

LFCPS Exercise Class 7

Comprehensive CPS Correctness

ModelPlex

Exercise 1. Derive a runtime monitor for the hybrid program:

$$\{u := *; ?(0 < u \leq 1); \{x' = v, v' = u, t' = 1\}\}.$$

How can you formally prove the correctness of this monitor?

Exercise 2. Consider the following model of a water tank system:

$$f := *; ?(-1 \leq f \leq (m - x)/\epsilon); t := 0; \{x' = f, t' = 1 \ \& \ x \geq 0 \wedge t \leq \epsilon\},$$

where:

- x represents the water level,
- m is the maximum water level, and
- ϵ is the time trigger constant.

Assuming $\epsilon \neq 0$:

1. Prove that the water level never exceeds the maximum m .
2. Derive a runtime monitor for this model.

Virtual substitution

Exercise 3. Normalize the following formulas

1. $\exists x x^2 = a \vee b > 0$
2. $\exists x x^2 = a \vee x^2 > b$
3. $\exists x ((\exists y x^5 > y^4) \wedge \forall z (z + a \leq b \rightarrow z^5 \geq a))$

Exercise 4. Substitute the square root expression ε into the formula F , i.e. compute F_x^ε . Then find an equivalent formula in the language of real arithmetic.

1. $F \equiv 3x^2 > 2x \wedge x \neq 3$ and $\varepsilon \equiv \sqrt{2}/2$
2. $F \equiv 4x^3 < a \wedge ax \neq 0$ and $\varepsilon = a + \sqrt{a}$

Exercise 5. Consider polynomials $p = ax^2 + bx + c$ and $q = dx + e$. When do $p = 0$ and $q = 0$ have simultaneous solutions?

Exercise 6. Let p, q be as before. When does the system $p = 0, q > 0$ have a solution?

Exercise 7. Use Virtual substitution to eliminate the quantifier in

$$\exists x (ax + b \leq 0 \wedge cx - d < 0)$$