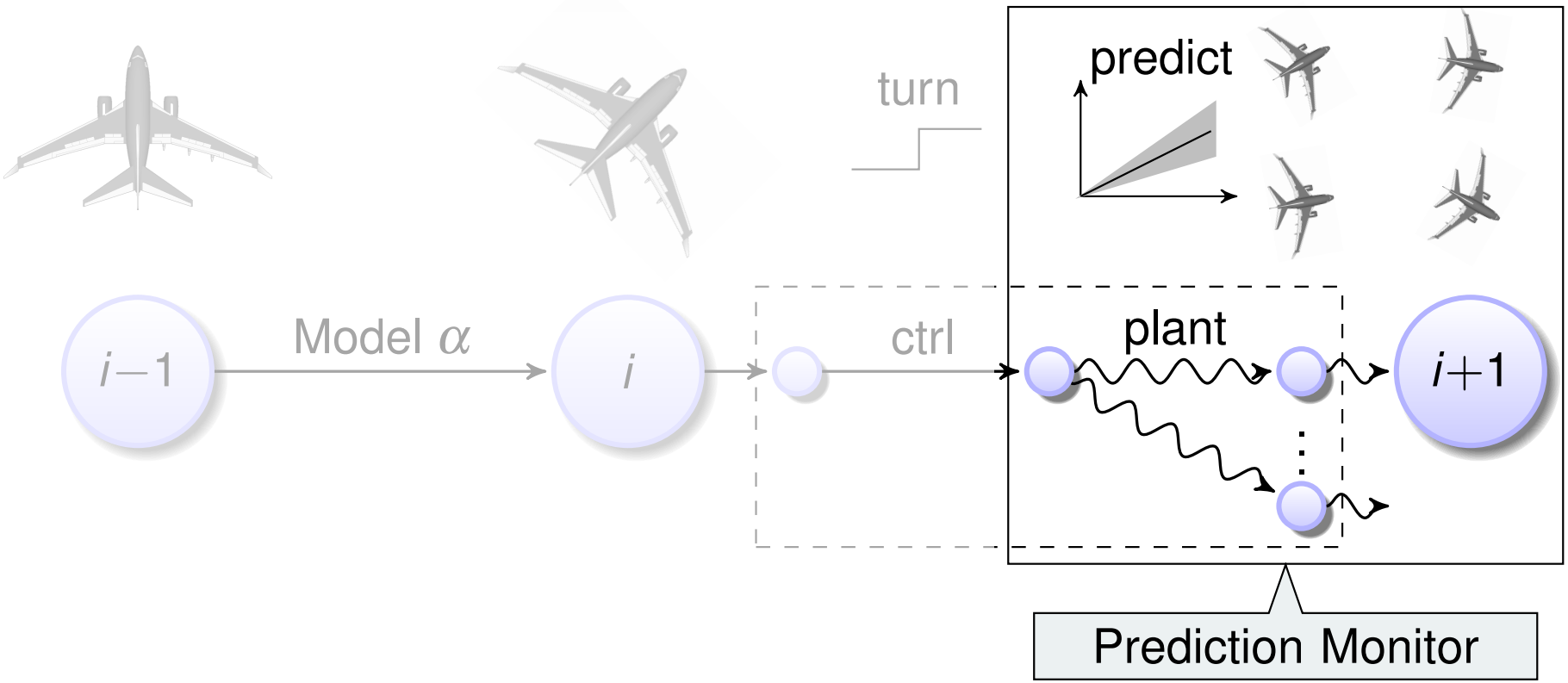
Immediate detection of unsafe control before actuation
$\rightsquigarrow$ Safe execution of unverified implementations
in perfect environments

Arithmetical: $(\omega, \nu) \models F(x, x^+)$ — check at runtime (efficient)

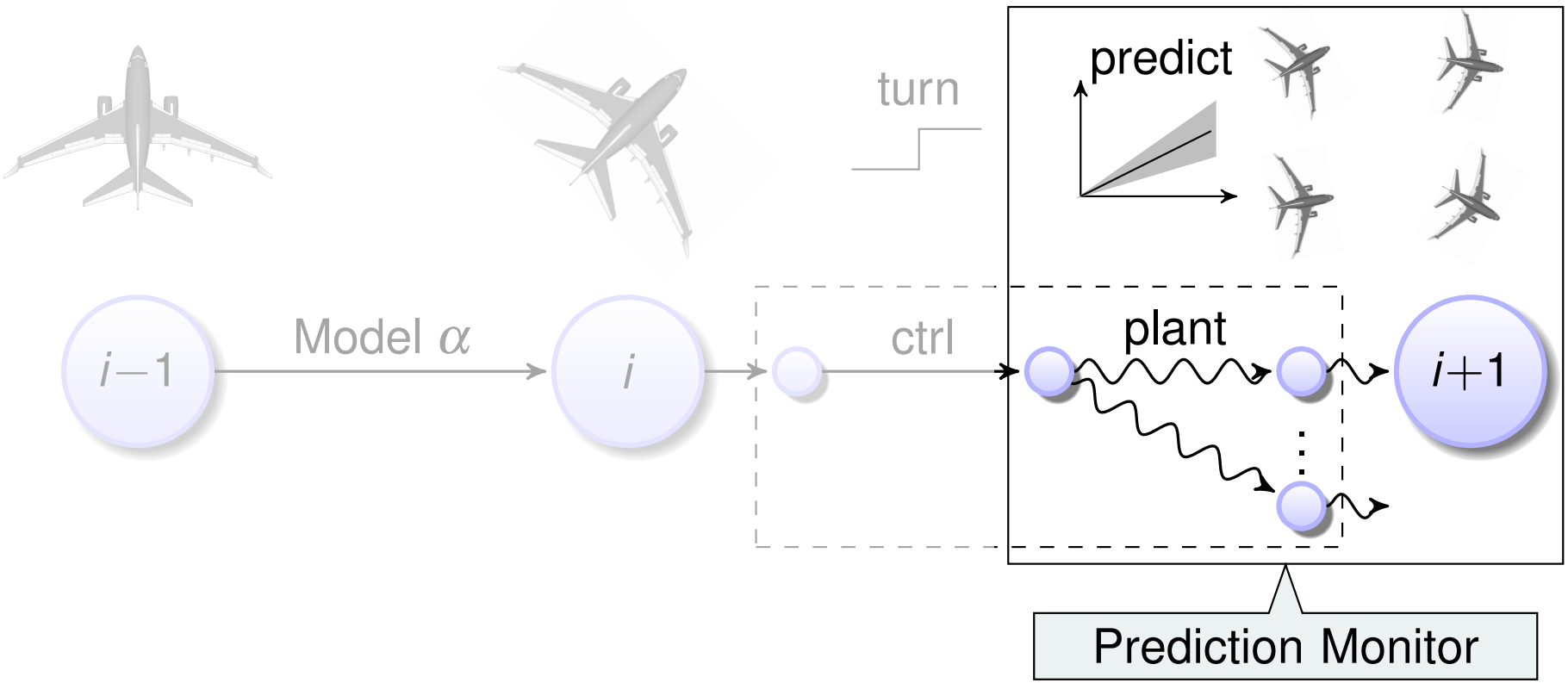
Theorem (Controller Monitor Correctness) (FMSD'16)

Controller safe and in plant bounds as long as monitor satisfied.

Safe despite evolution with disturbance?

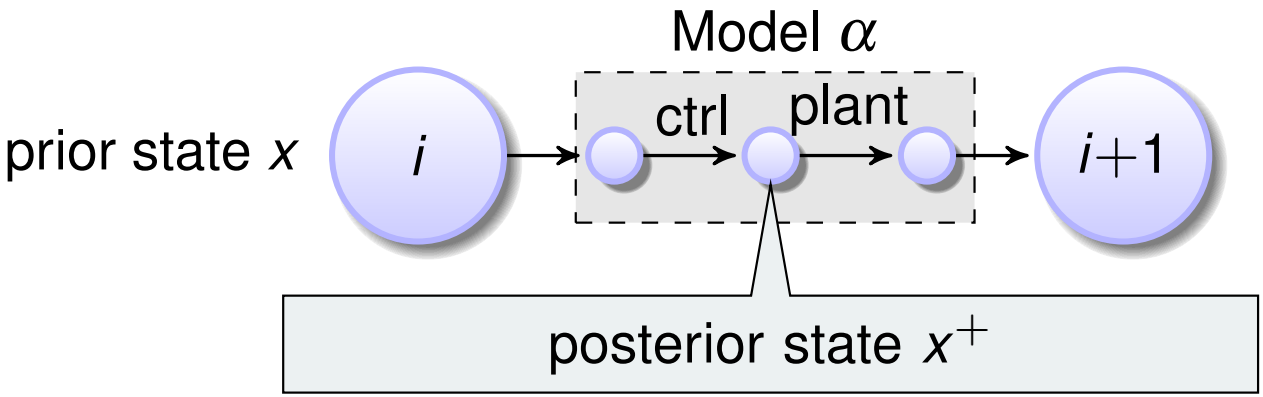


Safe despite evolution with disturbance?

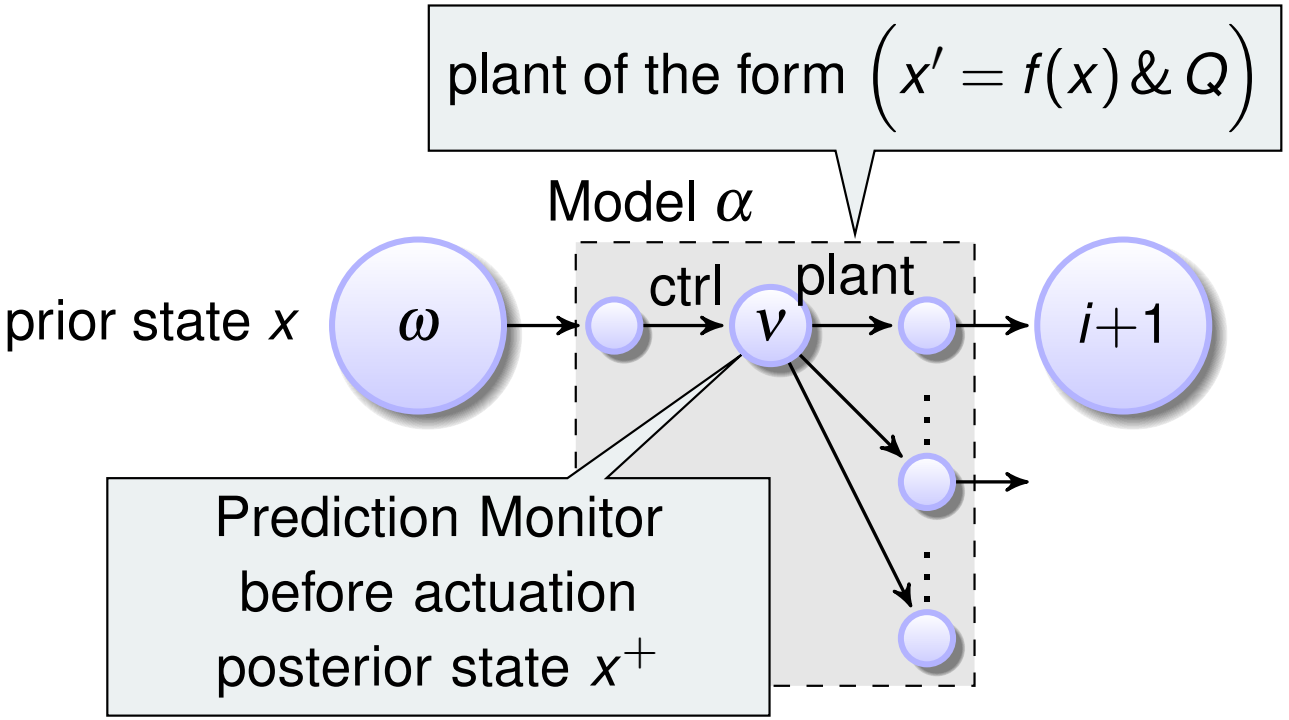


"Prediction is very difficult, especially if it's about the future." [Nils Bohr]

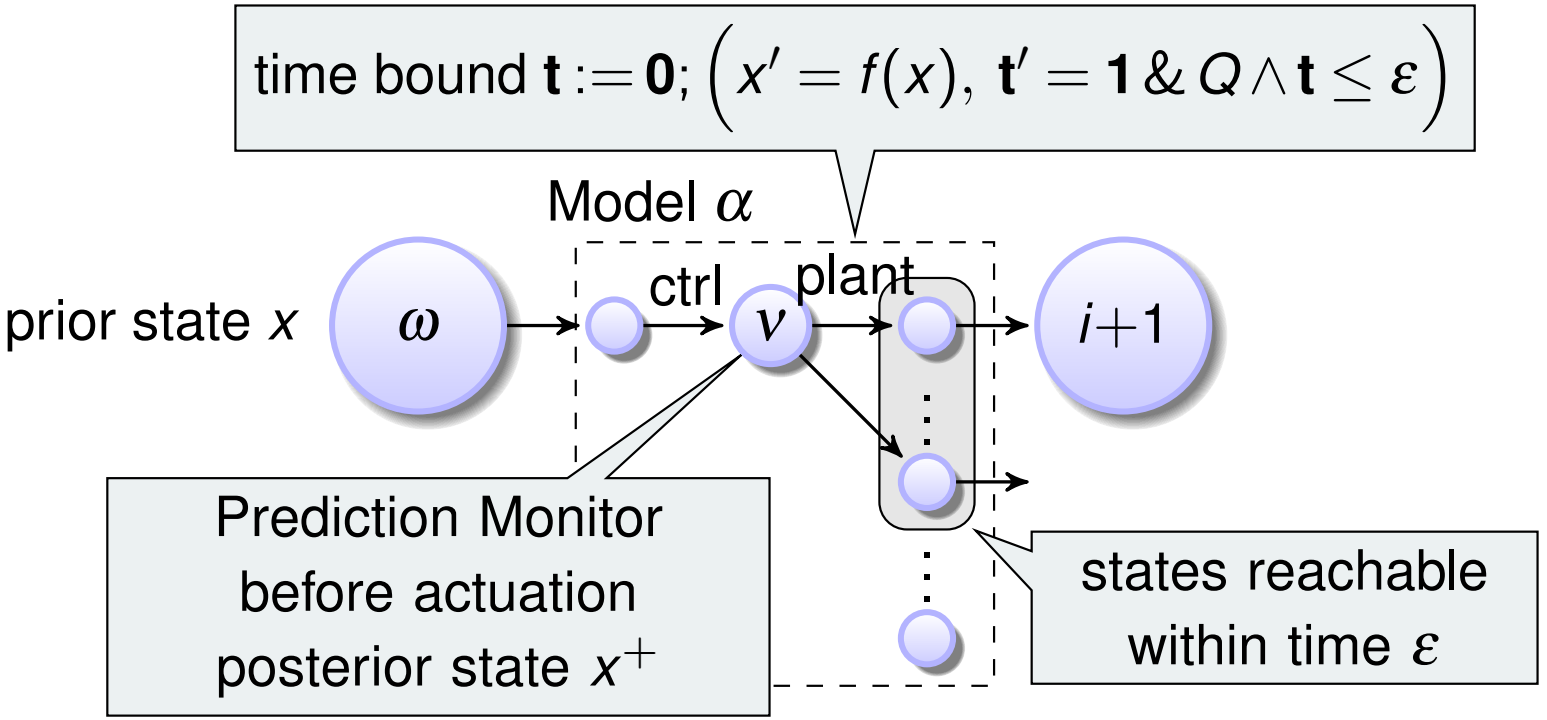
Prediction Monitor: Compliance with Disturbance



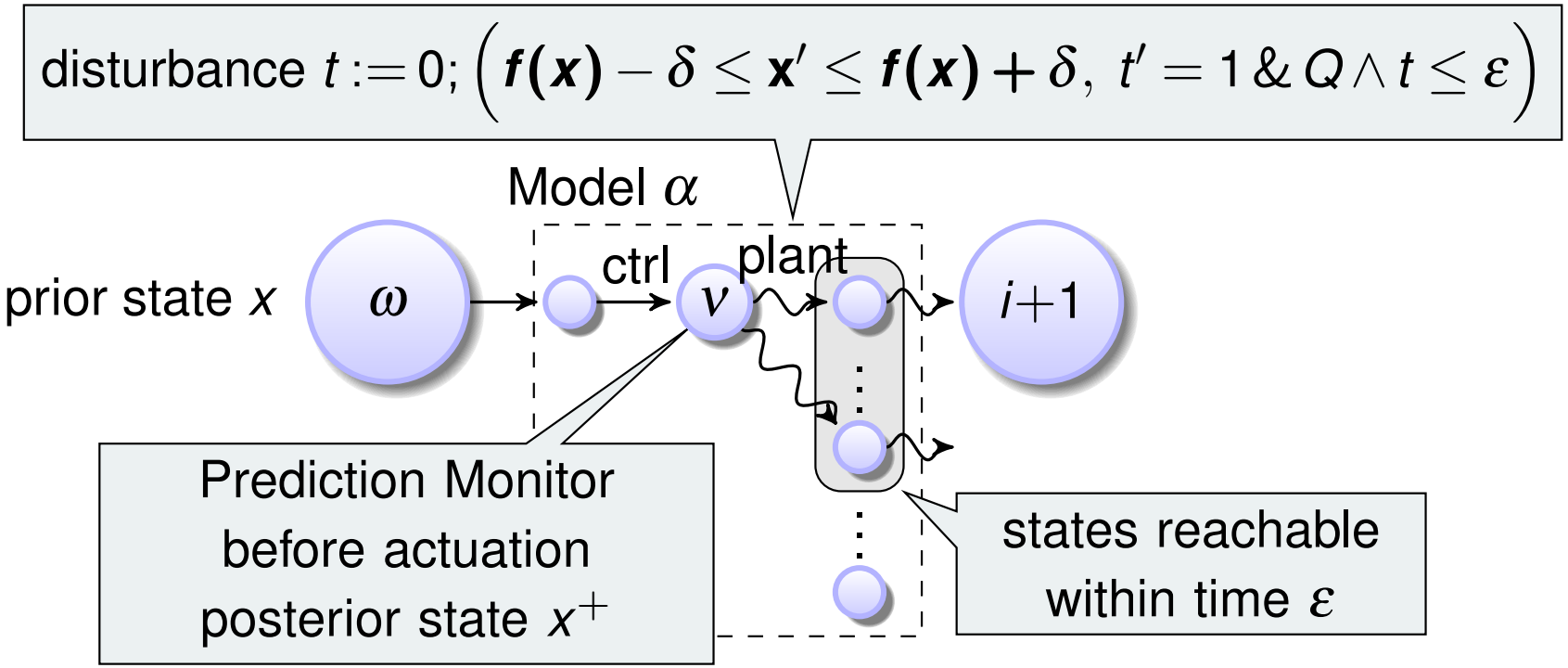
Prediction Monitor: Compliance with Disturbance



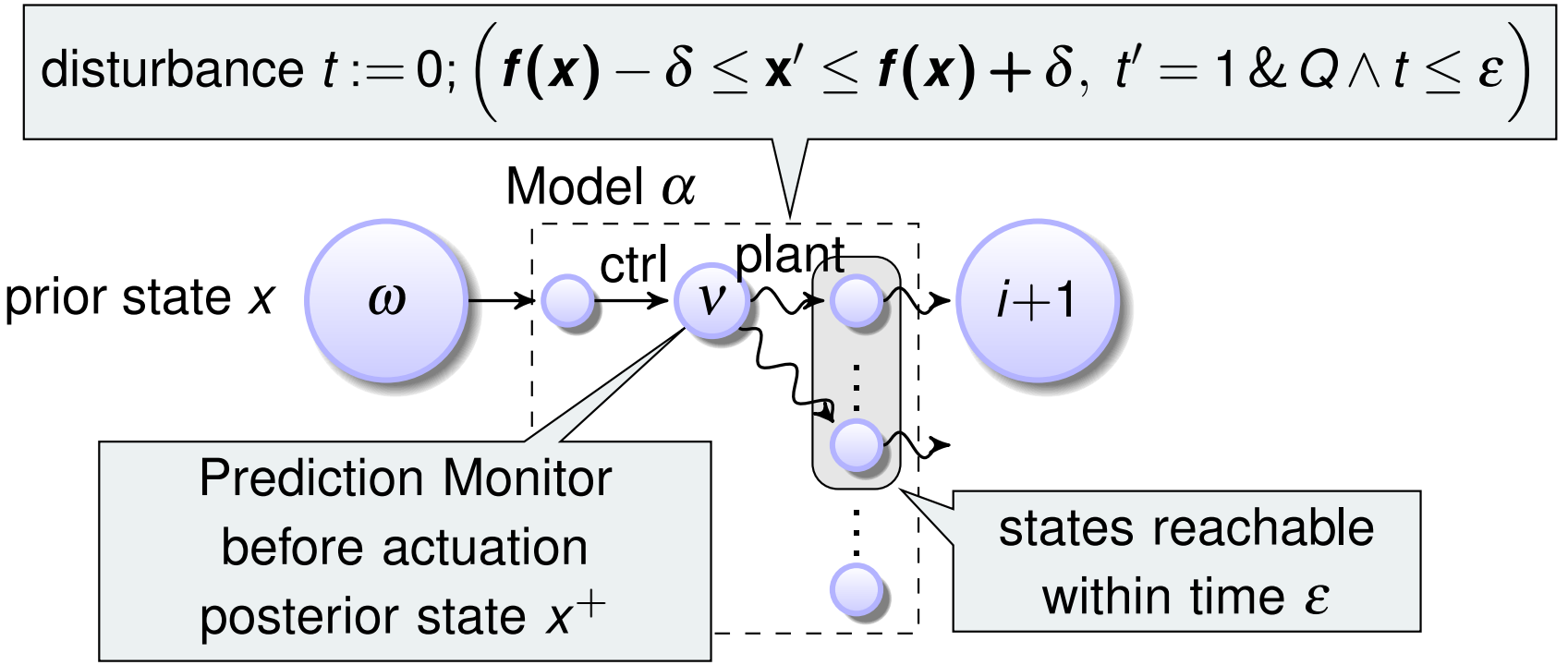
Prediction Monitor: Compliance with Disturbance



Prediction Monitor: Compliance with Disturbance



Prediction Monitor: Compliance with Disturbance



Offline

Logical dL: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+ \wedge [\text{plant}] J)$

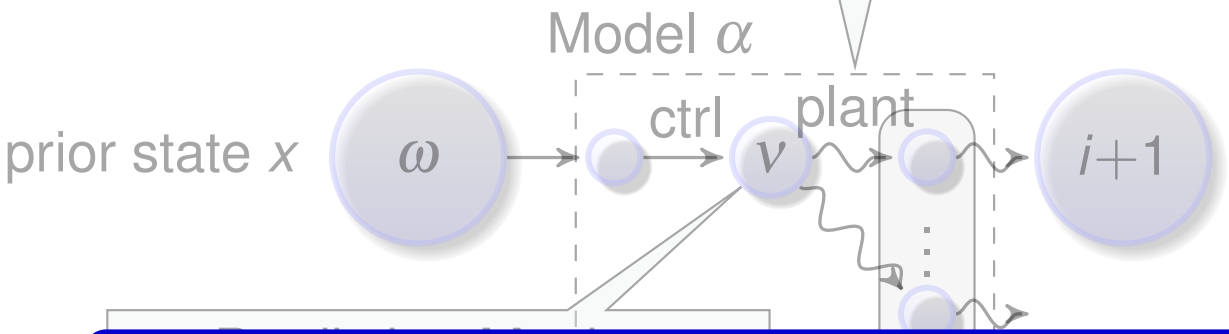
$\Uparrow$ dL proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

Invariant J implies safety S
(known from safety proof)

Prediction Monitor: Compliance with Disturbance

$$\text{disturbance } t := 0; \left(\boldsymbol{f(x)} - \delta \leq \boldsymbol{x'} \leq \boldsymbol{f(x)} + \delta,\ t' = 1 \,\&\, Q \wedge t \leq \varepsilon\right)$$



Prediction Monitor with Disturbance

Detect unsafe control before actuation despite disturbance

$\rightsquigarrow$ **Safety in realistic environments**

Offline

Logical dL: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+ \wedge [\text{plant}] J)$

$\Uparrow$ dL proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

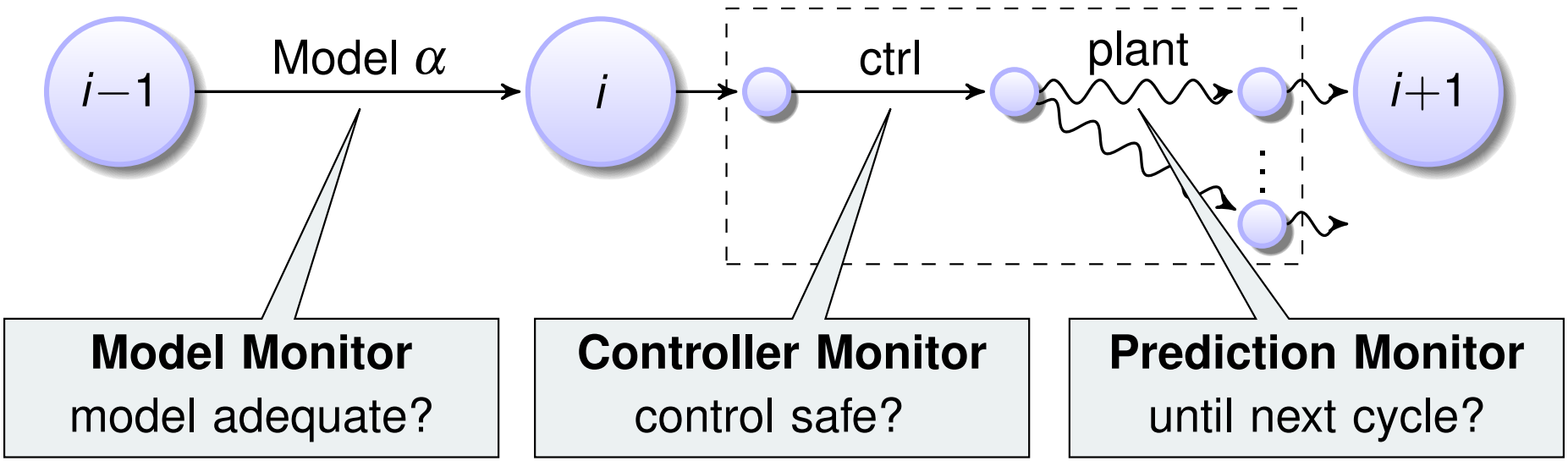
Invariant J implies safety S
(known from safety proof)

Outline

Summary

ModelPlex ensures that proofs transfer to real CPS

- Validate model compliance
- Characterize compliance with model in logic
- Prover transforms compliance formula to executable monitor
- Provably correct runtime model validation by offline + online proof



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