Modeling Safer Traffic Light Transitions and an Intersection Control System for Autonomous Vehicles

Kyle Kauffman (kkauffma@andrew.cmu.edu)

I. ABSTRACT

Traffic control systems are designed to reduce the rate of accidents while promoting more efficient travel. As technology advances, it is particularly important to reconsider common infrastructure and safety system implementations for possible improvements. In this project I analyze these two objectives, proposing a safer traffic light system for manned vehicles and separately a more efficient control system for autonomous vehicles. This paper explains the notions behind the verifiable proof of safety for a reactive traffic control system which is aimed at reducing red-light running accidents. It additionally explores the modeling requirements of an intersection control system for autonomous vehicles with the goal of eliminating the need for stopping at intersections, though much of this paper focuses on the safer traffic light transition application with some discussions of the shortcomings in the autonomous traffic control system model.

II. MOTIVATION AND BACKGROUND

In 2018, 846 people were killed and roughly 139,000 injured in crashes as a result of running a red light with over half of these deaths involving pedestrians or cyclists hit by the red-light runners[1]. Red light violations are a serious concern for drivers and especially pedestrians when navigating intersections. While the advent of red-light cameras can certainly dissuade these red-light perpetrators, with a 40% reduction of violation rates shown in a 1999 study, these cameras do not prevent red-light runners from running into opposing traffic.[2] What is even more striking is that nearly all accidents resulting from running red lights occur soon after the light has turned red. A U.S. Department of Transportation report from March 2006 assessed that from the period of May 1999 to June 2003 in Sacramento, CA, "94 percent of the violations occurred within 2 seconds after the onset of the red light and only 3 percent of these violations were recorded 5 seconds after the onset of the red light."[3]

My first proved model verifies the safety of a traffic control system aimed at reducing the frequency of these red-light accidents. This theoretical system observes vehicle dynamics within a small-time window immediately after the light has changed and ensures that an opposing light will not turn from red to green within this window if an oncoming vehicle is not braking appropriately before the intersection. The light will only turn green within this time window, T, if the oncoming car whose light just turned red is braking at a rate which would allow it to stop before the intersection with this deceleration. This model implicitly considers real world assumptions like driver behavior and awareness to combat accidents. In combination with current red-light running countermeasures, the implementation of this verifiably safe traffic light control system would further reduce accidents by targeting the most frequent red-light running occurrences.

While perhaps not as compelling in terms of increasing the safety of roadways, the intersection control system for autonomous vehicles strives to represent a more efficient, safe intersection system with only autonomous vehicles. The goal is like that of a recent startup out of CMU, Rapid Flow Technologies, which uses advanced sensing techniques to predict traffic flows and decrease commute times[4]. This model strives to improve upon this notion of smart traffic signals in order to indirectly reduce commute time and improve vehicle efficiency.

The second model, if finished, would guarantee that no vehicles collide in the intersection provided the assumption that each vehicle is following the published speed limit and functioning correctly, having equal braking and acceleration potential. By allowing the intersection system to communicate with the autonomous vehicles and control their accelerations, this model strives to prove that autonomous vehicles will not always need to stop when crossing an intersection under constrained circumstances.

The focus of this project is not to implement these models, but rather prove the safety of these theoretical systems and provide evidence for their viability. That is not to say that these systems must remain purely theoretical. The safer traffic light transition control system is conceivable with today's technology. The only requirements would be a device which is able to observe the dynamics of vehicles and relay this information to the traffic light's SCADA system. As stated, companies like Rapid Flow Technologies are already using vehicle dynamics to influence roadway efficiency. My model suggests that this technology could be easily adapted to increase intersection safety by reducing red-light accidents.

III. MODELING APPROACH AND IMPLEMENTATION

A. Safer Traffic Light Transition:

1. Model Structure

The safer traffic light transition model successfully formalizes the problem of detecting red light runners by predicting if a driver will stop before the intersection. This dL verification model accounts for one vehicle driving straight, approaching an intersection with a position x, velocity v, and acceleration a. The guarantee is that within time T, the opposing light will never be green if the car is in the intersection. This guarantee clearly illustrates the safety of the system: the opposite intersection will never receive the signal to go if a car running the red light will be in the intersection at any point in the interval [0, T]. The general structure of the model is as follows:

$preconditions \rightarrow$
$[\{\, assignments; \{linear\ dynamics\ \&\ Q\}\} * @invariant]$
$Postcondition$

2. General Assumptions

Modeling a safe traffic intersection can be extremely complicated. This model reduces the complexity of a true traffic light intersection by omitting intricacies in favor of clarity. While this model breaks down the dynamics of a traffic intersection into smaller components, the simplicity allows for easier understanding of the issue the model is attempting to solve. Additionally, without being overly precise, this model still manages to present a legitimate use for existing traffic light observation systems that monitor speed, acceleration and position of vehicles.

Another key assumption is the fact that the light for the approaching vehicle is red. In a more robust implementation of this model, the light would have various states, including red, yellow, and green. This model only attempts to represent the state in which the light has just changed as the focus of this project is reducing the occurrences of red-light accidents.

A more subtle assumption of the model is the assumed driver behavior. This model assumes that when a driver makes the decision to brake at a rate which would allow them to stop before the intersection, that indicates they see the light is red and maintain this deceleration. If this assumption was omitted, then there would be no mechanism preventing the driver from accelerating once again after braking and running the light once the other light has turned green.

The model further assumes that the vehicle's movement is linearly constrained. Detecting if a vehicle is going to turn or their intended direction is much more complicated and would require lane detection, turning rate analysis and more. As such, this model does not include other dimensions such as the y or even z dimensions, though in reality these would be important when developing a true traffic light system. At intersections where the dynamics do not begin some fixed, linear distance away from the traffic light monitoring system, for instance around a curve, the y direction would need to be included.

Ultimately this model proves the safety of an intersection to prevent collisions resulting from running straight red lights, not in turns, within the first T seconds after a light has turned.

3. Explanation of Precondition

This model relies on several suppositions, ignoring more minute intricacies in order to develop a coherent and understandable model. The preconditions for this model are that the light just turned red and the opposite light is red, the time window T is greater than t, meaning non-zero, that the vehicle is not yet in or past the intersection x<=-w, and the vehicle is traveling forward or stopped with v>=0. The following is the formal precondition:

$$t = 0 \; \&$$
$$currLight = 0 \;\&\; v \geq 0 \;\&\; x \leq -w \;\&$$
$$T > t \;\&\; w > 0 \;\&\; oppositeLight \; = \; 0$$

Table 1 lists all the variables used in the model:

Variable Name	Description
T	The time for which the stop light system measures oncoming vehicle dynamics
t	The current time, starting at 0 and progressing to at most t.
x	The position of the car in the x direction.
v	The linear velocity of the car.
a	The linear acceleration of the car.
w	The width of half the intersection. -w on the left, w on the right of the origin.
oppositeLight	The color of the opposite light, 0 for red and 1 for green.
currLight	The color of the light. Assumed to be red as the light just changed.

Table 1: Model variables.

While realistically the acceleration of a vehicle is constrained by numerous factors, including the type of brakes, road conditions, tires, torque, horsepower and more, this model treats the acceleration choice as a nondeterministic assignment. This follows from the notion that the stop light system is not controlling the acceleration of the vehicle, only that the system is observing its characteristics.

$$a \coloneqq *;$$

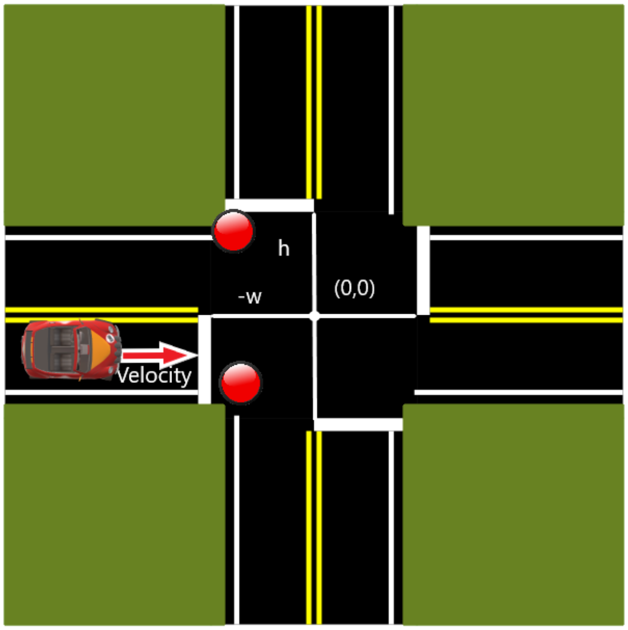

Fig. 1. Depiction of the model starting state where -w is the position of the beginning of the intersection. The model only reflects the behavior of the vehicle under linear motion in the x direction.

4. Assignments and Control

```
/*The driver is only able to change the acceleration when the
opposite light is still red. Assume a rational driver, meaning the
car will keep braking once it starts braking.*/

if(oppositeLight=0)
{
     a:=*;
}
/*   The car is braking and braking at a rate which puts it at or
before the intersection*/
if((a<0 & -v^2/(2*a)<=-w-x))
{
     oppositeLight:=1;
}
```

The traffic light transition control system relies on several assumptions as stated above. In the first assignment, the implicit assumption of driver behavior is formalized in that the driver of the car can only change acceleration when the opposite light is red. As soon as maintaining the acceleration would result in the car's ability to stop before the intersection, the vehicle is assumed to maintain this acceleration. Any other system would result in either the light changing from red to green to red any number of times depending on how the driver chose to accelerate. Essentially this assignment declares that the driver is rational, and the action of braking indicates the driver sees the light is red.

The second assignment illustrates the light changing from red to green, meaning it is safe for other cars to go. The braking formula is constructed using the kinematic equation:

$$v^2 = v_0^2 + 2a\Delta \mathrm{x}$$

To calculate the braking distance, set v to 0 and solve for Δx:

$$\Delta \mathrm{x} = (0 - v_0^2)/(2a)$$

For a to be deemed a safe deceleration, the total distance between the car and the intersection should be greater than the braking distance. Here v is the same as v_0. Additionally, a cannot be 0 or else the formula would be undefined:

$$(\mathrm{a} < 0 \ \& \ -\mathrm{v}\hat{\ }2/(2 * \mathrm{a}) <= -\mathrm{w} - \mathrm{x})$$

5. Invariant

The invariant for this program represents the formula that remains true for any run of the program. The semantics of the nondeterministic repetition, *, indicates a hybrid program can be run any n-times for any $n \epsilon N$. In this case, what remains true is that the oppositeLight is either red, or the oppositeLight is green and the car is decelerating at a rate which results in a braking distance less than its current distance from the start of the intersection:

$$@invariant(oppositeLight = 0 \mid (oppositeLight = 1 \& (a < 0 \& -v\hat{}2/(2*a) \leq -w - x)))$$

6. Safety

While there are conceivably several ways to write the safety guarantee of this system, the conceptual idea is that the car should not be in the intersection at any point within time T if the oppositeLight is 1, meaning green. This can be rephrased as an implication, since when the light is not 1, there is no safety concern:

$$oppositeLight = 1 \rightarrow (x \leq -w \mid x \geq w)$$

Though this safety guarantee is seemingly simple, its simplicity is what makes it such a viable way to improve traffic safety. Often problems perceived as complex, like reducing intersection accidents, may have simple solutions like this modeled system.

7. Potential Improvements

There exist several potential improvements to this model to create a more robust system that verifies the safety of such a traffic light intersection. Two key aspects would be accounting for driver behavior and physical constraints of the environment. Currently the model assumes that a driver who brakes at a rate which would allow the car to stop before the intersection maintains this deceleration. This is a difficult issue to solve because predicting driver behavior is not a simple problem. However, another theoretical, much more advanced system could classify the driver based on the vehicle's license plate and associated past behavior at intersections. If the driver is classified as safe, then the standard for turning the light green may be relaxed slightly, for instance perhaps the length that the light observes the vehicle dynamics. While such an effort could increase efficiency, it would lead to a tradeoff of guaranteed safety.

An additional improvement would be factoring in the constraints of the physical environment, including factors such as vehicle type and assumed braking ability. If a car is currently decelerating at a rate which would enable it to stop, that does not necessarily mean it can maintain that rate of deceleration. Road conditions or even braking endurance would factor in to whether the calculated stopping distance could actually be maintained.

These are just a couple of many improvements that could be made to this system in order to improve its efficiency, safety, and accuracy.

B. Intersection Control System for Autonomous Vehicles:

1. Model Structure

In full transparency, this model is unproved and thus lacks some critical logic that ensures the safety of this system. The structure of the model is like that of the previous model and follows the outline:

$preconditions \rightarrow$
$[\{\ assignments; \{linear\ dynamics\ \&\ Q\}\} * @invariant]$
$Postcondition$
2. General Assumptions

The key assumptions for this model are like that of the previous model with some variation. Each vehicle is treated as a point in space with their respective velocity, acceleration, and position. Each vehicle is traveling along either the x-axis or y-axis approaching a common intersection. This assumption can easily be modified to be more realistic by factoring in a buffer distance that accounts for the size of each car.

Another key assumption is that the traffic intersection control system can control each vehicle's braking in order to prevent a collision if one will occur. This problem is extremely complex and dealing with many of the factors required to accurately model such a system is beyond the scope of this project.

One key assumption is that the cars can brake at any rate less than 0, when in fact this is unrealistic. A control strategy which factors in the actual braking capability of the cars would prove difficult and require extensive research. This would mean the actual monitoring of the vehicles would need to start at a distance that is a factor of the vehicle's speed, acceleration and braking ability, and even weather conditions.

3. Explanation of Precondition

The required preconditions for this unproved model are that both vehicles start outside the intersection and that one vehicle starts on the x axis to the left of the intersection, and another starts below the intersection on the y axis, both with velocities greater or equal to 0. While the vehicles' positions could be flipped, starting at a positive x and y value respectively and with opposite velocities, it makes more sense to maintain consistency from the previous model. The formalized preconditions are as follows:

$$(t = 0\ \&\ v1 \geq 0\ \&\ v2 \geq 0\ \&\ x1 \leq -w\ \&\ y1 = 0\ \&\ x2 = 0$$
$$\&\ y2 \leq -h\ \&\ T > t\ \&\ x1 \neq x2\ \&\ y2 \neq y1\ \&\ h > 0\ \&\ w > 0\ \&$$
$$intersectionT1 = -1\ \&\ intersectionT2 = -1)$$

The last two preconditions, that intersectionT1 = -1 and intersectionT2 = -1 account for the uncertainty if a collision will occur or not. Within the program, intersectionT1 and T2 represent the time solutions for when the two cars cross. While there is only one time for which the vehicles could collide, the two time values correspond to the potential solutions to the quadratic equation explained in the assignment and control section.

4. Assignments and Control

While this model is not yet proven, the math behind the necessary assignments leans towards a provable safety condition. In order to detect if a collision will occur given each car's respective velocity, position, and acceleration, the time at which $x_1(t) = y_2(t)=0$ must be determined. If no such solution exists or if the solution is negative, then no collision occurs. To factor in the size of the vehicles would require adjusting this equation to be not when they are equal, but if the resulting point is less than a certain threshold. $x_1(t) = y_2(t)$ simplifies to $(x_1-y_0) + t*(v_1-v_2) + ½ * t\text{^}2 * (a_1-a_2) = 0$. Solving for t results in t = (-(v1-v2)+-((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-y2))^0.5)/(2*(1/2*a1-1/2*a2)). The +- means there are two solutions if no division by 0 occurs and the equation under the square root does not evaluate to a negative value. However, there may still be one solution since a value of zero means the accelerations are equal, resulting in a linear solution. Additionally, the solution to the quadratic is not that they intersect, but rather that the x and y coordinates are equal at a given time. This is not the solution to the problem however, because they must be equal and also equal to zero. Perhaps a better way to solve this problem would be to calculate if the hypotenuse, or diagonal distance between the two vehicles ever equals zero, or in the case of the buffer distance less than the buffer. Unfortunately given the time constraints of this project, the following explains the achieved progress, not necessarily a successful outcome.

```
/*(-b+- sqrt(b^2-4ac))/(2a). Only assign if not dividing by 0
and not taking square root of negative*/

if ((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-y2)>=0 & 2*(1/2*a1-
1/2*a2)!=0 )
{
     intersectionT1:= (-(v1-v2)+((v1-v2)^2-4*(1/2*a1-
1/2*a2)*(x1-y2))^0.5)/(2*(1/2*a1-1/2*a2));
     intersectionT2:= (-(v1-v2)-((v1-v2)^2-4*(1/2*a1-
1/2*a2)*(x1-y2))^0.5)/(2*(1/2*a1-1/2*a2));
}
else{
     intersectionT1:=-1;
     intersectionT2:=-1;
}
```

Code Excerpt 2. Illustrates the attempt at assigning intersection solutions. Lacks accounting for if each solution occurs when x1=y2=0. Only checks for x1=y2 and does not account for when "a" in the quadratic formula is 0 and a solution still exists.

If a positive solution exists, meaning either intersectionT1>0 or intersectionT2>0, then a collision will occur in the intersection given the car's speed. To eliminate the possibility of collision, one of the vehicles must slow down. This decision is arbitrary as to which car must slow down, though in this model the vehicle which is furthest from the light must be the one to slow down. This conceptually makes sense because the intersection control system will have a maximum point at which it can communicate with oncoming vehicles, so the car that reaches that threshold first should not have to reduce speed since the intersection system would be unaware of any potential collision at that point. Only once the second car reaches that threshold would any concern for safety arise and it would be that second car's responsibility to brake, as commanded by the intersection control system.

The conceptual idea is to assign a new acceleration such that the acceleration is less than 0, meaning braking, and using the new acceleration ensures no positive solution exists for a collision where $x_1(t) = y_2(t)$. This is logically equivalent to the following case where car 1 arrives to the intersection first and the second car is required to brake:

$$if\ (-w - x1 < -h - y2)$$
$$\{a2:=*;?((\forall\ t\ (t <= T\ \&\ t >= 0\ \&\ (x1 + v1 * t + 1/2 * a1 * t\text{^}2! = x2\ |\ y2 + v2 * t + 1/2 * a2 * t\text{^}2! = y1)))\ \&\ a2 <= 0);\}$$

The mathematical modeling portion of the control system is as follows:

```
if (intersectionT1>0 | intersectionT2>0){

          /*Control Strategy. Simply based on distance, since
this is most likely. Once car is within range, it sends a signal
to the intersection alerting of its presence.*/

          /*Case 1: Car 1 is closer to intersection then car 2
must reduce speed to prevent collision.*/

            if (0-w-x1<0-h-y2)
            {
                a2:=*;?(a2<0 & (   ((v1-v2)^2-4*(1/2*a1-
1/2*a2)*(x1-y2)<0 | 2*(1/2*a1-1/2*a2)=0) |
                ((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-y2)>=0 &
2*(1/2*a1-1/2*a2)!=0 &
                (-(v1-v2)+((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-
y2))^0.5)/(2*(1/2*a1-1/2*a2))<0
                & (-(v1-v2)-((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-
y2))^0.5)/(2*(1/2*a1-1/2*a2))<0)));
            }
             /*Case 2: Car 1 is further or equal from
intersection then car 1 must reduce speed*/
            else
            {
                a1:=*;?(a1<0 & (((v1-v2)^2-4*(1/2*a1-
1/2*a2)*(x1-y2)<0 | 2*(1/2*a1-1/2*a2)=0) |
                ((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-y2)>=0 &
2*(1/2*a1-1/2*a2)!=0 &
                (-(v1-v2)+((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-
y2))^0.5)/(2*(1/2*a1-1/2*a2))<0
```

```
                  & (-(v1-v2)-((v1-v2)^2-4*(1/2*a1-1/2*a2)*(x1-
y2))^0.5)/(2*(1/2*a1-1/2*a2))<0)));
             }

}
```

The reason this proof is not yet completed is that the math required to determine if there exists a solution is much more complicated than initially anticipated. A time t may exist where there is a collision, but this might occur because braking with a given acceleration would cause the vehicle to reverse and then collide which should not be possible in the theoretical system. Further, by using the quadratic formula to solve for if a collision exists only checks if x1=y2 and does not account for if the cars pass through the origin at the same point. As proposed above, perhaps a more viable solution would be calculating the distance between the two points and determining if the time at which the minimum distance occurs, found by calculating the second derivative of the distance between the cars, ever equals 0 or is less than the threshold in the case where a buffer is included.

5. Safety

As stated, this model has not been proven, therefore the appropriate safety condition has not been solidified. However, the strict condition that no collision will occur, meaning $x_1 \neq x_2$ | $y_1 \neq y_2$ is likely the most concise solution to the problem. The issue with this safety condition given the current model is that using a dI proof method, the rate of change of the velocity does not equal zero. dI states that for two conditions to be not equal, they must be not equal in the beginning and have a rate of change that is equal. However, the rate of change is not the issue in this scenario, it is the time at which they would intersect. So, for ease of proving the safety of this system, this safety condition may not be the most appropriate.

7. Shortcomings and Potential Improvements

The control system for autonomous vehicles navigating through an intersection is not thoroughly complete. While the conceptual model works out the necessary logic and control, the mathematical implementation is not yet proved. The complication is that I need to find a way to determine if the cars will collide given their given accelerations, velocity and position while constraining their potential intersection to occur only when they are traveling forward or stopped.

This system attempts to model the dynamics of two vehicles approaching an intersection. This means that like the previous model, both cars will need to start outside the intersection. The difficult part about this formulation is coming up with the precondition that ensures that changing the acceleration values can prevent a collision. In other words, if the maximum rate of deceleration is -5 m/s^2 but in order to avoid a crash, it would require -10 m/s^2, the start distance between each car and the intersection is not great enough given their respective velocities.

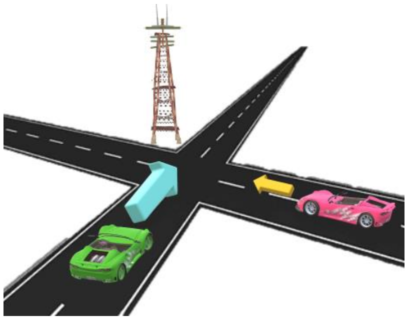

Fig. 2. Depiction of two autonomous vehicles navigating an intersection, controlled by an intersection control system.

The unproven model toward a correct system described previously is one where each vehicle is treated as a point bypasses this problem, allowing the model to focus on how the respective accelerations of both vehicles are manipulated to avoid a crash. Detecting a crash in this scenario is resolved by finding if the x and y coordinates of both vehicles are equal at a given time t. In this model, the vehicles are traveling such that their intersection point will be (0,0), meaning detecting a collision is conceptually as follows:

$$if \begin{pmatrix} \exists t \text{ such that } t \leq T \text{ and } t \geq 0 \text{ and} \\ \mathrm{x1} + \mathrm{v1} * \mathrm{t} + \frac{1}{2} * \mathrm{a1} * \mathrm{t}^2 = \mathrm{x2} \text{ and} \\ \mathrm{y2} + \mathrm{v2} * \mathrm{t} + \frac{1}{2} * \mathrm{a2} * \mathrm{t}^2 = \mathrm{y1} \end{pmatrix}$$
$$\{\mathrm{crashOccurs} \coloneqq 1;\}$$

However the necessary mathematics to solve if a time exists is more complicated than initially assumed and more time would be needed to solve this problem.

III. RELATED WORK

A. Safe Traffic Light:

This project is not the only attempt to improve the safety or efficiency of traffic intersections. One of the first attempts at deterring drivers from running red lights was New York City's Red-Light Camera program, launched in 1994 and the first instance of using photo violation monitoring systems at intersections[5]. These photos were used as evidence to issue a Notice of Liability, essentially a fine to the registered owner of the vehicle, thus dissuading red light running and reducing violations at these intersections by 75 percent. While this approach reduces red light running, it only works for drivers that are paying attention to their environment, realizing the light has turned red. Unfortunately, it does not solve the issue of distracted drivers running the light shortly after the light has changed, the time period where the most violations occur.

While the red-light camera solution uses the threat of fines to dissuade potential red-light violators, other approaches have focused on the physical dynamics of vehicles to predict the occurrence of red-light running violations. The authors of the paper *Predicting Driver Behavior during the Yellow Interval Using Video Surveillance,* published in the December 2016 issue of the

International Journal of Environmental Research and Public Health developed a system to detect speed, acceleration, and vehicle position using video cameras placed at traffic intersections[6]. By observing the behavior of drivers and their stop and go decisions during a yellow light, these researchers were able to effectively predict whether a vehicle would stop or continue through a yellow light and thus better infer if a red light violation would occur based on the vehicle's speed, acceleration, and distance from the intersection. This research illustrates the importance of vehicle position, speed, and acceleration in determining whether a red-light violation would likely occur, however it does not implement any safety guarantee at the intersection, only the likelihood that a violation would occur.

This observing of the physical dynamics of inbound vehicles has also been utilized in modeling the safety at stop-sign intersections, alerting the driver whether it is safe to proceed through the intersection, or if the driver should wait. Like this project, the cooperative intersection collision avoidance system for stop-sign assist (CICAS-SSA) utilizes sensor data to determine overall system safety while recognizing the numerous factors that would contribute to such a model, and instead decomposes the model into simpler systems. Unlike the CICAS-SSA, this project will focus only on the physics-based model while necessarily neglecting factors such as reaction time, weather conditions, and sensor reliability in order to develop a simpler and more easily provable model.

B. Intersection Control System for Autonomous Vehicles:

Cornell Researchers have already made headway in the area of autonomous vehicle control at intersections. These researchers have developed a model to control autonomous traffic at no-stop, one-way intersections by using consecutive platoons of synchronized vehicles with gaps between the arrival and departure at intersections to prevent collisions. They additionally implemented an adjustment operation for when the platoons are out of sync. The main delineation between this accomplished research and this project is scope and ultimate outcome. This project will focus on the safety postcondition of no collisions of two autonomous vehicles approaching a single one-way intersection, while potentially relying on aspects of these researchers' model for collision avoidance, however; most of this research is beyond the scope of this project.

The most similar research to the autonomous vehicle intersection problem is the Autonomous Intersection Management (AIM) project conducted by the Learning Agents Research Group at the University of Texas at Austin. [7]

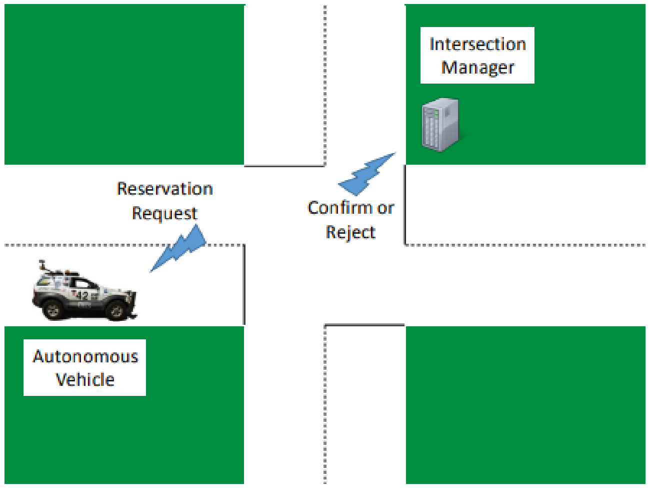

Fig. X. AIM reservation-based intersection problem. Credit: Learning Agents Research Group

Their approach uses a space-time reservation system to prevent collisions. This project is very similar to that of these researchers, the main delineation being the scope and the objective of proving the safety of a simple collision avoidance system using formal differential logic.

REFERENCES

[1] “Red Light Running.” IIHS, February 2020. https://www.iihs.org/topics/red-light-running.
[2] Retting, Richard A, Allan F Williams, Charles M Farmer, and Amy F Feldman. Rep. *Accident Analysis & Prevention* . Insurance Institute for Highway Safety.
[3] Yang, David, and Wassim G Najm. Rep. *Analysis of Red Light Violation Data Collected from Intersections Equipped with Red Light Photo Enforcement Cameras* . Cambridge, MA: Volpe National Transportation Systems Center, 2006.
[4] “About Rapid Flow: Adaptive Traffic Signal Control Technology.” Rapid Flow. Surtrac, 2018. https://www.rapidflowtech.com/about.
[5] Rep. *New York City Red Light Camera Program: 2018 Report. Program Review 1994-2017*. New York City, NY: New York City DOT, 2018.
[6] Li, Juan; Jia, Xudong; Shao, Chunfu. 2016. "Predicting Driver Behavior during the Yellow Interval Using Video Surveillance." *Int. J. Environ. Res. Public Health* 13, no. 12: 1213.
[7] Tsz-Chiu Au, Shun Zhang, and Peter Stone. Autonomous Intersection Management for Semi-Autonomous Vehicles. In Dusan Teodorovi'c, editors, *Handbook of Transportation*, pp. 88–104, Routledge, 2016.