Quarterback Safety in American Football

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Nice to Meet You!

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 Agenda

- Intro to American Football
- Motivation
- Approaches to Modelling
- Proofs and Insights
Intro to American Football

- Line of Scrimmage
- Quarterback
- Offensive Linemen
- Defensive Linemen
- Wide Receiver
- Linebacker
Intro to American Football

Game of Advancing the Ball Down the Field. Played in Increments
Intro to American Football (Line of Scrimmage)

Current Lengthwise Position of the Ball
Intro to American Football (Quarterback)

Quarterback: Responsible for Passing the Ball
Intro to American Football (Linemen)

Offensive Line Protects QB, Defensive Line Attacks QB
Wide Receivers Run and Catch, Linebackers Prevent This
Motivation

- **Safety** ⇒ QB is not tackled
- **Efficiency** ⇒ Pass is made
- Stepping stone to more multi-agent systems
Approaches to Modelling

- Simplifications
- Modeling Scenarios
  - Linemen Collision (Safety)
  - Passing (Efficiency)
- KeyMaera X Model
Simplifications

Less Players + One Dimensional
Lineman Collision
Lineman Collision
(Visualization)

\[ vy_D - vy_A \]

- QB
Lineman Collision (Movement)

Pre-collision: run towards each other

Perfectly inelastic collision

Post-collision: move with dampened speed towards QB

QB Safety 15/23
Lineman Collision  (Conservation of Momentum)

\[ m_A \cdot vy_{Ai} \cdot dy_{Ai} + m_D \cdot vy_{Di} \cdot dy_{Ai} = m_A \cdot vy_f \cdot dy_f + m_D \cdot vy_f \cdot dy_f \]

\[ m(vy_{Ai} \cdot dy_{Ai} + vy_{Di} \cdot dy_{Ai}) = 2 \cdot m \cdot vy_f \cdot dy_f \]

\[ m(vy_{Ai} - vy_{Di}) = 2 \cdot m \cdot vy_f \cdot dy_f \]

\[ vy_{Ai} - vy_{Di} = 2 \cdot vy_f \cdot dy_f \]

\[ \frac{vy_{Ai} - vy_{Di}}{2} = -vy_f \]

\((m = m_A = m_D)\)

\((vy_{Ai} = 1, vy_{Di} = -1)\)

(eliminate m)
Passing
Passing (Passing $\leftrightarrow$ Open)

- **Efficiency** $\Rightarrow$ Pass is made
- Assume Quarterback can *instantaneously* pass as soon as our Wide Receiver is *open*
- **Lemma:** isOpen(...) == ballPassed(...)
- **Why?**
  - Focus on QB Safety
  - Helps us define “pass is made”
Passing (Defensive Considerations)

**Man:** Wide Receiver must **run past** the Linebacker in order to catch the ball

\[
\text{Bool } isOpen(WR,LB) \leftrightarrow LB < WR
\]

**Zone:** Wide Receiver must **catch before** getting tackled by the Linebacker

\[
\text{Bool } isOpen(WR,LB) \leftrightarrow WR < LB
\]
Keymaera X Model

- Variation
- Pre-collision Movement
- Post-collision Movement
- isOpen() == ballPassed()
- QB Unhurt + Ball is Passed

vyA := vyD - diffLine;
yyLB := vyWR - diffPass;
t := 0;

/* Pre-collision movement */
{ yQB' = dyQB*vyQB,
  yA' = dyA*vyA,
  yD' = dyD*vyD,
  yWR' = dyWR*vyWR,
  yLB' = dyLB*vyLB,
  t' = 1
  & yA <= yD /* Pre-collision */
  & t <= T /* Realism */
}

/* Keep evolving */
{ yQB' = dyQB*vyQB,
  /* Dampened Movement */
  yD' = (dyA*vyA + dyD*vyD)/2,
  yWR' = dyWR*vyWR,
  yLB' = dyLB*vyLB,
  t' = 1
  & yA >= yD /* Collided */
  & t <= T /* Realism */
}

>( yQB < yD /* QB unhurt */
  & isOpen(yWR, yLB, buffer) /* Passed */
  & onField(yQB) & onField(yD)
  & onField(yWR) & onField(yLB)
  & t <= T /* Within 40 second play clock */
)
Proofs and Insights

- Tactic
- Finding the Time
Proof: ✔ All goals closed

Provable( \implies yQB=150&yA=yQB+15&yD=yA+36&yWR=yA&yLB=(yD+300)/2&\theta < \text{diffLine}&\text{diffLine} < 260 < \text{diffPass}&\text{diffPass} < 2&\text{buffer}=0&v_yQB=4.6 
&v_yD=23.72&v_yWR=26.79->v_yA=v_yD-\text{diffLine};&v_yLB=v_yWR-\text{diffPass};t:=0;\{yQB'=(-1)*v_yQB,yA'=1*v_yA,yD'=(-1)*v_yD,y_WR'=1*v_yWR,yLB'=(-1)*v_yLB, 
&t'=1&yA<=yD&y_t<=40}\{yQB'=(-1)*v_yQB,yD'=(1+yA+(-1)*v_yD)/2,y_WR'=1*v_yWR,yLB'=(-1)*v_yLB,t'=1&yA>=yD&\{yQB<yD&y_WR+\text{buffer}<y_{LB}&(0< 
&yQB&yQB<=300)\}&(0<=yD&y_t<=300)\&(0<=y_WR&y_WR<=300)\&(0<=yLB&yLB<=300)\&(t<=40)\} \text{ proved})

Tactic to Reproduce the Proof

\text{expandAllDefs ; unfold ; assignd(1) ; composed(1) ; solve(1.1) ; solve(1) ; QE}

\[
T_{1c} = T_{\text{lineman collision}} = \frac{yD - yA}{dyD * vyD - dyA * vyA} = \frac{yD - yA}{2*vyD - \text{diffLine}}
\]

\[
T_{\text{zone}} = T_{\text{wr open zone}} = \frac{yLB - yWR}{dyLB * vyLB - dWR * vyWR} = \frac{yLB - yWR}{2*vyWR - \text{diffPass}}
\]

\[
T_{\text{man}} = T_{\text{wr open man}} = \frac{yWR - yLB}{dWR * vyWR - dyLB * vyLB} = \frac{yWR - yLB}{2*vyWR - \text{diffPass}}
\]
Questions?