AI for verification and verification for AI

Overview and discussion

KIT, February 16th 2026

Lecture Overview

- **Verification for AI**
  - Shielding
  - Neural-network verification
- **AI for Verification**
  - Imitation learning of theorem proving
  - Reinforcement learning of theorem proving
  - Formal Methods in the Era of Large Language Models

Machine-Learning Primer

Supervised Learning as an Optimization Problem

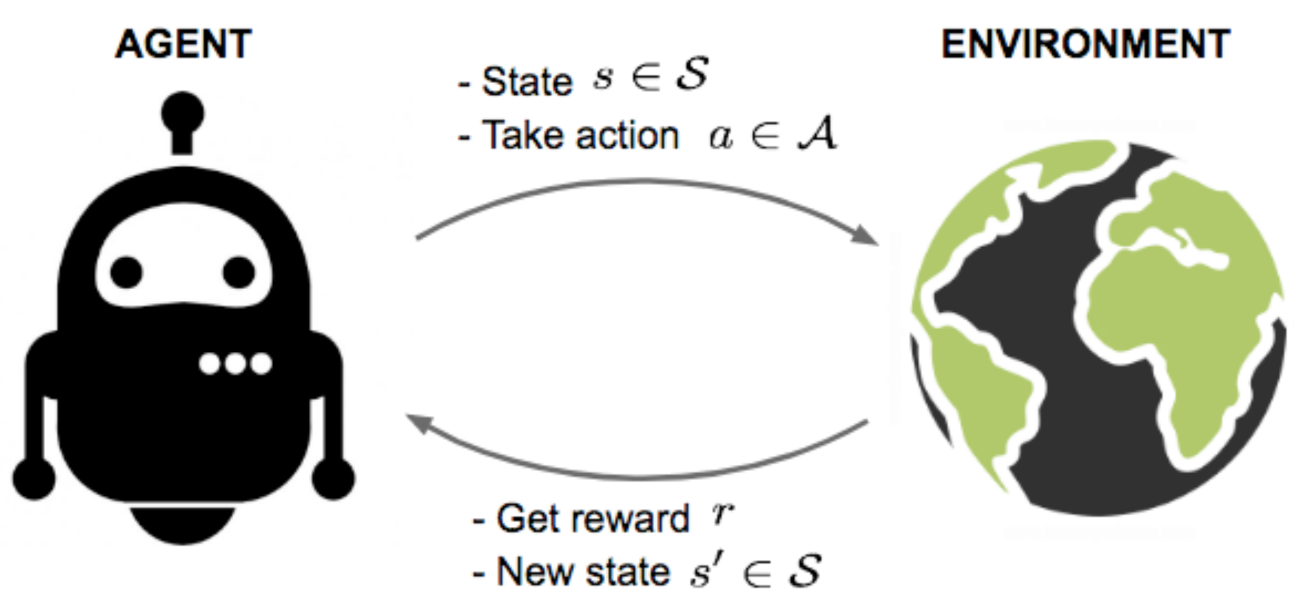
Problem: learning an unknown function from many input/output examples.

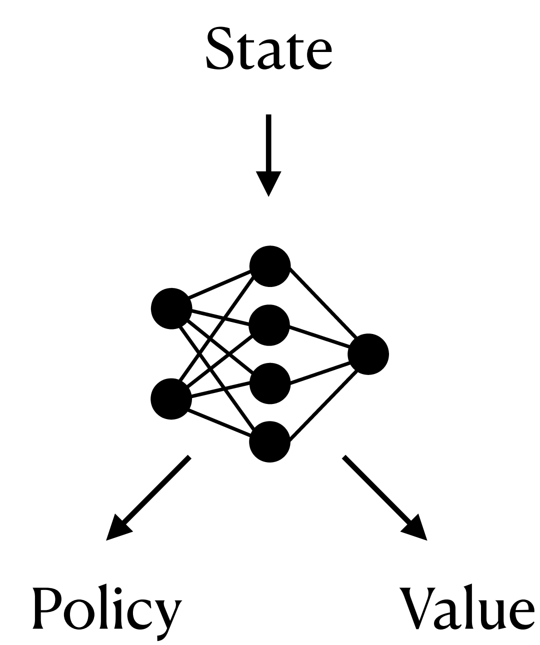
$$L(W) = \frac{1}{|D|} \sum_{x,y \in D} (y - f_W(x))^2$$

$$f_W(x) = W_3 \cdot \sigma(W_2 \cdot \sigma(W_1 \cdot x))$$

$$W_{t+1} \leftarrow W_t + \lambda \nabla_{W_t} L$$

Reinforcement Learning





AI for Verification

```
init → [(ctrl; plant)*] safe
```

Shielding

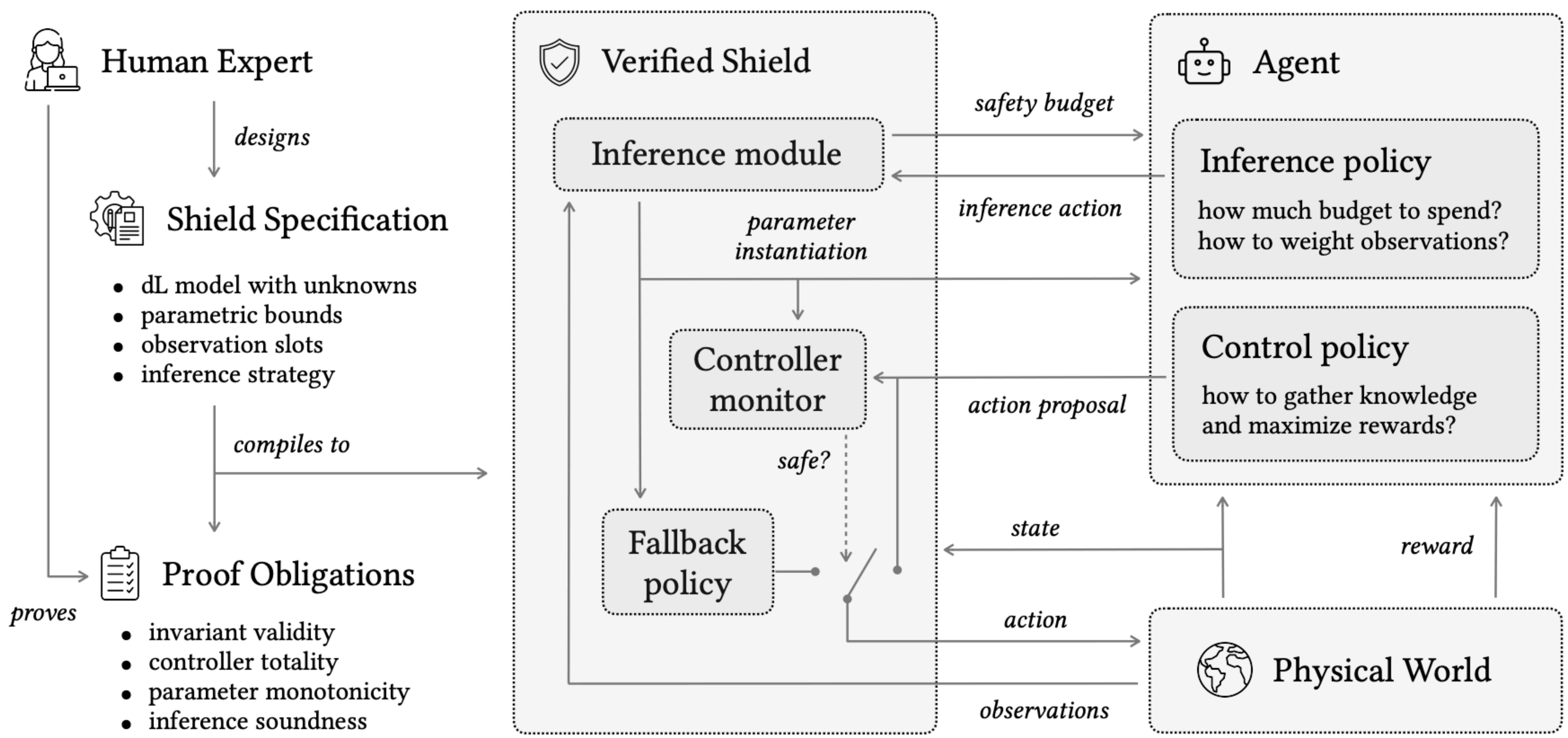
Approach: do not trust the neural-network, sandbox it!

```
init → [{
   {?safeAccel; accel
    ∪ brake};
   t:=0; {pos’=vel,vel’=acc}
}*](pos < stopSign)
```

Safe Reinforcement Learning via Formal Methods, Fulton and Platzer, AAAI 2018

Adaptive Shielding

- **Problem:** what if the environment is not fully known offline?
- **Solution:** write and verify **parametric safety proofs** offline, and estimate parameters online, allowing increasingly permissive shielding.



Adaptive Shielding via Parametric Safety Proofs, Feng et al., OOPSLA 2025

Example 1: estimating scalar parameters

CONSTANT A, B, T, σ

UNKNOWN θ, φ

ASSUME $A > 0,\ B > 0,\ T > 0,\ \sigma > 0,\ \theta > 0$

BOUND $\underline{\theta} : \underline{\theta} \leq \theta,\ \bar{\theta} : \bar{\theta} \geq \theta,\ \bar{\varphi} : \bar{\varphi} \geq \varphi$

CONTROLLER

$$u := * \ ; \ ?(-B \leq u \leq A) \ ; \ ?(x + vT + (\bar{\theta}u + \bar{\varphi})T^2/2 + (v + (\bar{\theta}u + \bar{\varphi})T)^2/2(\underline{\theta}B - \bar{\varphi}) \leq e)$$

PLANT $t := 0 \ ; \{x' = v, v' = \theta u + \varphi, t' = 1 \ \& \ t \leq T \land v \geq 0\}$

SAFE $x \leq e$

INVARIANT $(\underline{\theta}B - \bar{\varphi} > 0) \land (x + v^2/2(\underline{\theta}B - \bar{\varphi}) \leq e)$

NOISE $\eta \sim \mathcal{N}(0, \sigma^2)$

OBSERVE $\omega = \theta u + \varphi - \eta$

INFER

$$\underline{\theta}, \bar{\theta} := \text{AGGREGATE } i, j : (\omega_j - \omega_i)/(u_j - u_i) \text{ AND } (\eta_j - \eta_i)/(u_j - u_i) \text{ WHEN } u_j > u_i \ ;$$

$$\bar{\varphi} := \text{AGGREGATE } i : \omega_i - \bar{\theta}u_i \text{ AND } \eta_i \text{ WHEN } u_i \leq 0$$

Example 2: handling functional unknowns with local bounds

CONSTANT A, B, F, k, σ

UNKNOWN $f(*)$

ASSUME

$$A > 0,\ B > 0,\ T > 0,\ k > 0,\ \sigma > 0,\ F < B,\ A + F > 0,$$
$$(\forall x\ -A \le f(x) \le F),\ (\forall x\, \forall y\, |f(x) - f(y)| \le k|x - y|)$$

BOUND $\bar{f} : f(x) \le \bar{f}$

CONTROLLER

$$y := \min(y, \bar{f})\ ;$$
$$((a := -B) \cup$$
$$(?(x + vT + \tfrac{1}{2}(A+F)T^2 + \mathsf{Bdist}_{v+(A+F)T}(B - \min(F,\ y + k(vT + \tfrac{1}{2}(A+F)T^2) +$$
$$k \cdot \mathsf{Bdist}_{v+(A+F)T}(B - F))) \le e)\ ;\ a := A)$$

PLANT $t := 0\ ; \{x' = v, v' = a + f(x), y' = kv, t' = 1\ \&\ t \le T \wedge v \ge 0\}$

SAFE $x \le e$

INVARIANT $(v \ge 0) \wedge (y \ge f(x)) \wedge (x + \mathsf{Bdist}_v(B - \min(F,\ y + k \cdot \mathsf{Bdist}_v(B - F))) \le e)$

NOISE $\eta \sim \mathcal{N}(0, \sigma^2)$

OBSERVE $\omega = f(x) - \eta$

INFER $\bar{f} := F\ ;\ \bar{f} :=$ BEST $i : \bar{f}_i + k|x - x_i|\ ;\ \bar{f} :=$ AGGREGATE $i : \omega_i + k|x - x_i|$ AND η_i

Example 2: handling functional unknowns with local bounds

NOISE $\eta \sim \mathcal{N}(0, \sigma^2)$

OBSERVE $\omega = f(x) - \eta$

INFER $\bar{f} := F$; $\bar{f} :=$ BEST $i : \bar{f}_i + k|x - x_i|$; $\bar{f} :=$ AGGREGATE $i : \omega_i + k|x - x_i|$ AND η_i

Proof obligations:

$$\mathsf{Init} \wedge \mathsf{Inv} \wedge \bar{f} = F \rightarrow f(x) \leq \bar{f}$$

$$\mathsf{Init} \wedge \mathsf{Inv} \wedge \bar{f} = \bar{f}_i + k|x - x_i| \wedge f(x_i) \leq \bar{f}_i \rightarrow f(x) \leq \bar{f}$$

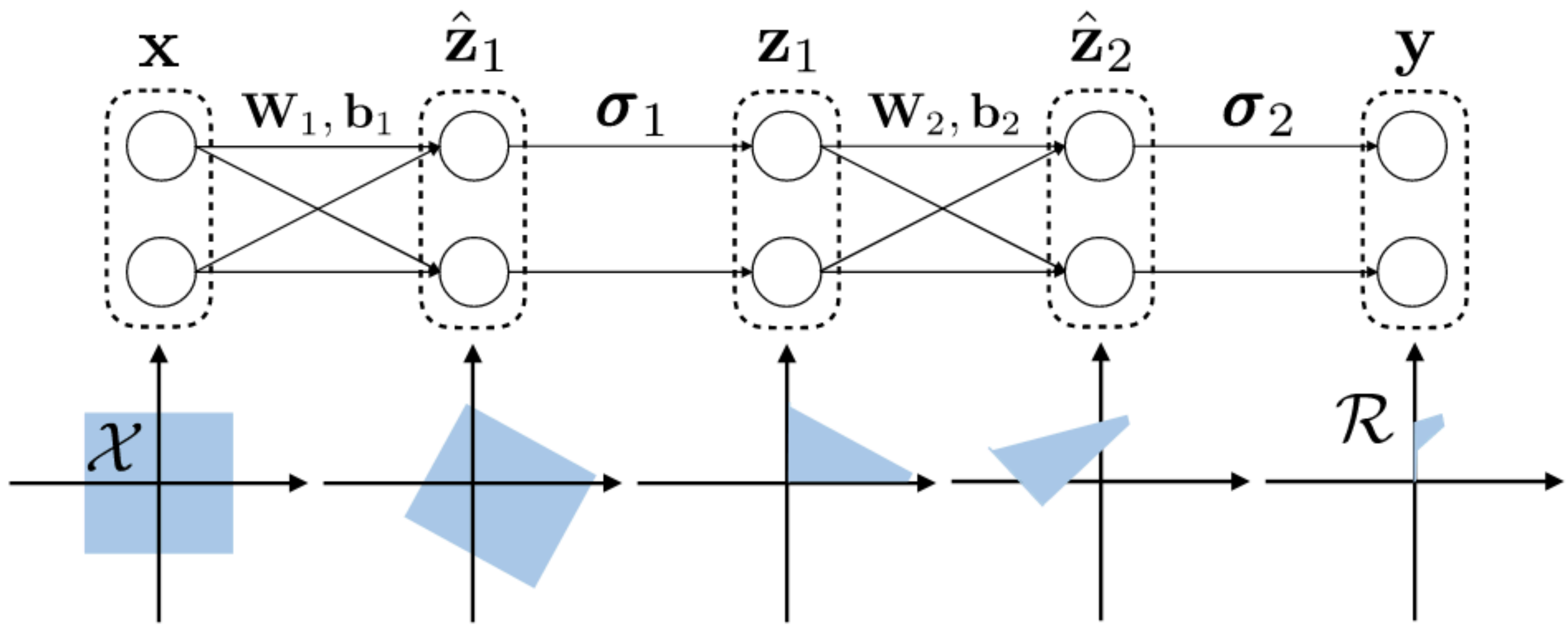
$$\mathsf{Init} \wedge \mathsf{Inv} \wedge \bar{f} = \omega_i + k|x - x_i| + \eta_i \wedge \omega_i = f(x_i) - \eta_i \rightarrow f(x) \leq \bar{f}.$$

Closed-Loop Network Verification

```
init → [(
    u := NN(x);
    x := plant(x, u);
)*] safe
```

Reachability analysis of neural networks

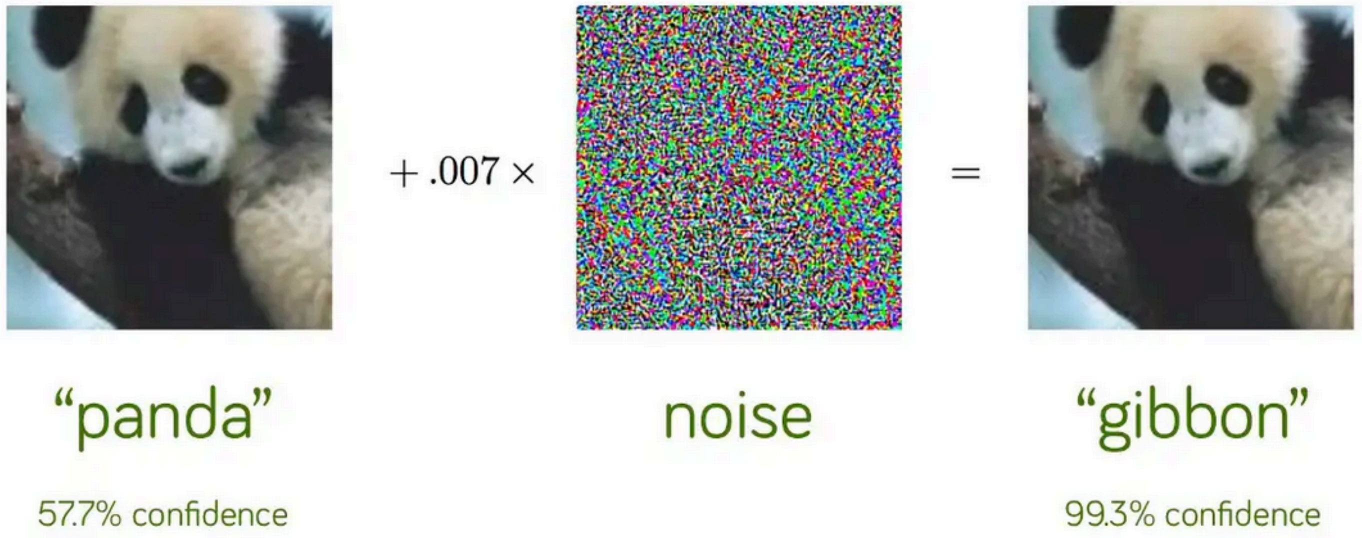
$\mathrm{NN}(x) = W_3 \cdot \sigma(W_2 \cdot \sigma(W_1 \cdot x))$



Survey: *Algorithms for Verifying Deep Neural Networks*, Liu et al.

Open-Loop Neural Network Verification

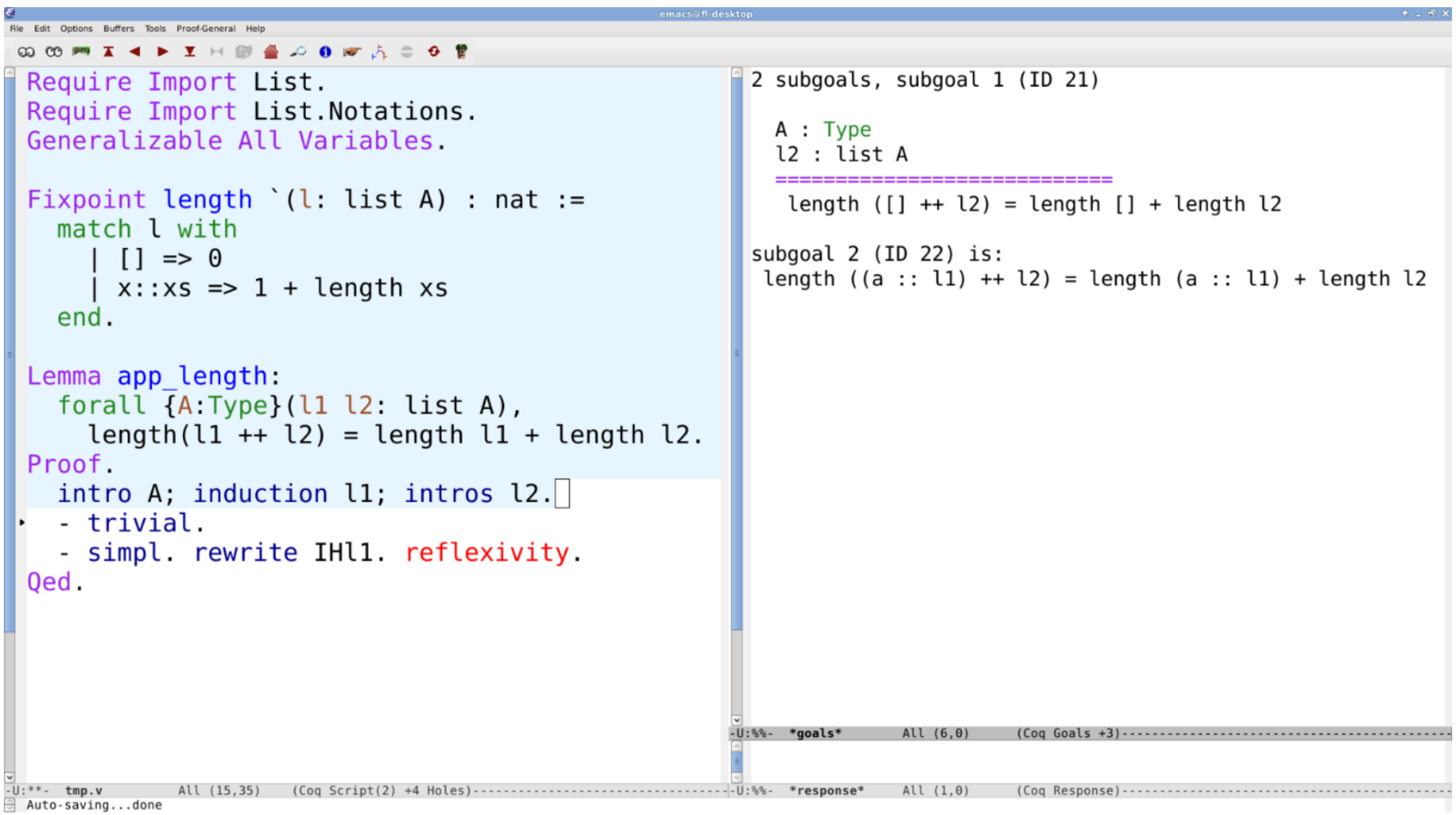
Example: robustness verification



$$\forall (x, y) \in D,\ \forall x',\ \|x - x'\|_\infty < \epsilon \ \longrightarrow F(x') = y$$

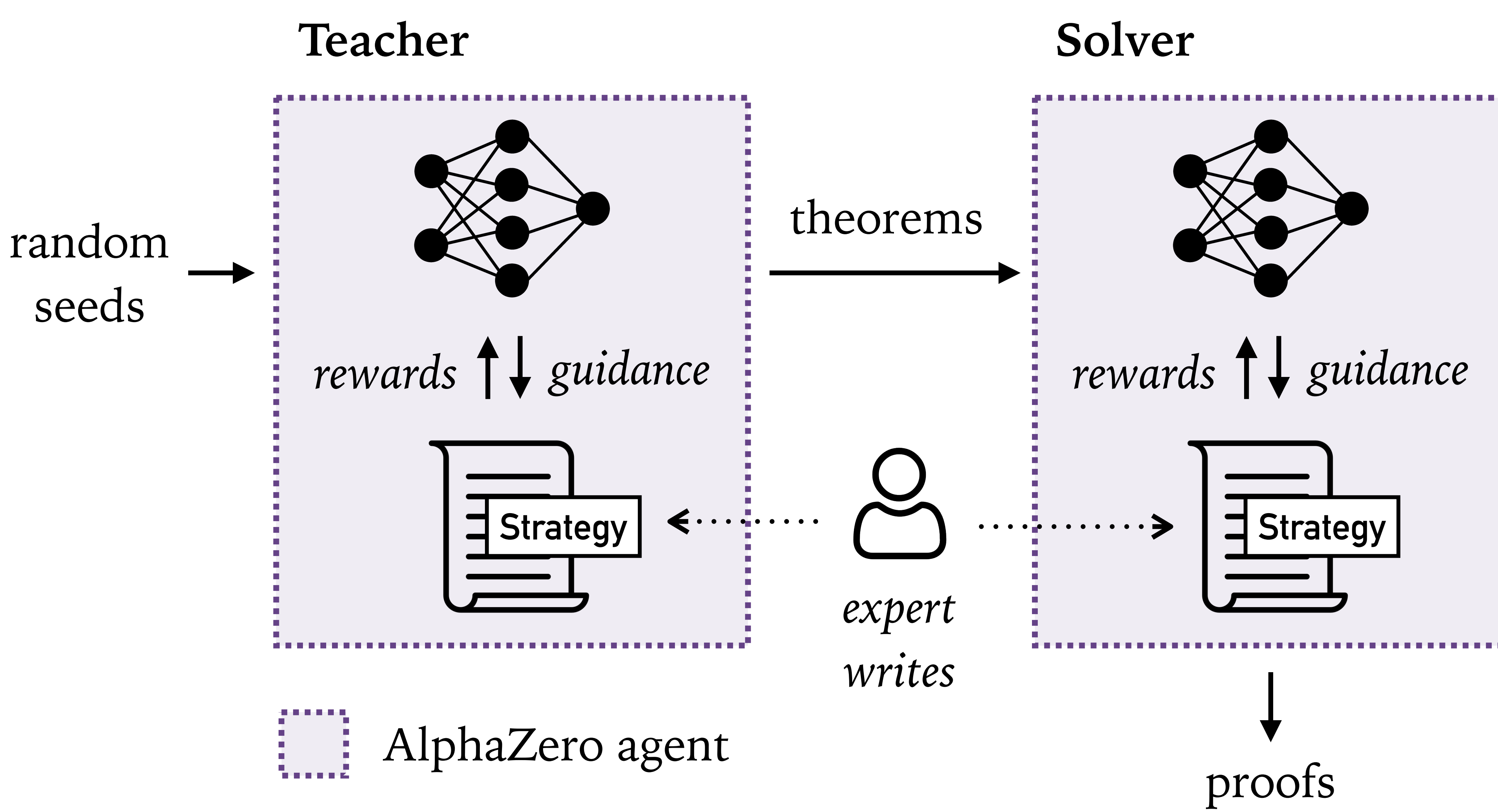
Verification for AI

Can we train an AI to interact with a theorem prover?



LEARNING TO FIND PROOFS AND THEOREMS BY LEARNING TO REFINE SEARCH STRATEGIES

Jonathan Laurent, André Platzer (NeurIPS 2022)



LOOP INVARIANT SYNTHESIS

- Training data unavailable and hard to generate!
- No pre-existing deep-learning agent capable of generalizing across instances.

```
assume x ⩾ 1
y = 0
while y < 1000 {
    x = x + y
    y = y + 1
}
assert x ⩾ y
```

To prove the final assertion, one must find a loop invariant that:

1. is true before the loop
2. is preserved by the loop body (when the loop guard holds)
3. implies the final assertion (when the loop guard does not hold)

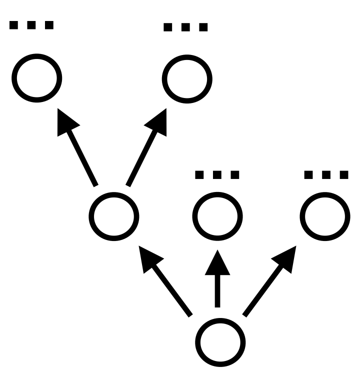
Invariant: $x \geq y \land x \geq 1 \land y \geq 0$

A LANGUAGE FOR EXPRESSING STRATEGIES

We define a strategy language based on **choose** and **event** operators.

Expert strategy

compiled into



MDP amenable to RL and neural-guided search

```
def solver(
    init: Formula, guard: Formula,
    body: Program, post: Formula) -> Formula:
    def prove_inv(inv: Formula) -> List[Formula]:
        assert valid(Implies(init, inv))
        ind = Implies(And(guard, inv), wlp(body, inv))
        event(PROVE_INV_EVENT)
        match abduct(ind):
        case Valid:
            return [inv]
        case [*suggestions]:
            aux = choose(suggestions)
            return [inv] + prove_inv(aux)
inv_cand = choose(abduct(Implies(Not(guard), post)))
inv_conjuncts = prove_inv(inv_cand)
return And(*inv_conjuncts)
```

▲ A **solver strategy** for invariant synthesis

GENERATING TRAINING PROBLEMS

- Generating interesting theorems is harder than proving those!
- **Our approach:** refining **conditional generative strategies** using RL.

```
def teacher(rng: RandGen) → Prog:
    cs = sample_constrs(rng)
    p = generate_prog(cs)
    p = transform(p, rng)
    p = hide_invariants(p)
    return p

def generate_prog(cs: Constrs):
    p = Prog("
        assume init;
        while (guard) {
          invariant inv_lin;
          invariant inv_aux;
          invariant inv_main;
          body; }
        assert post;")
    p = refine_guard(p, cs)
    p = refine_inv(p, cs)
    p = refine_body(p, cs)
    assert valid(inv_preserved(p))
    p = refine_post(p, cs)
    assert valid(inv_post(p))
    p = refine_init(p, cs)
    assert valid(inv_init(p))
    penalize_violations(p, cs)
    return p

def transform(p: Prog, rng: RandGen):
    p = shuffle_formulas(p, rng)
    p = add_useless_init(p, rng)
    ...
    return p
```

▲ Outline of a **teacher strategy** for invariant synthesis

Formal Methods in the Era of

Large Language Models

Language Models are Few-Shot Learners

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