Driving down Forbes Ave

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Motivation
Model Overview

- Two distinct points of driving situation
- Time triggered models
- Modeled through differential logic and hybrid systems
Point 1: Turning onto Forbes Ave
Formalizing Model for Point 1
Assumptions

- Vehicle turns at a perfect 90 degree angle
- Incoming traffic comes at constant speed
- Vehicle initial position is l away from the street
- Incoming traffic becomes irrelevant if they pass the vehicle’s position
- Vehicle is safe after turning is complete
\[
\begin{align*}
  v' &= a, \quad x' = -y \ast \frac{v}{4l}, \quad y' = \frac{vx}{4l}, \quad t' = 1, \quad \text{rogy}x1' = \text{rogy}v, \\
  \text{rogy}x2' &= \text{rogy}v \ast (v \geq 0 \land t \leq T \land x \geq 0) \\
  \text{/* turning radius assume to be the distance between car location to the center of the lane, if we are to turn with only 1 lane */}
\end{align*}
\]

\[
\{ x' = -v \land x \leq 0 \land v \geq 0 \}
\]

\[
\text{/* we turned into the lane already, go straight */}
\]
Safety Property

\[(x \geq 0 \implies (x^2 + y^2 = 16*l^2 \quad \& \quad \max(\text{abs}(x - \text{rogx1}), \text{abs}(2*l - y)) > \text{buffer} \quad \& \quad \max(\text{abs}(x - \text{rogx2}), \text{abs}(4*l - y)) > \text{buffer})\)
Control for Committing the Turn

- Both incoming vehicle can not pass our initial position by the time we complete the turn with maximum acceleration.
Point 2: Lane Switching on Forbes
Formalizing Model for Point 2
Assumptions

- Vehicle lane switches at a perfect 60 degree angle
- Front and side-lane traffic comes at constant speed
- Vehicle travels at constant speed < front vehicle before lane switching
- Incoming side-lane traffic becomes irrelevant if they pass the vehicle’s position
- Vehicle is safe after lane switching is complete
Dynamics

\{x' = 0, y' = v, sidey' = rogv, fronty' = rogv, t' = 1 \& t \leq T\}\n
\{v' = a, x' = v/2, y' = v*(3)^{(0.5)}/2, sidey' = rogv, fronty' = rogv, t' = 1 \& t \leq T \& v \geq 0 \& x \geq 0 \& x \leq 2*l\}\n
\{y' = v \& v \geq 0 \& x \geq 2*l\}\n
Safety Property

$$(((x \geq 0 \& x < 2*l) \rightarrow \text{fronty} - y > \text{buffer}) \& ((x > 0 \& x \leq 2*l) \rightarrow y - \text{sidey} > \text{buffer}))$$
Control for Lane Switch

- Enough space with front car
- Side lane traffic won’t reach vehicle’s current y position if our vehicle travels at maximum acceleration after the current control cycle

```
? (x >= 0 & x < 2*l &
fronty - y >= (3)^0.5*(2*l - x) + buffer/∗ enough space to make the lane switch ∗/);
a := ∗; ? (0 <= a & a <= A); /∗ only acceleration allowed for lane switch ∗/

{? (2*l - x >= 0);
switchT := (2*A*(2*l - x))^0.5/A;
? (sidey + rogv*T + rogv*switchT < y - buffer); /∗ enough space on the side ∗/}
```
Conclusion

- Both models successfully proven with KeYmaeraX
- Formally verifying driving control is hard!
- Future Work:
  - Loosen up assumption constraints
  - Verifying the efficiency of model through simulation
Questions?