Modeling of K-Pop Dance Choreography as a Synchronized Multi-Agent Hybrid System

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Background and Motivation

- Synchronization of autonomous agents is hard
- Spatial and temporal constraints
- Stepping stone for industrial applications

Background

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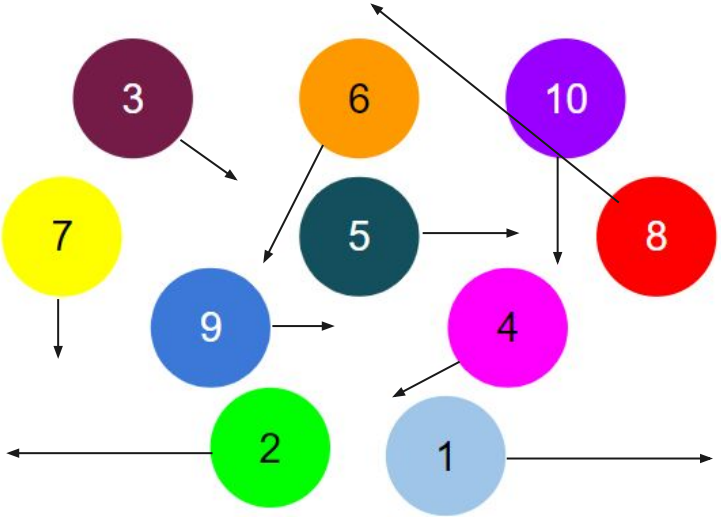

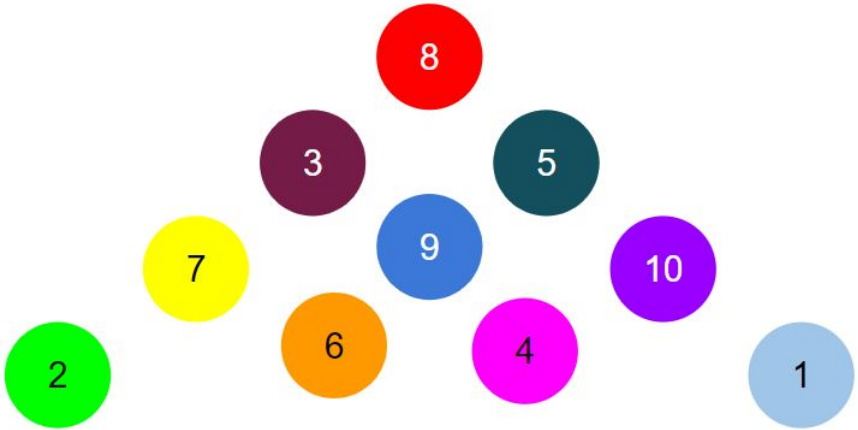

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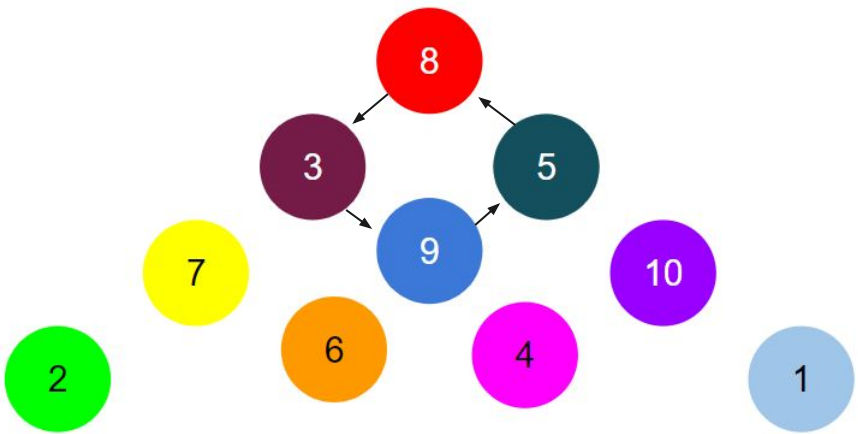
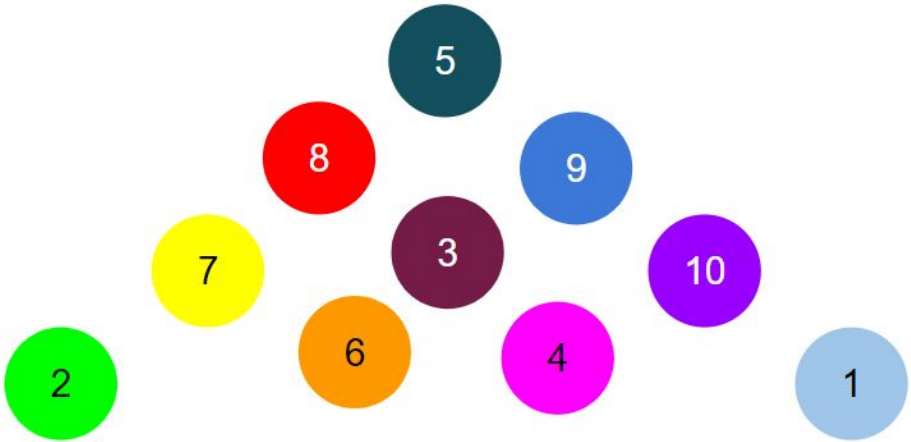
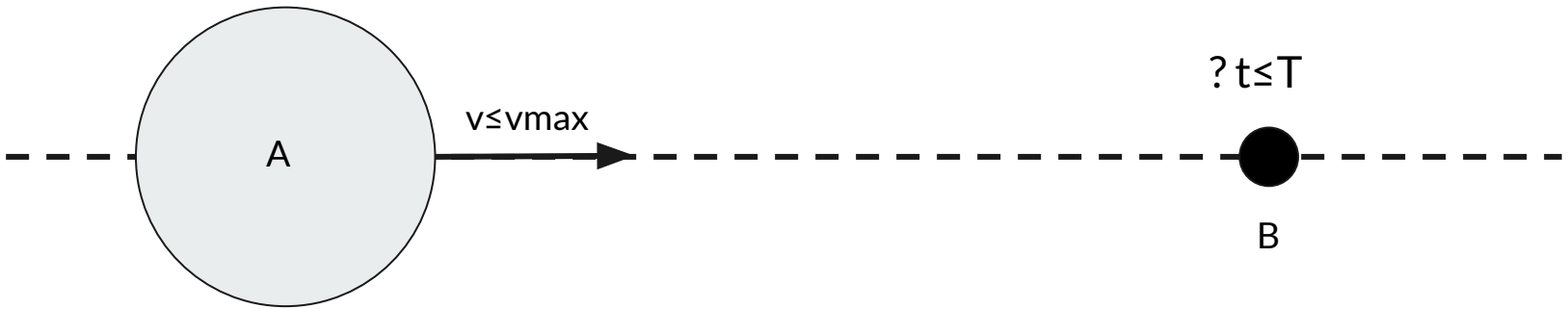




Modeling Choices

- Event triggered
- Point intersection
- Synchronous motion
- Velocity vector-based control
- $|vx|,|vy| \leq vmax$
- Velocity changes freely - no acceleration

Linear Model

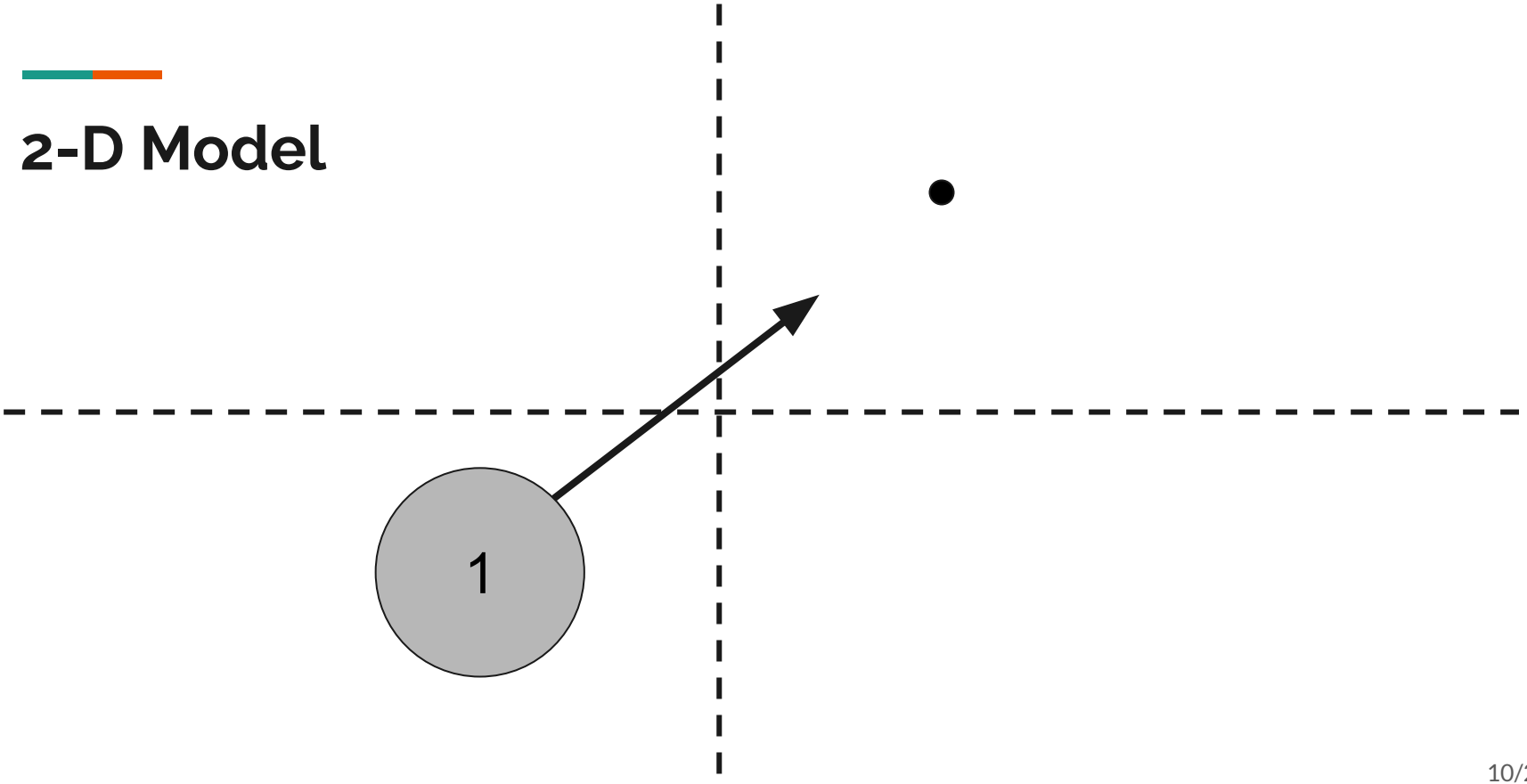


Proving a Lower Bound on T in 1-D

- Diamond notation: $\exists\, t \geq T$ reachable
  - Iterate
  - Assign $v := vmax$ and $t := T$
- Box notation: $\forall\, t < T$ unreachable
  - Loop invariant: $pos \leq A + (t * vmax)$

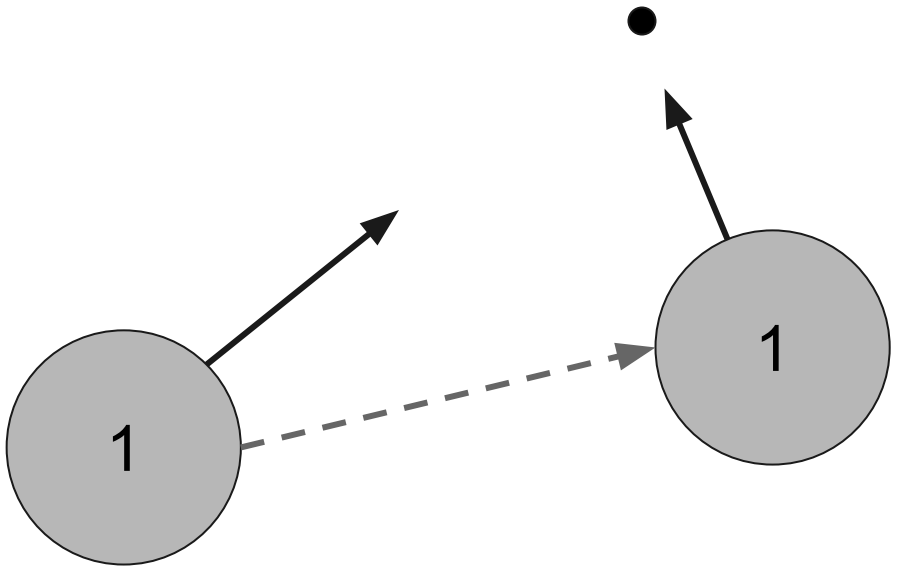
$$T = \frac{B - A}{vmax}$$

2-D Model



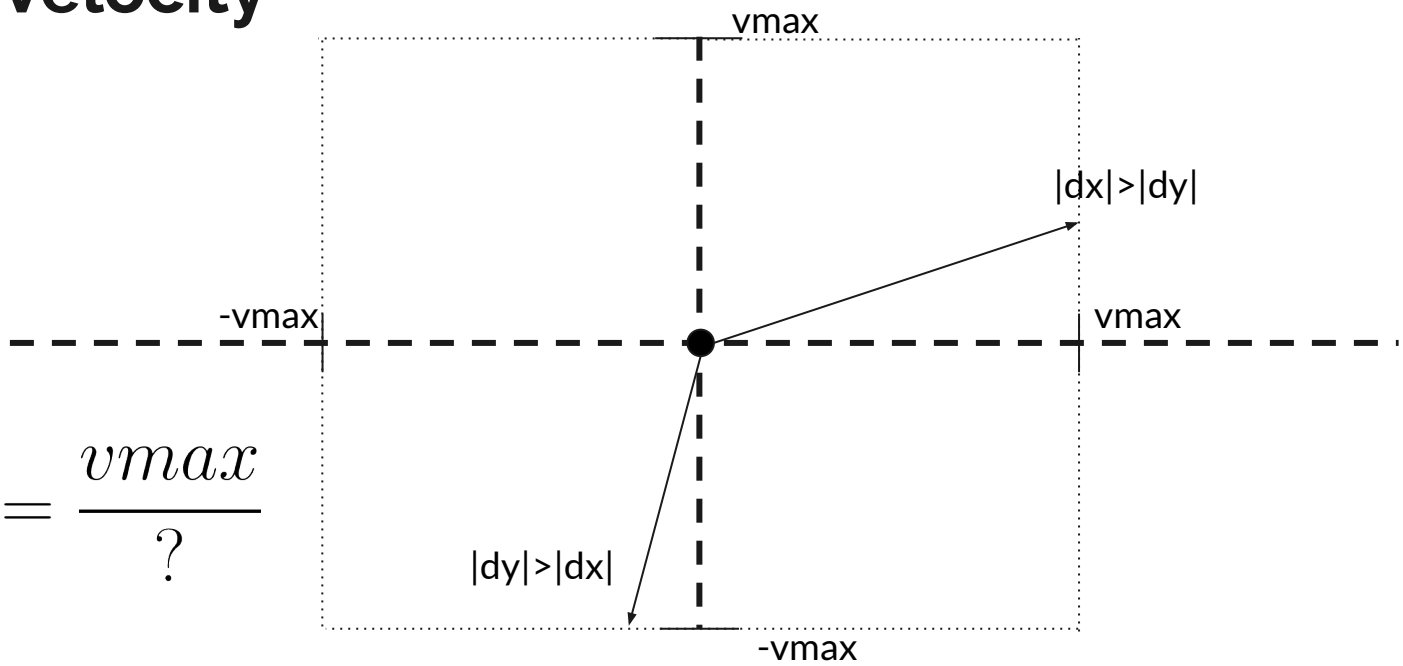
Proving Spatial Reachability with 2D Controller

- Diamond proof strategy: convergence
- Choose vector in direction of target
- Generalization of {a}* to any point in space



Assigning Velocity

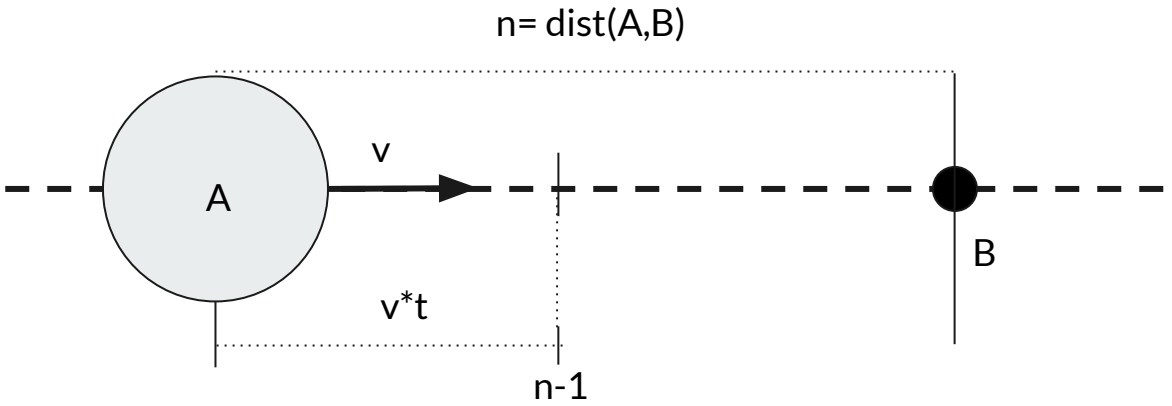
$$\frac{max(dx, dy)}{min(dx, dy)} = \frac{vmax}{?}$$



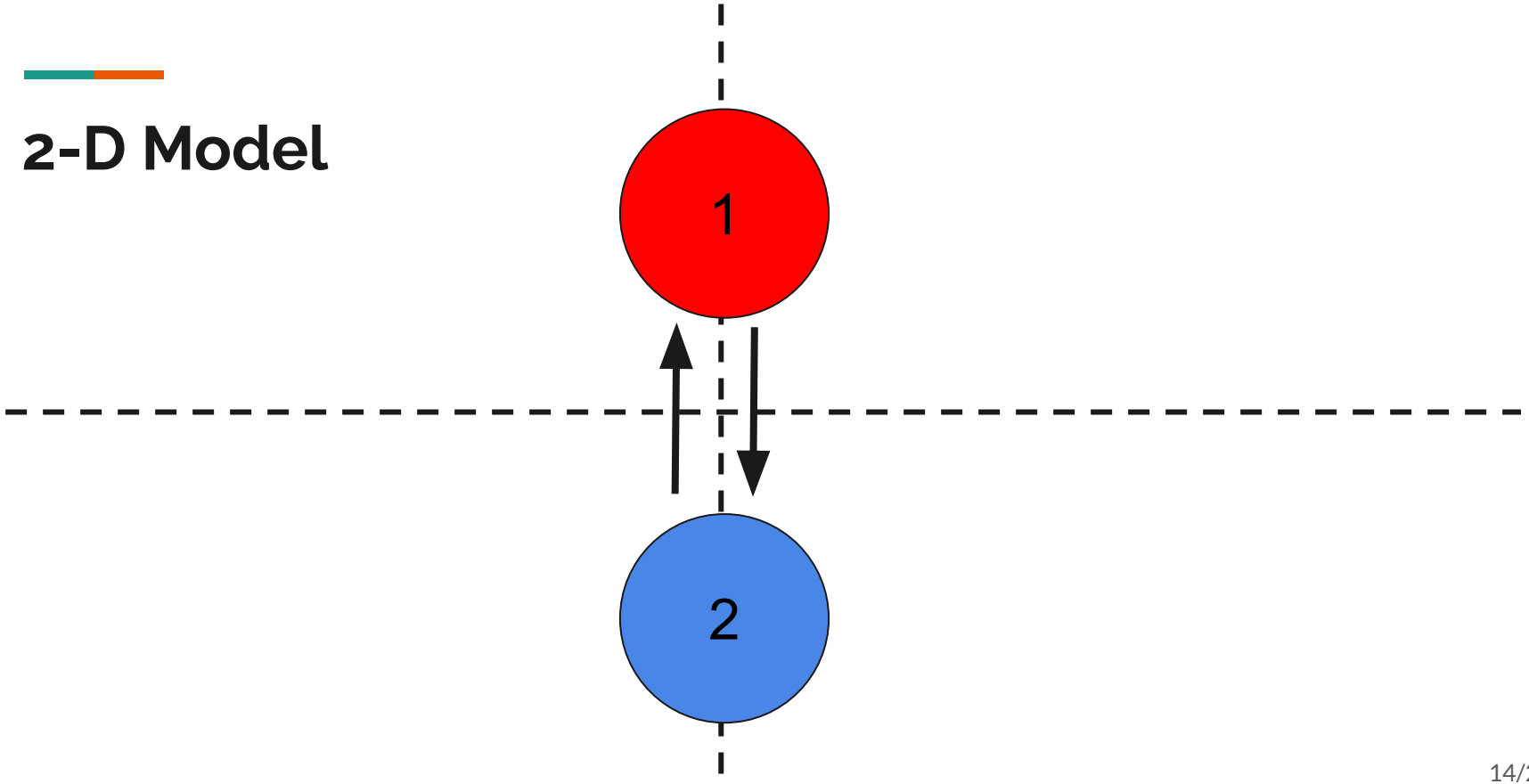
Proof Strategy: Convergence

1. $\Gamma \vdash \exists n\ J(n)$
2. $n \leq 0, J(n) \vdash P$
3. $n>0, J(n) \vdash \langle a \rangle J(n-1)$

- n = “distance”
- $\exists t\ vt=1$



2-D Model



Intersection

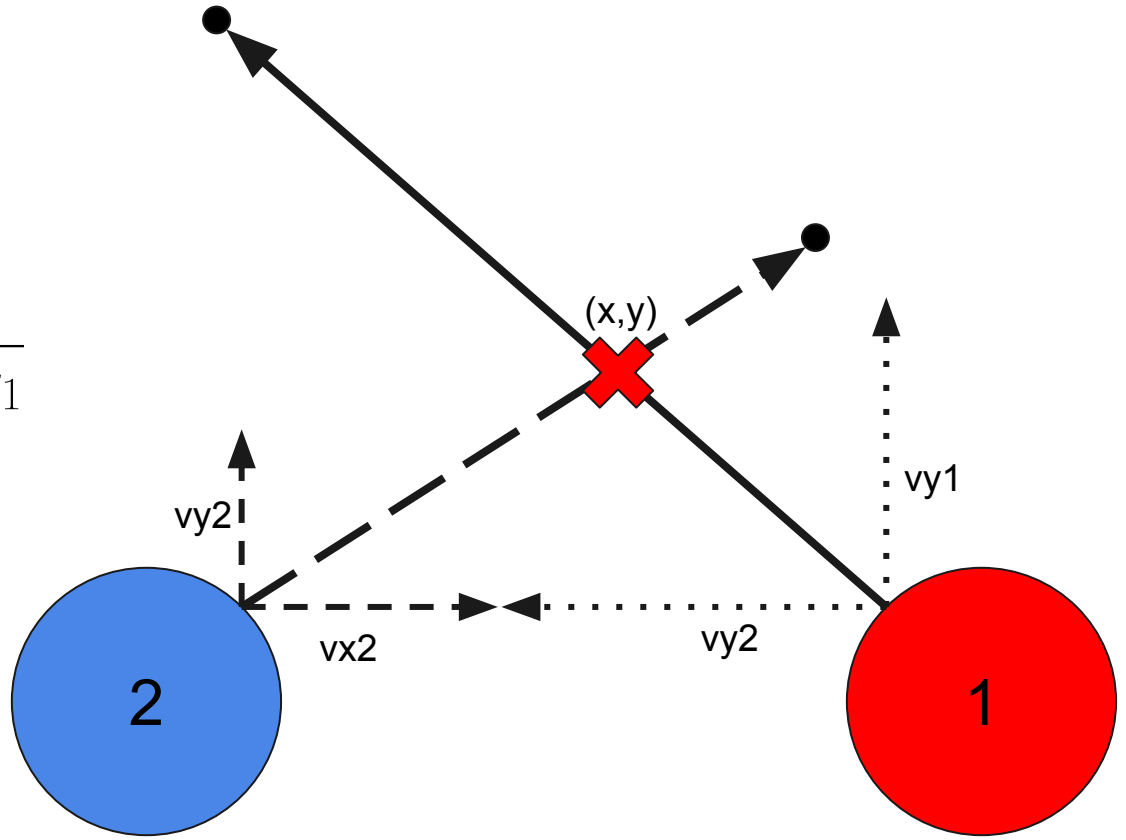
$$\frac{x_1 - x_2}{vx_2 - vx_1} = \frac{y_1 - y_2}{vy_2 - vy_1}$$

$$t > 0$$

$$(x_1 - x_2) * (vx_2 - vx_1) > 0$$

$$\vee$$

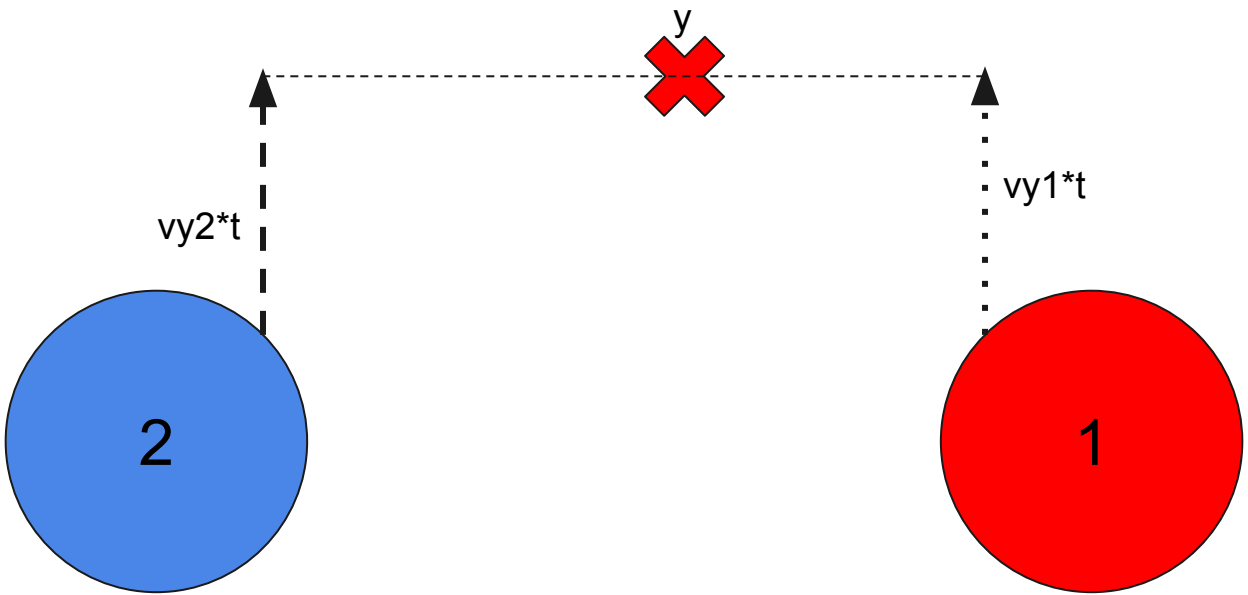
$$(y_1 - y_2) * (vy_2 - vx_1) > 0$$



$$y_1 + (vy_1 * t) = y$$

$$y_2 + (vy_2 * t) = y$$

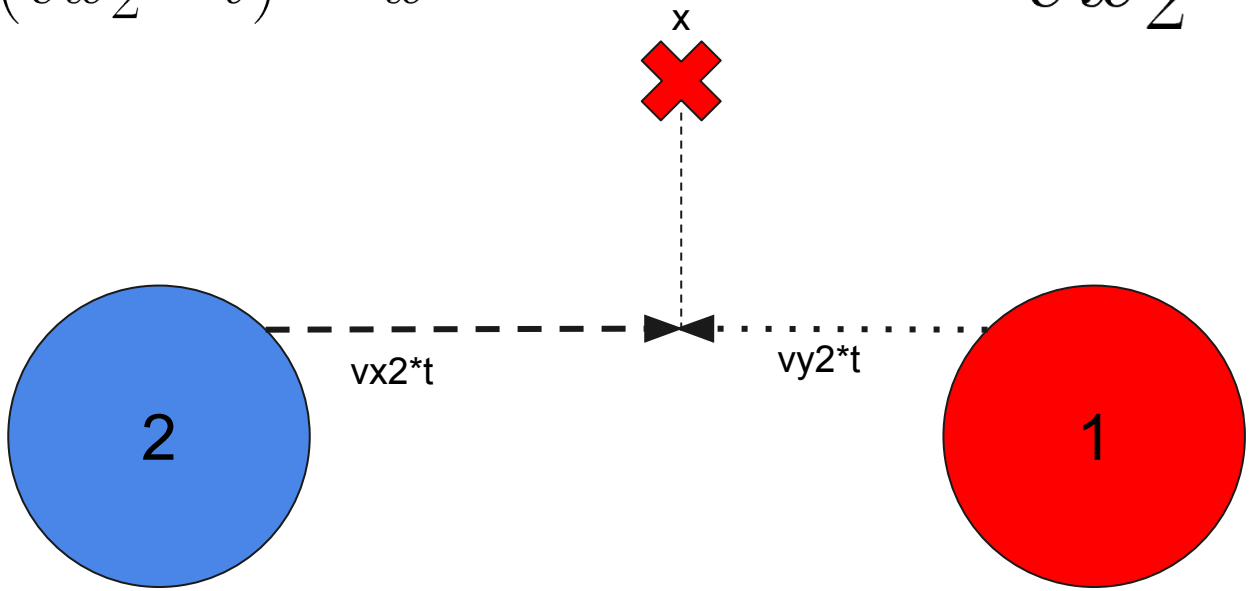
$$t = \frac{y_1 - y_2}{vy_2 - vy_1}$$

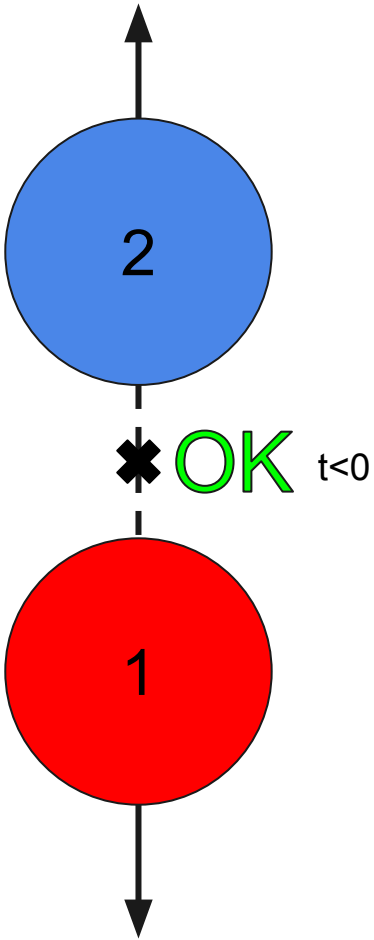


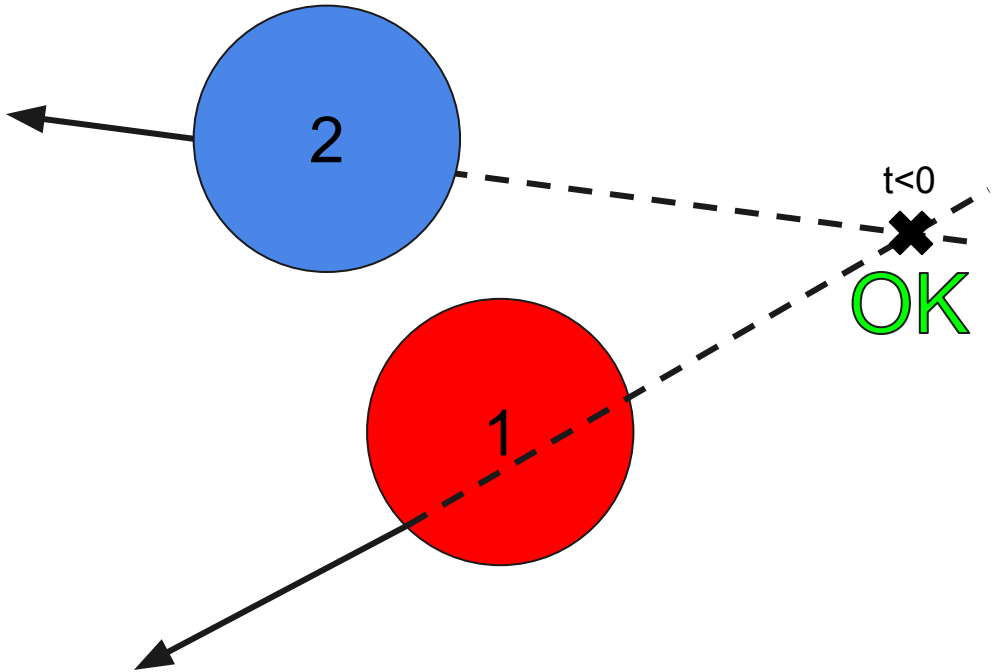
$$x_1 + (vx_1 * t) = x$$

$$x_2 + (vx_2 * t) = x$$

$$t = \frac{x_1 - x_2}{vx_2 - vx_1}$$




2
OK
t<0
1


2
t<0
OK
1

Proving a Collision-Free Controller

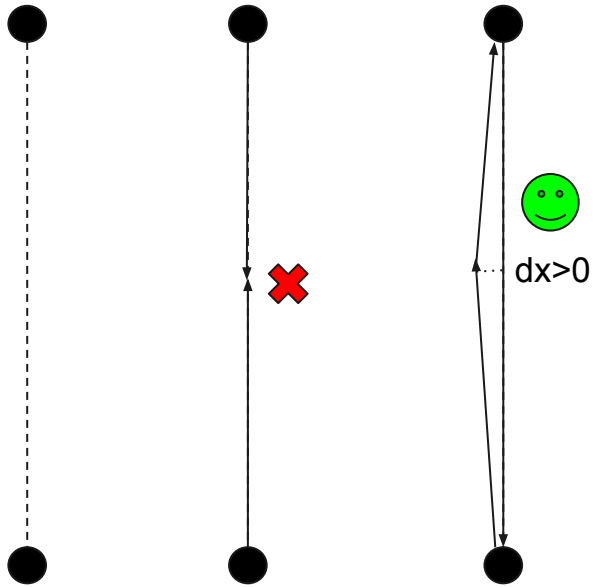
- Invariants
  - Dancers strictly moving away from each other
  - Intersection equations remain unequal
  - Time of intersection grows increasingly distant
- Cuts
  - Position of node 2 in relation to node 1

$$T < \frac{starty1 - starty2}{vmax}$$

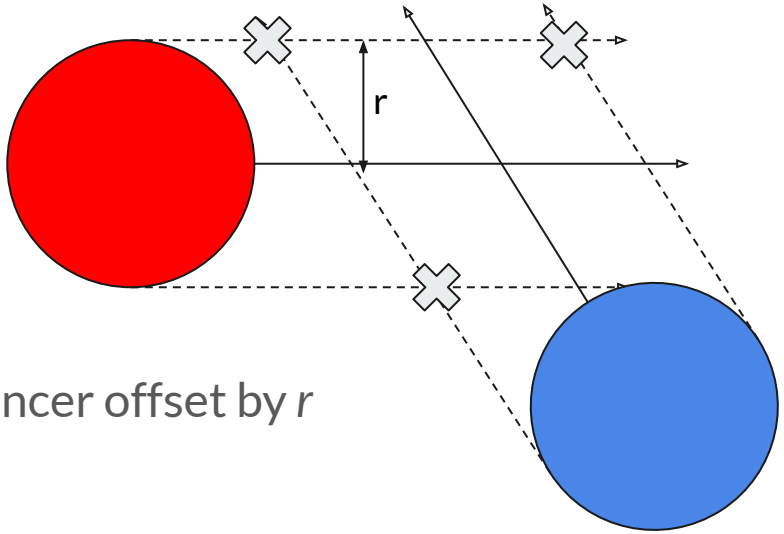
$$T > \frac{starty1 - starty2}{vmax}$$

Proving a Lower Bound on T in 2-D

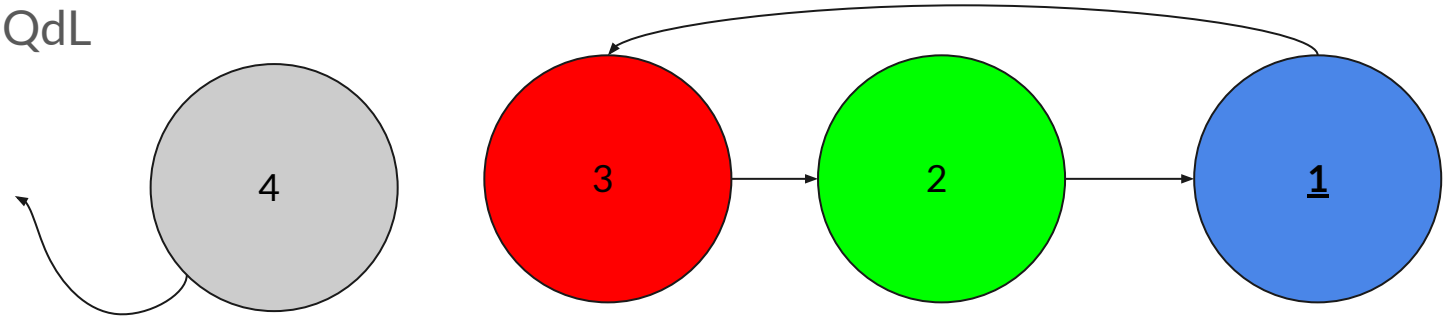
- Similar to 1-D
- Box notation: ∀ *t<T* unreachable
  - Loop invariants: *y1>starty2*, *y2<starty1*, *y2<=starty2+t*vmax*
- Diamond notation: ∃ *t≥T* reachable
  - Iterate
  - Vector magnitude: *vmax*



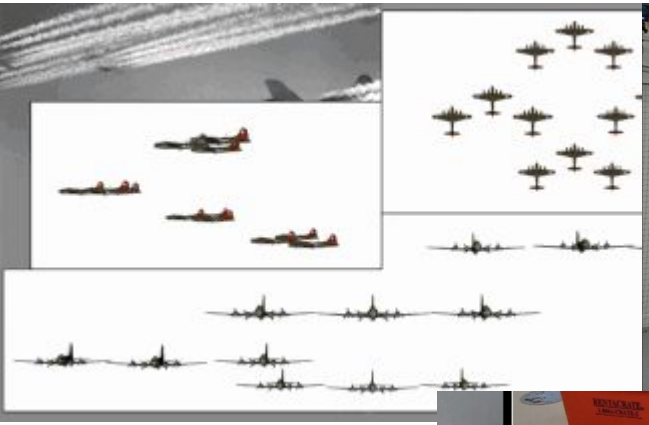
Possible Extensions

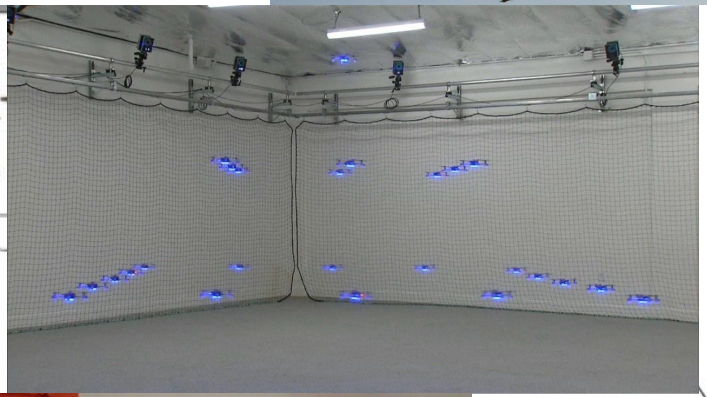


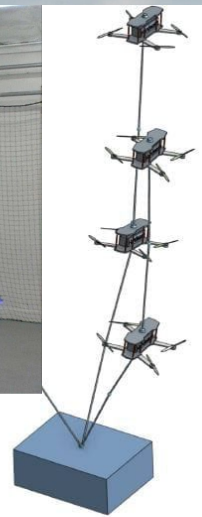
- Radial buffer
  - Generates two rays per dancer offset by *r*
- Adding more dancers
- Optimizing dancer labeling
- QdL

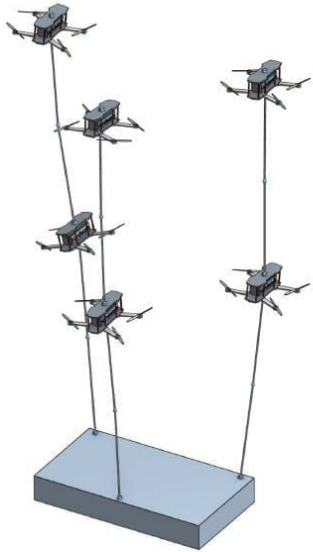


Outside Applications









Conclusion

- Proved properties:
  - Dancer can reach point on line within given time bound
  - A safe multi-agent 2D controller
  - Dancers can switch positions on an intersecting path
- Conjecture with proof strategy:
  - Dancers can switch positions within given time bound
  - Additional safety properties