

Minimizing Sequents to Find Modeling Errors in KeYmaera X

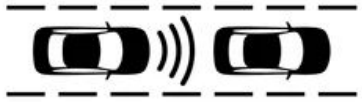
By Ben Gafford and Myra Dotzel



Modeling mistakes happen!

Don't let them get the best of you.

Problem domain

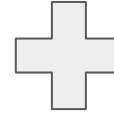


Model of controller and dynamics

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Proof of safety properties

Proof: ✓ All goals closed

Provable($\implies A() > 0 \& B() > 0 \& T() > 0 \&$
 $(())^2 + vC * T() + 1/2 * A() * T()^2 \rightarrow [\{ \{ ? \}$
 $1/2 * A() * T()^2 ; aC := A() ; ++aC := -B() ; -$

A model for adaptive cruise control

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))); aC := A;}
11     {?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20       (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

A model for *provably safe* adaptive cruise control!

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?vC>=vL -> (pL-pC > vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-
11     {?vC=0;aC := 0;}
12     ++{aC := -B;}
13     }
14     /* Continuous dynamics and event triggers */
15     {
16     { pL' = vL, pC' = vC, vC' = aC &
17     (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20     (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
21     }
22     }*
23   ]
24   /* Safety condition */
25   ( pC <= pL )
```

Proof: ✓ All goals closed

Provable(==> A()>0&B()>0&T()>0&
()^2)+vC*T()+1/2*A()*T()^2->[{}?
1/2*A()*T()^2;aC:=A();++aC:=-B();-

...oh oops sorry one second

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))); aC := A;}
11     {?vC=0;aC := 0;}
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16     { pL' = vL, pC' = vC, vC' = aC &
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18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20       (vC>=vL -> (pL-pC >= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

...now, a *provably safe* adaptive cruise control!

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL > 0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B)))}
11     {?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
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19     { pL' = vL, pC' = vC, vC' = aC &
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21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

Proof: ✓ All goals closed

Provable($\implies A()>0 \& B()>0 \& T()>0 \& (vC \geq vL \rightarrow (pL - pC > vC * (vC - vL) / -B + 1/2 * A() * T()^2 - [vL * ((vC - vL) / -B)]) \& vC \geq 0$)
 $\& (vC \geq vL \rightarrow (pL - pC \geq vC * (vC - vL) / -B + 1/2 * A() * T()^2 - [vL * ((vC - vL) / -B)]) \& vC \geq 0$) ;
 $aC := A(); ++aC := -B();$

...sorry this is embarrassing

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))); aC := A;}
11     ++{?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20       (vC>=vL -> (pL-pC >= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

...sorry this is even more embarrassing

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
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11     ++{?vC=0;aC := 0;}
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21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```


Motivation

- Problem
 - Difficult for programmers to catch their own errors
 - Over-constrained models often lead to weak models
 - Erroneous models are dangerous and can be expensive to cope with
- Our solution
 - A really good analysis.

How can we identify these over-constraining errors?

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))); aC := A;}
11     ++{?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
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21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

How can we identify these over-constraining errors?

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))}; aC := A;}
11     ++{?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))) & vC >= 0 }
18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20       (vC>=vL -> (pL-pC >= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))) & vC >= 0 }
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```

How can we identify these over-constraining errors?

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?true; aC := A;}
11     {?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B)))) & vC >= 0 }
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20       (vC>=vL -> (pL-pC >= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B)))) & vC >= 0 }
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```

Proof: ✓ All goals closed

Provable(==> A()>0&B()>0&T()>0&
()^2)+vC*T()+1/2*A()*T()^2->[{{?}
1/2*A()*T()^2;aC:=A();++aC:=-B();-

How can we identify these over-constraining errors?

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3  (vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))))
4  ->
5  [
6  {
7    /* Control */
8    {
9      /* Safely assign acceleration or braking for the controlled car */
10     {?(vC>=vL -> (pL-pC > vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))); aC := A;}
11     ++{?vC=0;aC := 0;}
12     ++{aC := -B;}
13   }
14   /* Continuous dynamics and event triggers */
15   {
16     { pL' = vL, pC' = vC, vC' = aC &
17       (vC>=vL -> (pL-pC <= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))) & vC >= 0 }
18     ++
19     { pL' = vL, pC' = vC, vC' = aC &
20       (vC>=vL -> (pL-pC >= vC*(vL-vC)/-B + 1/2*-B*((vL-vC)/-B)^2 - (vL * ((vL-vC)/-B))) & vC >= 0 }
21   }
22   }*
23 ]
24 /* Safety condition */
25 ( pC <= pL )
```

How can we identify these over-constraining errors?

```
1  /* Initial conditions */
2  ( A > 0 & B > 0 & vL >0 & pC < pL &
3    true )
4  ->
5  [
6    {
7      /* Control */
8      {
9        /* Safely assign acceleration or braking for the controlled car */
10       {?true; aC := A;}
11       ++{?true;aC := 0;}
12       ++{aC := -B;}
13     }
14     /* Continuous dynamics and event triggers */
15     {
16       { pL' = vL, pC' = vC, vC' = aC &
17         (vC>=vL -> (pL-pC <= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
18       ++
19       { pL' = vL, pC' = vC, vC' = aC &
20         (vC>=vL -> (pL-pC >= vC*(vC-vL)/-B + 1/2*-B*((vC-vL)/-B)^2 - (vL * ((vC-vL)/-B))) & vC >= 0 }
21     }
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Overview of Implementation

Proof tree analysis

- Witnessed facts?
- Used facts?
- Unused facts?

New tactics

- minQE
- minAuto
- minAutoXtreme

Minimizing sequents: Guess & Check

Starting sequent: $\vdash x > 0 \& y > 0 \rightarrow [?x \geq 1 ; ++?x < 1 ; \{x' = 1 \& y > 0\}] x > 0$

Candidate mutations:

Minimizing sequents: Guess

Starting sequent: $\vdash x > 0 \& y > 0 \rightarrow [?x \geq 1 ; ++?x < 1 ; \{x' = 1 \& y > 0\}] x > 0$

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\vdash **true** $\rightarrow [?x \geq 1 ; ++?x < 1 ; \{x' = 1 \& y > 0\}] x > 0$

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\vdash $x > 0 \& y > 0$ $\rightarrow [?x \geq 1; ++?x < 1; \{x' = 1 \&$ **true** $\}] x > 0$

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\vdash $x > 0 \& y > 0$ $\rightarrow [?x \geq 1; ++?$ **true** $;$ $\{x' = 1 \& y > 0\}] x > 0$

\vdash $x > 0 \&$ **true** $\rightarrow [?$ **true** $;$ $++?$ **true** $;$ $\{x' = 1 \&$ **true** $\}] x > 0$

Minimizing sequents: Guess & Check

Starting sequent: $\vdash x > 0 \& y > 0 \rightarrow [?x \geq 1; ++?x < 1; \{x' = 1 \& y > 0\}] x > 0$

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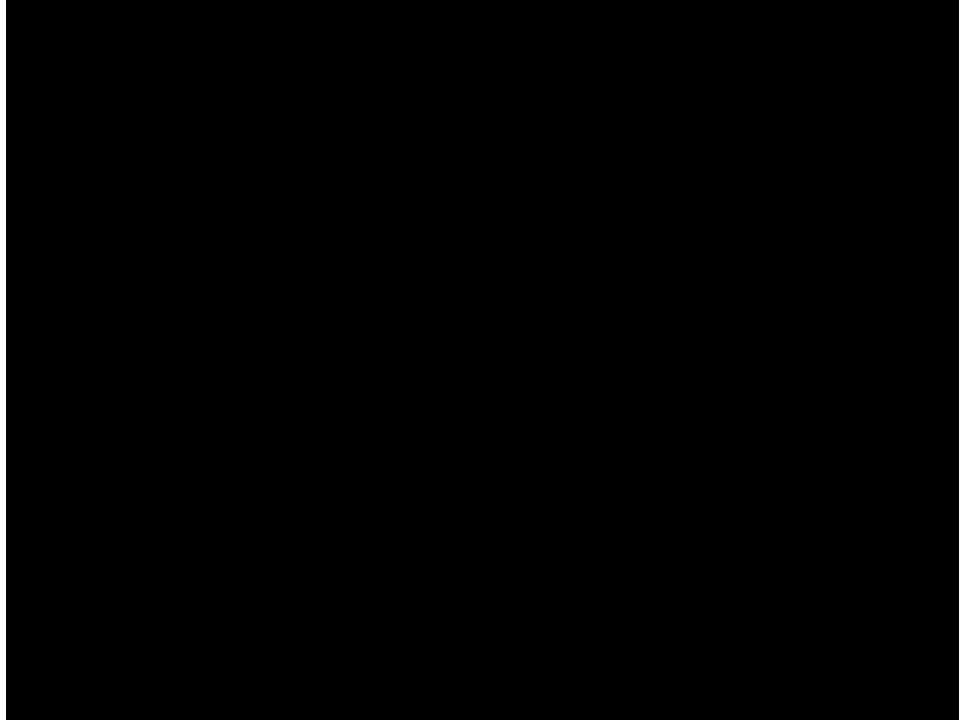
$\vdash x > 0 \& \mathbf{true} \rightarrow [?x \geq 1; ++?x < 1; \{x' = 1 \& y > 0\}] x > 0$

$\vdash x > 0 \& y > 0 \rightarrow [?x \geq 1; ++?x < 1; \{x' = 1 \& \mathbf{true}\}] x > 0$

$\vdash x > 0 \& y > 0 \rightarrow [?x \geq 1; ++? \mathbf{true}; \{x' = 1 \& y > 0\}] x > 0$

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Example/Demo



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$\vdash x > 0 \& \text{true} \rightarrow [? \text{true}; ++? \text{true}; \{x' = 1 \& \text{true}\}] x > 0$



Sequent Proof Example (Simple)

$$\vdash v^2 \leq 10 \wedge b > 0 \rightarrow b > 0 \wedge (\neg(v \geq 0) \vee v^2 \leq 10)$$

Witnessed: $v^2 \leq 10, b > 0, \neg(v \geq 0)$

Used:

Unused: $v^2 \leq 10, b > 0, \neg(v \geq 0)$



Sequent Proof Example (Simple)

$$\begin{array}{c} * \\ \text{id} \frac{}{v^2 \leq 10, b > 0 \vdash b > 0} \\ \wedge L \frac{}{v^2 \leq 10 \wedge b > 0 \vdash b > 0} \quad \frac{}{v^2 \leq 10 \wedge b > 0 \vdash \neg(v \geq 0) \vee v^2 \leq 10} \\ \wedge R \frac{}{v^2 \leq 10 \wedge b > 0 \vdash b > 0 \wedge (\neg(v \geq 0) \vee v^2 \leq 10)} \\ \rightarrow R \frac{}{\vdash v^2 \leq 10 \wedge b > 0 \rightarrow b > 0 \wedge (\neg(v \geq 0) \vee v^2 \leq 10)} \end{array}$$

Witnessed: $v^2 \leq 10, b > 0, \neg(v \geq 0)$

Used: $b > 0$

Unused: $v^2 \leq 10, \neg(v \geq 0)$



Sequent Proof Example (Simple)

$$\begin{array}{c}
 \text{id} \frac{}{v^2 \leq 10, b > 0 \vdash b > 0} \quad * \\
 \wedge L \frac{}{v^2 \leq 10 \wedge b > 0 \vdash b > 0} \\
 \wedge R \frac{}{v^2 \leq 10 \wedge b > 0 \vdash b > 0 \wedge (\neg(v \geq 0) \vee v^2 \leq 10)} \\
 \rightarrow R \frac{}{\vdash v^2 \leq 10 \wedge b > 0 \rightarrow b > 0 \wedge (\neg(v \geq 0) \vee v^2 \leq 10)} \\
 \end{array}
 \quad
 \begin{array}{c}
 * \\
 \text{id} \frac{}{v^2 \leq 10, b > 0 \vdash \neg(v \geq 0), v^2 \leq 10} \\
 \wedge L \frac{}{v^2 \leq 10 \wedge b > 0 \vdash \neg(v \geq 0), v^2 \leq 10} \\
 \vee R \frac{}{v^2 \leq 10 \wedge b > 0 \vdash \neg(v \geq 0) \vee v^2 \leq 10} \\
 \end{array}$$

Witnessed: $v^2 \leq 10, b > 0, \neg(v \geq 0)$

Used: $b > 0, v^2 \leq 10$

Unused: $\neg(v \geq 0)$



Sequent Proof Example (Simple)

$$\begin{array}{c}
 \text{id} \frac{}{v^2 \leq 10, b > 0 \vdash b > 0} \\
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 \end{array}
 \quad
 \begin{array}{c}
 * \\
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 \vee R \frac{}{v^2 \leq 10 \wedge b > 0 \vdash v^2 \leq 10 \vee \neg(v \geq 0)} \\
 \rightarrow R \frac{}{\vdash v^2 \leq 10 \wedge b > 0 \rightarrow v^2 \leq 10 \vee \neg(v \geq 0)}
 \end{array}$$

Witnessed: $v^2 \leq 10, b > 0, \neg(v \geq 0)$

Used: $b > 0, v^2 \leq 10$

Unused: $\neg(v \geq 0)$

Overview of Implementation

Proof tree analysis

Sequent Proof Example (Simple)

$$\begin{array}{c}
 \text{id} \frac{}{v^2 \leq 10, b > 0 \vdash b > 0} \quad * \\
 \text{AL} \frac{v^2 \leq 10, b > 0 \vdash b > 0}{v^2 \leq 10 \wedge b > 0 \vdash b > 0} \quad \text{VR} \frac{v^2 \leq 10 \wedge b > 0 \vdash \neg \neg v^2 \leq 10}{v^2 \leq 10 \wedge b > 0 \vdash v^2 \leq 10} \\
 \text{AR} \frac{v^2 \leq 10 \wedge b > 0 \vdash b > 0 \wedge (\neg \neg v^2 \leq 10)}{v^2 \leq 10 \wedge b > 0 \vdash b > 0 \wedge v^2 \leq 10} \\
 \text{→R} \frac{}{\vdash v^2 \leq 10 \wedge b > 0 \rightarrow b > 0 \wedge (\neg \neg v^2 \leq 10)}
 \end{array}$$

Witnessed: $v^2 \leq 10, b > 0, \neg(v \geq 0)$

Used: $b > 0, v^2 \leq 10$

Unused: $\neg(v \geq 0)$

New tactics

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$\vdash x > 0 \& y > 0 \rightarrow [?x >= 1; ++\text{true}; \{x' = 1 \& y > 0\}] x > 0$

$\vdash x > 0 \& \text{true} \rightarrow [?\text{true}; ++\text{true}; \{x' = 1 \& \text{true}\}] x > 0$

Rules for Suggested Mutations

Suggested Mutation (m)

$$\frac{\text{SM:ASSUMPTIONR} \quad A \vdash [\alpha]P}{m_1(A) \vdash [\alpha]P} \quad (a \in A)$$

$$m_1(\cdot) = \{\cdot \mapsto \cdot \setminus \{a\}\}$$

$$\frac{\text{SM:ASSUMPTIONW} \quad A \vdash [\alpha]P}{m_2(A) \vdash [\alpha]P} \quad (p(a_1, a_2) \in A)$$

$$m_2(\cdot) = \{p(a_1, a_2) \mapsto p'(a_1, a_2)\}$$

$$\frac{\text{SM:POSTA} \quad A \vdash [\alpha]P}{A \vdash [\alpha]m_7(P)} \quad (a \notin \text{const}(A), a \in A, a \notin P)$$

$$m_7(\cdot) = \{P \mapsto P \cup \{a\}\}$$

How m strengthens the model

- removes an assumption
- m -mutated dL formula is valid

- weakens an assumption by replacing $a_1 < a_2$ with $a_1 \leq a_2$
- m -mutated dL formula is valid

- adds a post-condition
- m -mutated dL formula is valid

Formal Guarantees

- **Termination:**
 - The analysis terminates.
 - Proof sketch: well-ordering argument over the # of proof steps, # of mutable facts, and remaining test time per fact
- **Soundness:**
 - m-mutated models...
 - are valid,
 - prove the same post-conditions with fewer facts, and
 - prove stronger post-conditions with the same facts.
 - Proof sketch: rule induction over suggested mutations

Contributions

- An analysis for diagnosing modeling errors
 - Implementation
 - <https://github.com/gaffordb/KeYmaeraX-release>
 - Examples
 - Candidate and suggested mutations
 - Formal guarantees
 - Soundness
 - Termination