Modeling of K-Pop Dance Choreography as a Synchronized Multi-Agent Hybrid System

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Background and Motivation

- Synchronization of autonomous agents is hard
- Spatial and temporal constraints
- Stepping stone for industrial applications
Background
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Modeling Choices

- Event triggered
- Point intersection
- Synchronous motion
- Velocity vector-based control
- $|v_x|, |v_y| \leq v_{max}$
- Velocity changes freely - no acceleration
Linear Model
Proving a Lower Bound on $T$ in 1-D

- Diamond notation: $\exists t \geq T$ reachable
  - Iterate
  - Assign $v := v_{\text{max}}$ and $t := T$
- Box notation: $\forall t < T$ unreachable
  - Loop invariant: $\text{pos} \leq A + (t \cdot v_{\text{max}})$

$$T = \frac{B - A}{v_{\text{max}}}$$
2-D Model
Proving Spatial Reachability with 2D Controller

- Diamond proof strategy: convergence
- Choose vector in direction of target
- Generalization of \{a\}^* to any point in space
Assigning Velocity

\[
\frac{\max(dx, dy)}{\min(dx, dy)} = \frac{v_{\text{max}}}{?}
\]
Proof Strategy: Convergence

1. $\Gamma \vdash \exists n J(n)$
2. $n \leq 0, J(n) \vdash P$
3. $n > 0, J(n) \vdash \langle a \rangle J(n-1)$

- $n = \text{“distance”}$
- $\exists t\, vt = 1$
2-D Model
Intersection

\[
\frac{x_1 - x_2}{vx_2 - vx_1} = \frac{y_1 - y_2}{vy_2 - vy_1}
\]

\( t > 0 \)

\[
(x_1 - x_2) \times (vx_2 - vx_1) > 0
\]

\( \lor \)

\[
(y_1 - y_2) \times (vy_2 - vx_1) > 0
\]
\[ y_1 + (vy_1 \times t) = y \]
\[ y_2 + (vy_2 \times t) = y \]

\[ t = \frac{y_1 - y_2}{vy_2 - vy_1} \]
\[ x_1 + (vx_1 \times t) = x \]
\[ x_2 + (vx_2 \times t) = x \]

\[ t = \frac{x_1 - x_2}{vx_2 - vx_1} \]
Proving a Collision-Free Controller

- **Invariants**
  - Dancers strictly moving away from each other
  - Intersection equations remain unequal
  - Time of intersection grows increasingly distant

- **Cuts**
  - Position of node 2 in relation to node 1
Proving a Lower Bound on $T$ in 2-D

- Similar to 1-D
- Box notation: $\forall \ t < T$ unreachable
  - Loop invariants: $y_1 > starty_2$, $y_2 < starty_1$, $y_2 <= starty_2 + t \cdot vmax$
- Diamond notation: $\exists \ t \geq T$ reachable
  - Iterate
  - Vector magnitude: $vmax$

$$T < \frac{starty_1 - starty_2}{vmax}$$

$$T > \frac{starty_1 - starty_2}{vmax}$$
Possible Extensions

- Radial buffer
  - Generates two rays per dancer offset by $r$
- Adding more dancers
- Optimizing dancer labeling
- QdL
Outside Applications
Conclusion

- Proved properties:
  - Dancer can reach point on line within given time bound
  - A safe multi-agent 2D controller
  - Dancers can switch positions on an intersecting path

- Conjecture with proof strategy:
  - Dancers can switch positions within given time bound
  - Additional safety properties