Nondeterminstic Discrete Stochastic Differential Dynamic Logic

Samuel Kim

December 10, 2019

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- Powerful dynamic logics:
  - Stochastic Differential Dynamic Logic (Sd$\mathcal{L}$)
  - Nondeterminstic Stochastic Differential Dynamic Logic (NSd$\mathcal{L}$)
- Problem: no implementations!

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- Compiler implementation

NDSd$\mathcal{L}$ Syntax

$$\text{Term } e_1, e_2 ::= x \mid c \mid e_1 + e_2 \mid e_1 \cdot e_2$$

$$\text{Formula } P, Q ::= e_1 \geq e_2 \mid e_1 = e_2 \mid \neg P \mid P \odot Q \mid \forall x P \mid \exists x P \mid [\alpha]P \mid \langle\alpha\rangle P$$
$$\mid \langle\!\langle \alpha \rangle\!\rangle P \leq p \qquad (\odot \in \{\wedge, \vee, \rightarrow, \leftrightarrow\})$$

$$\text{Program } \alpha, \beta ::= x := e \mid x := * \mid x := \{p_1 : e_1, \ldots, p_n : e_n\} \mid ?P \mid \alpha; \beta$$
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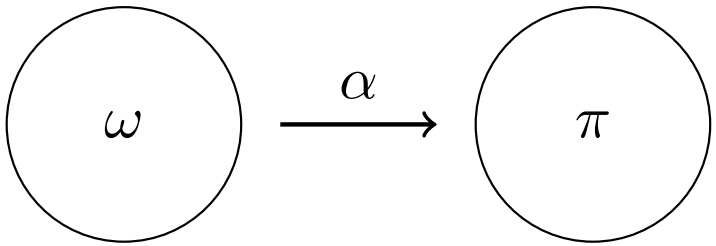
- $\alpha^{*:p}$: Probabilistic loop
- $\bigoplus_{i=1}^{n} p_i \alpha_i$: Probabilistic choice

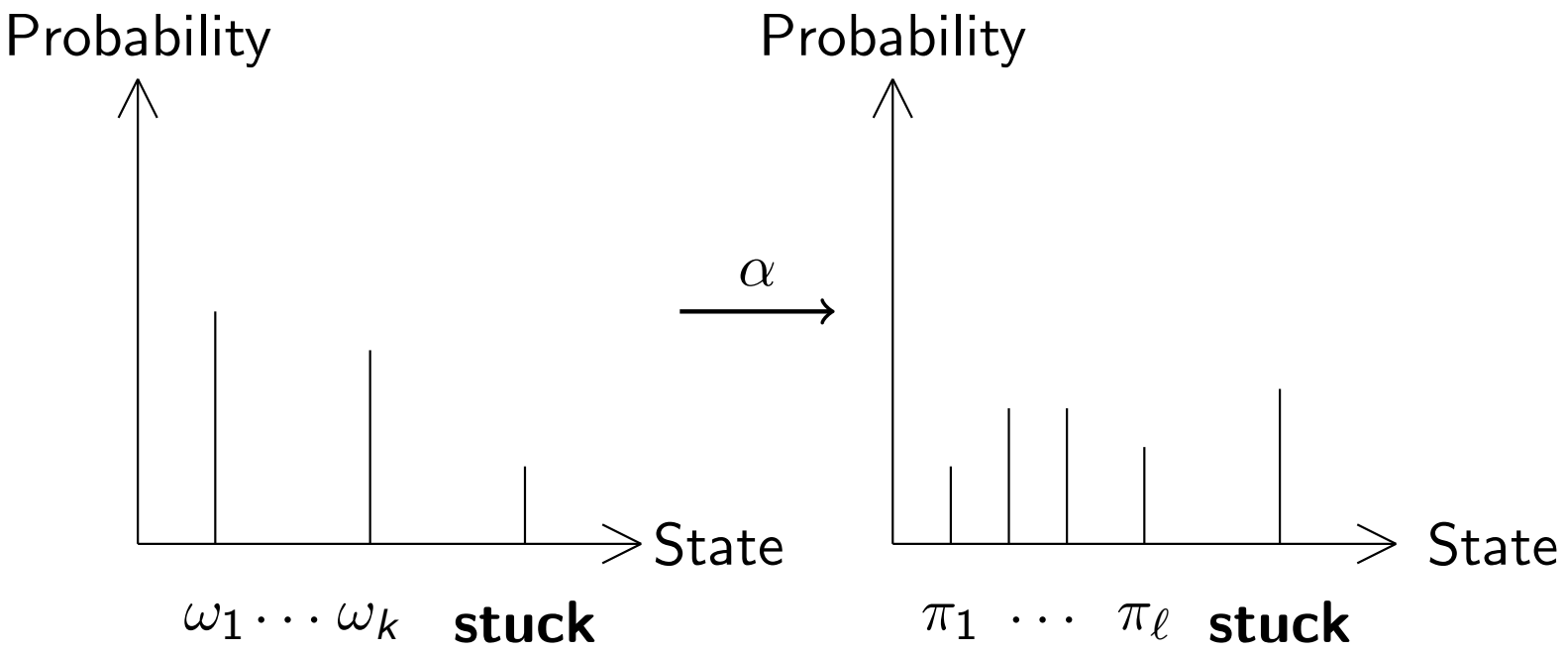
Program Semantics

- $\mathrm{d}\mathcal{L}$: transition between states



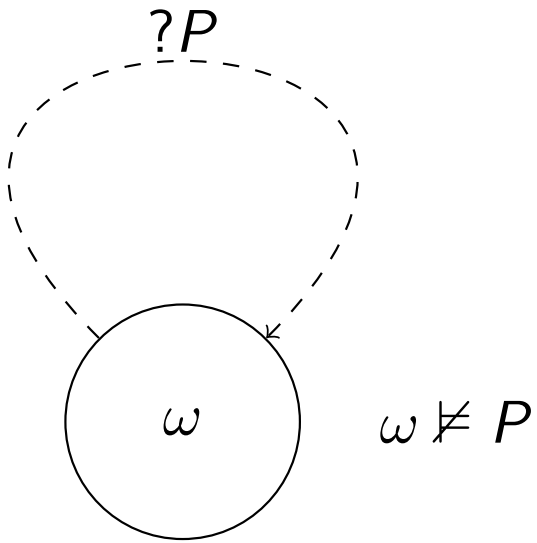
Program Semantics

- d$\mathcal{L}$: transition between states
- NDSd$\mathcal{L}$: transition between probability distributions of states



stuck?

- d$\mathcal{L}$: Failed test: no transition



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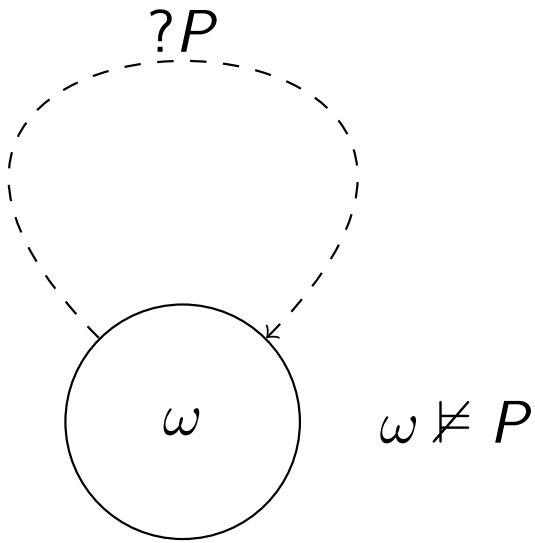

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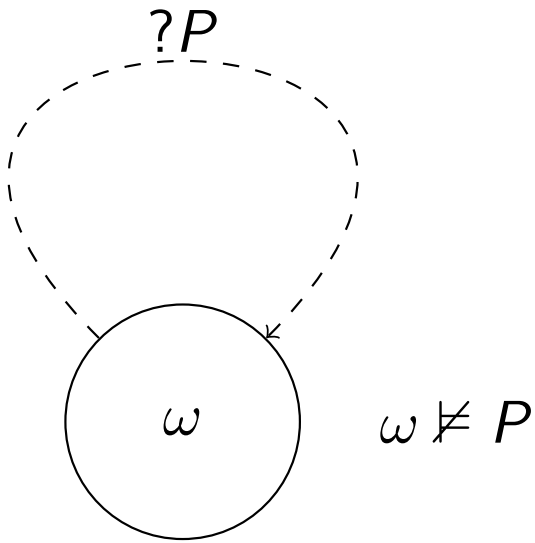

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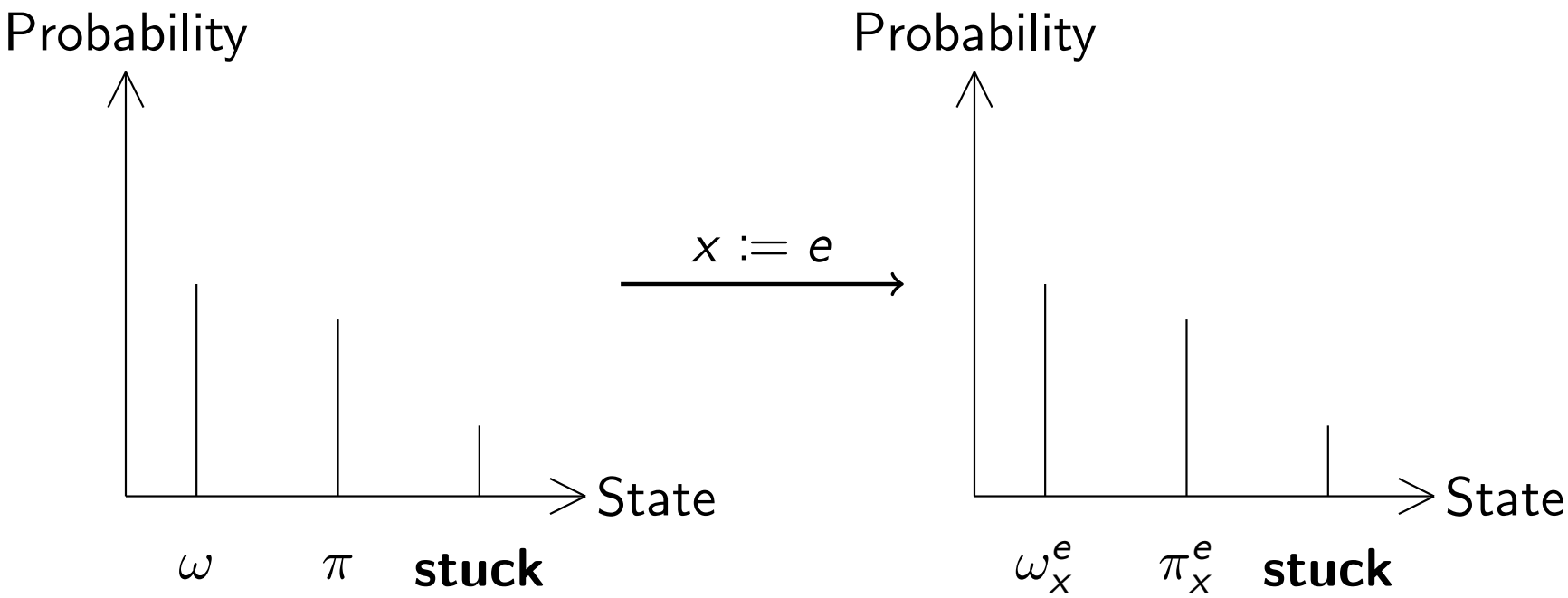
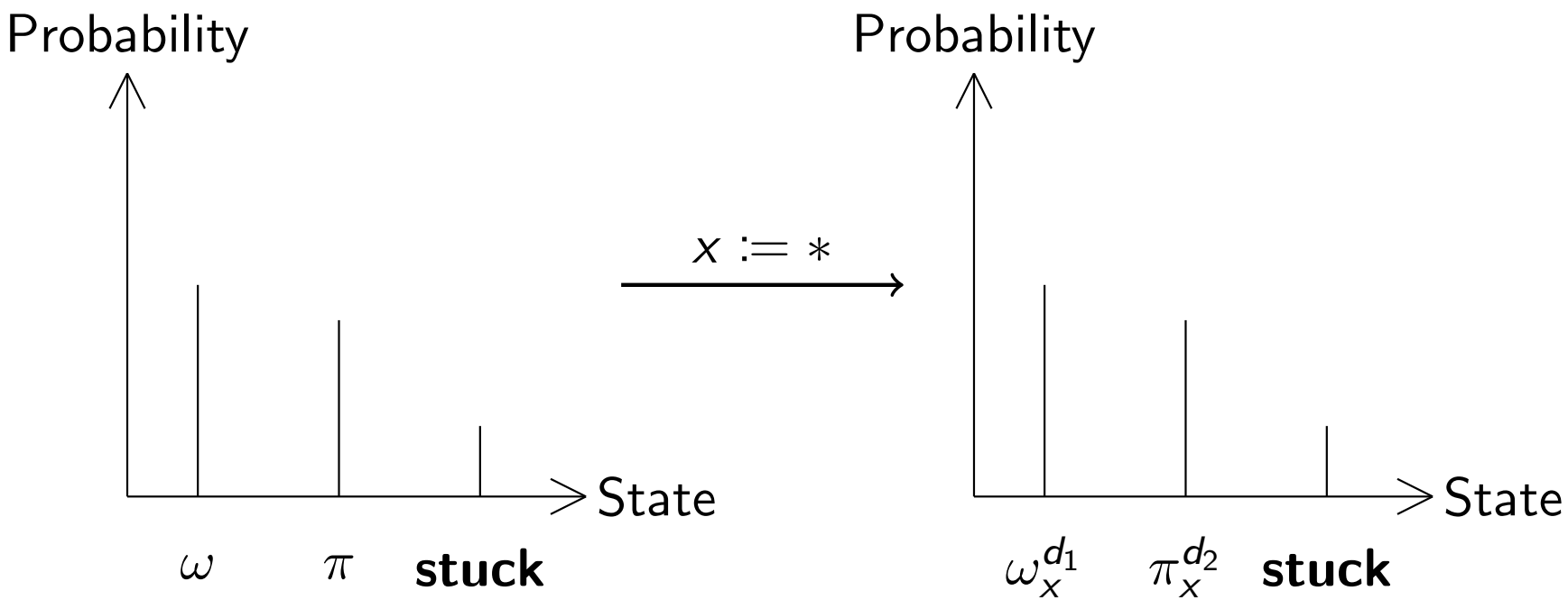
- **stuck** is not a state!

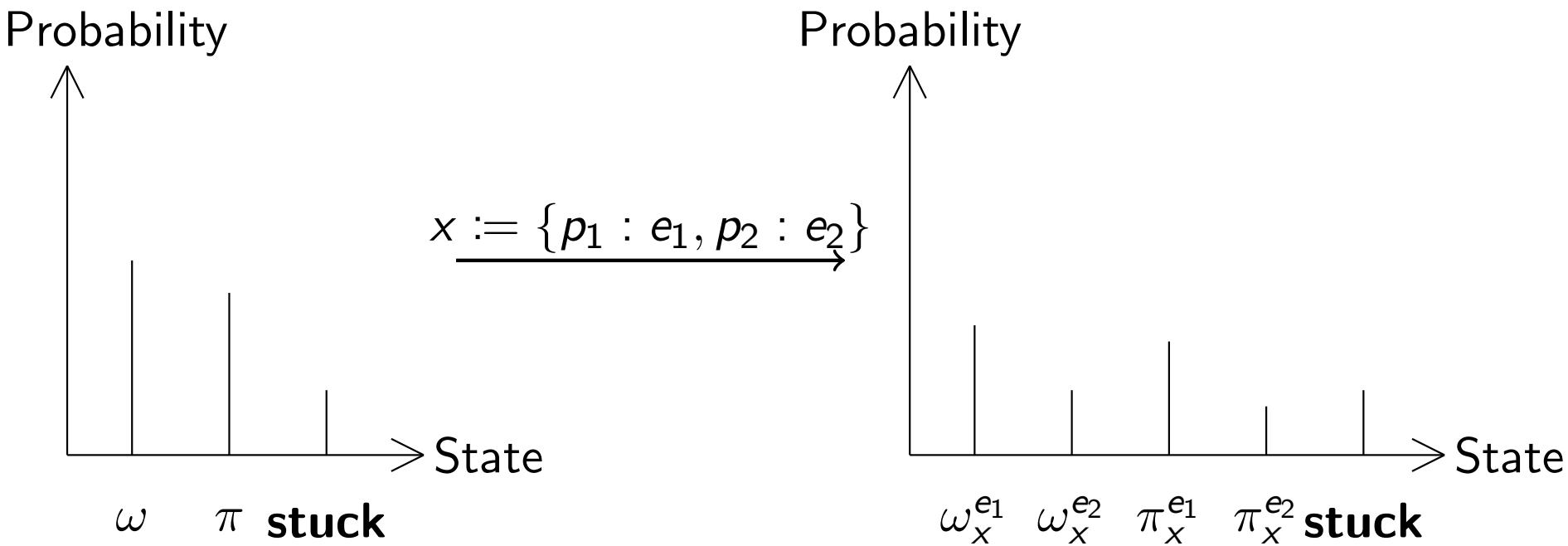
Program Semantics: Assignment



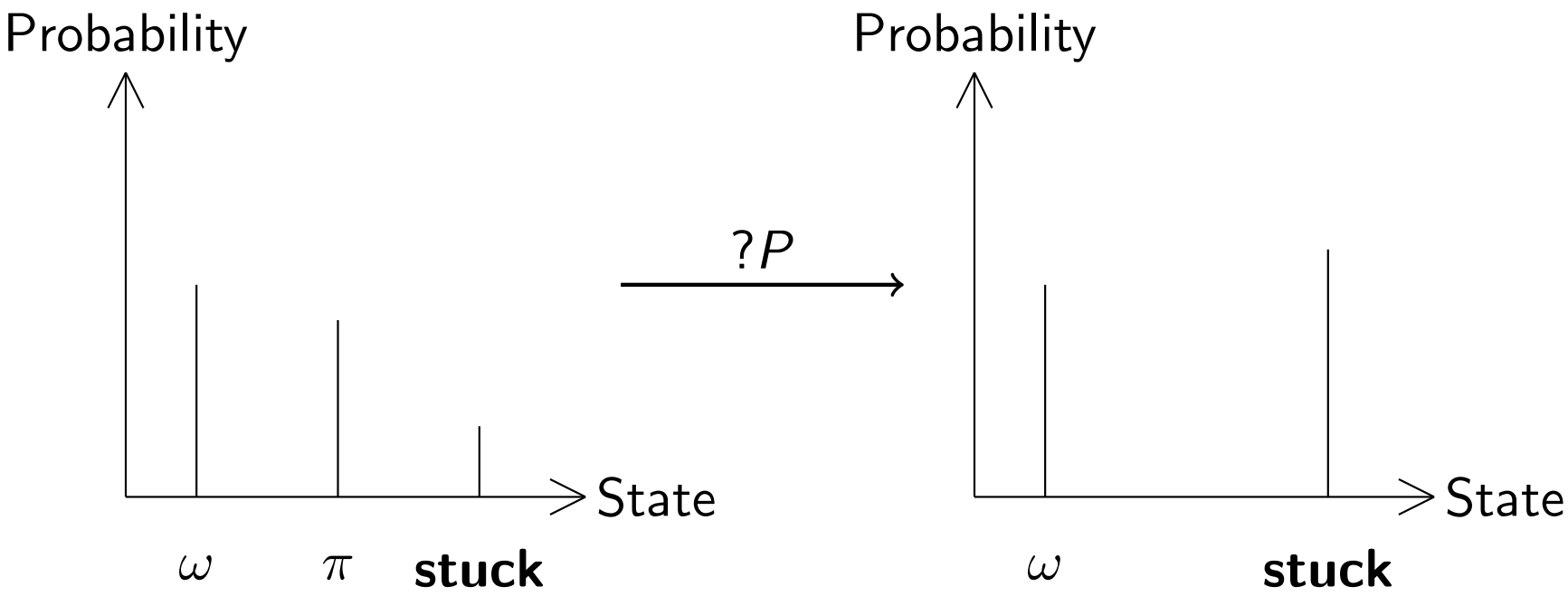
Program Semantics: Nondeterminstic Assignment



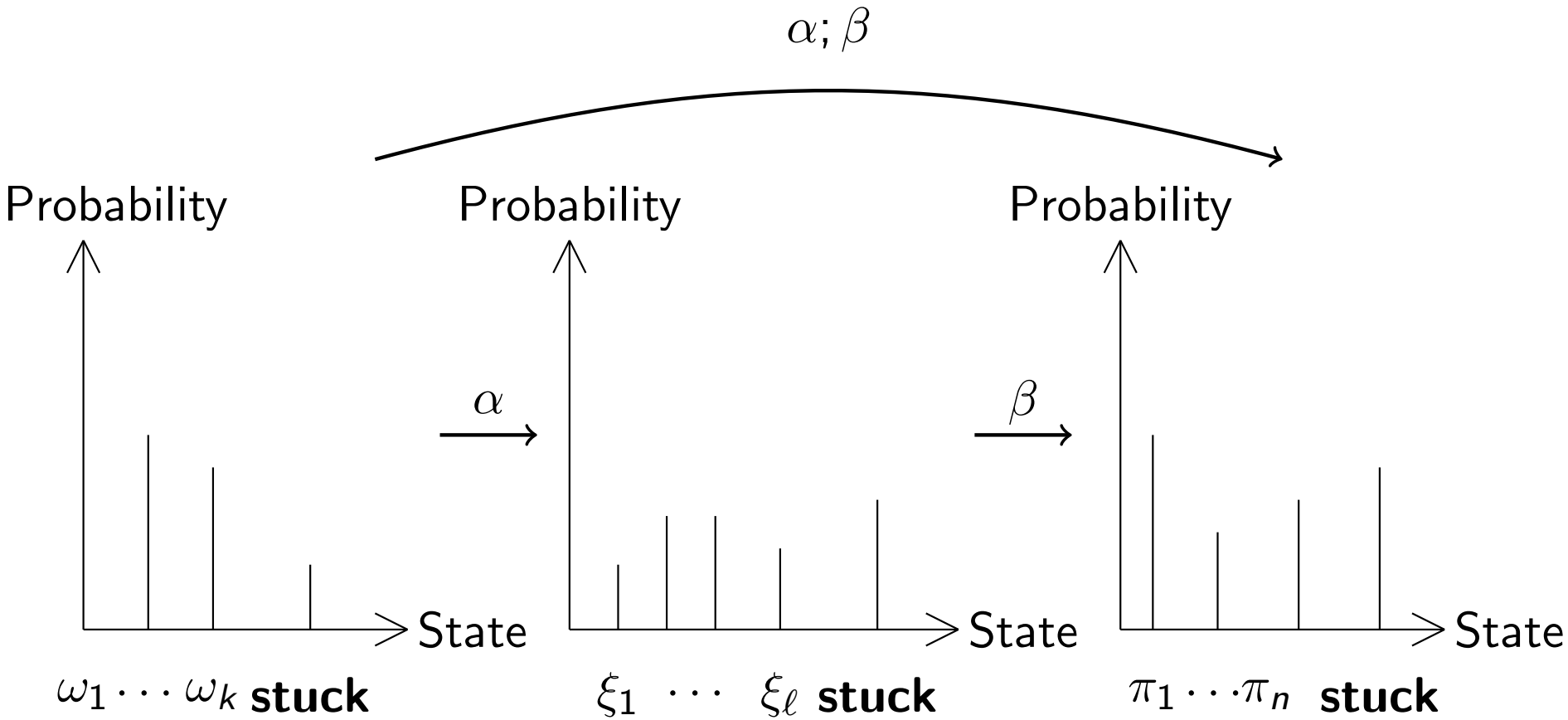
Program Semantics: Probabilistic Assignment



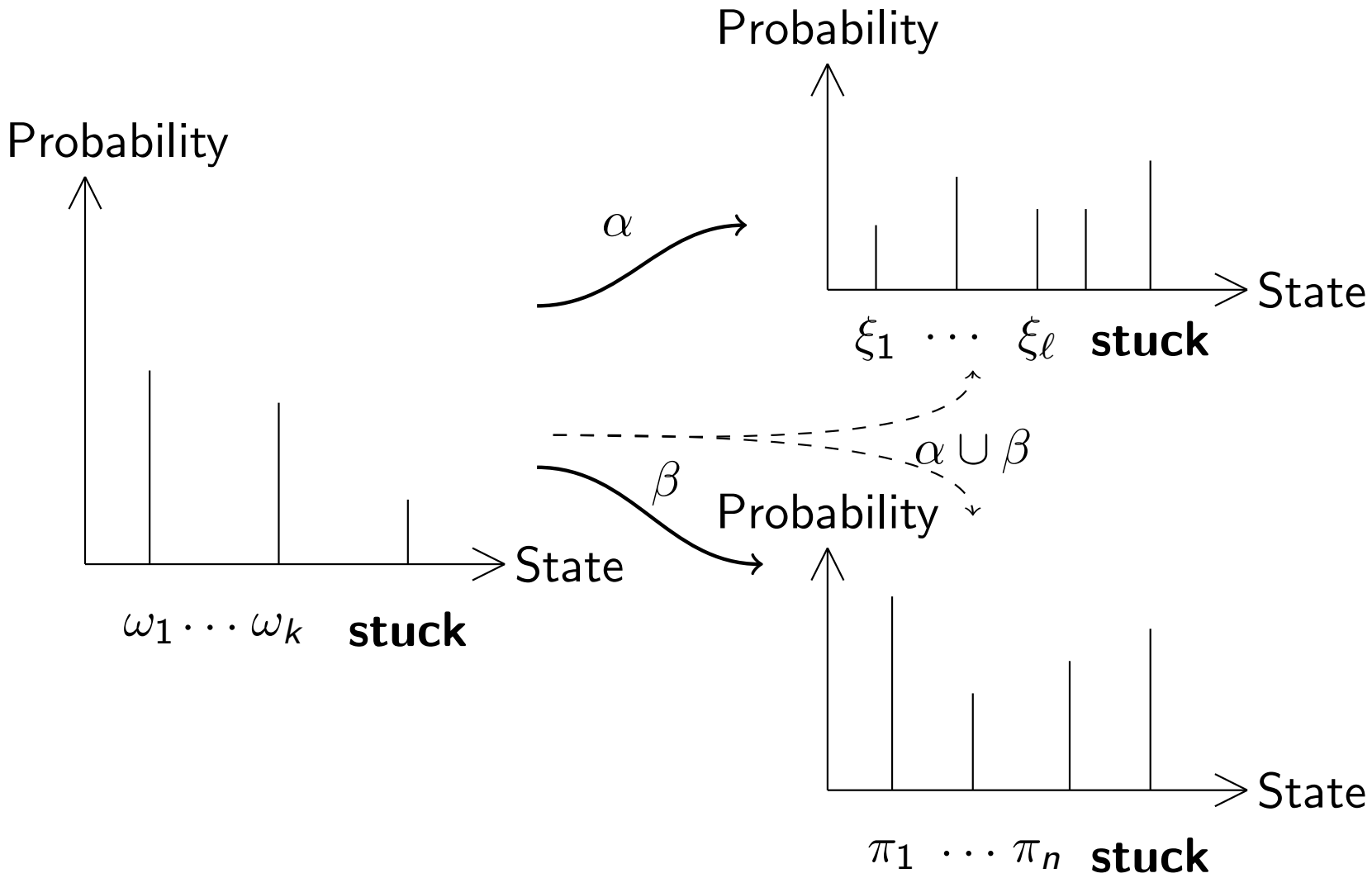
Program Semantics: Test



Program Semantics: Sequential Composition



Program Semantics: Nondeterministic Choice



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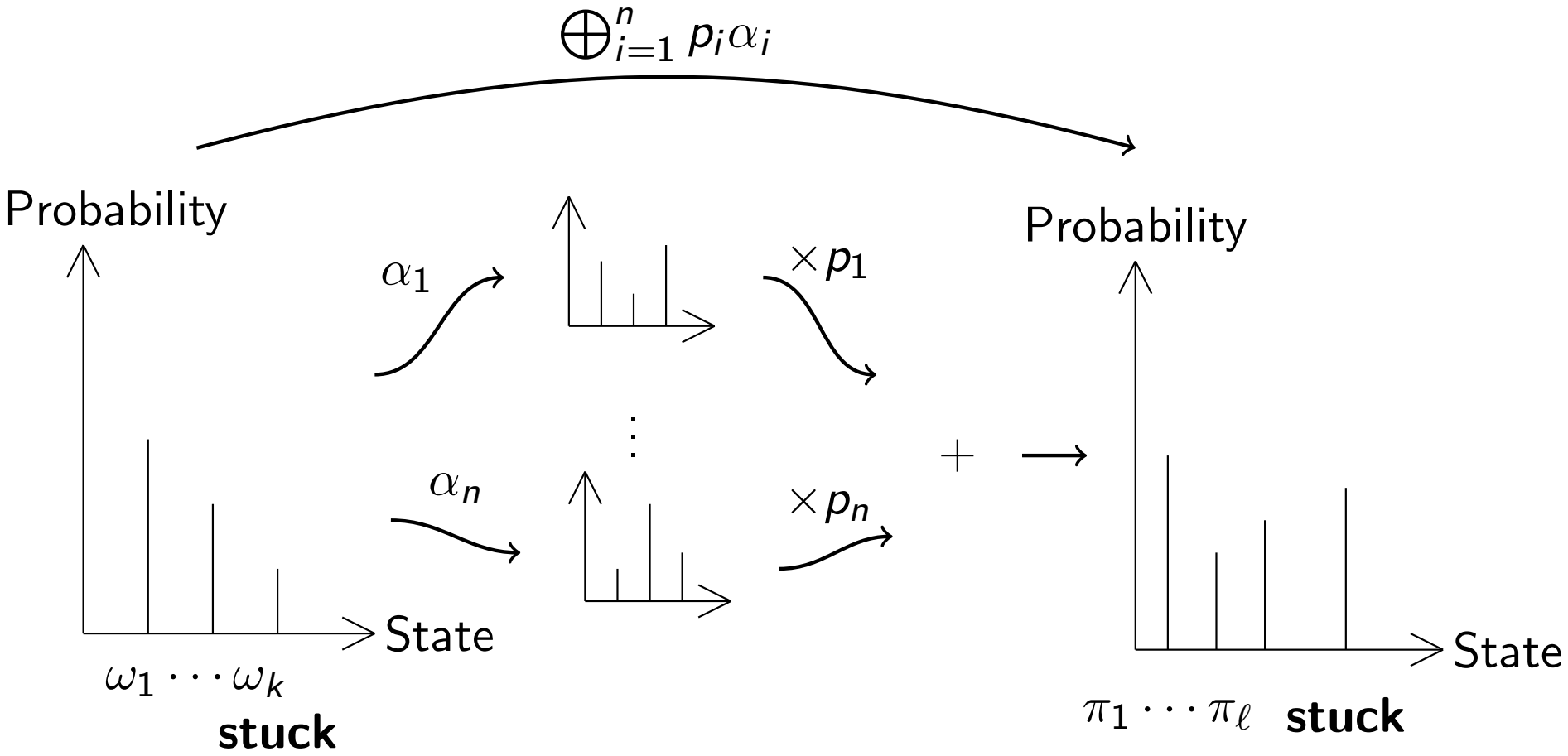
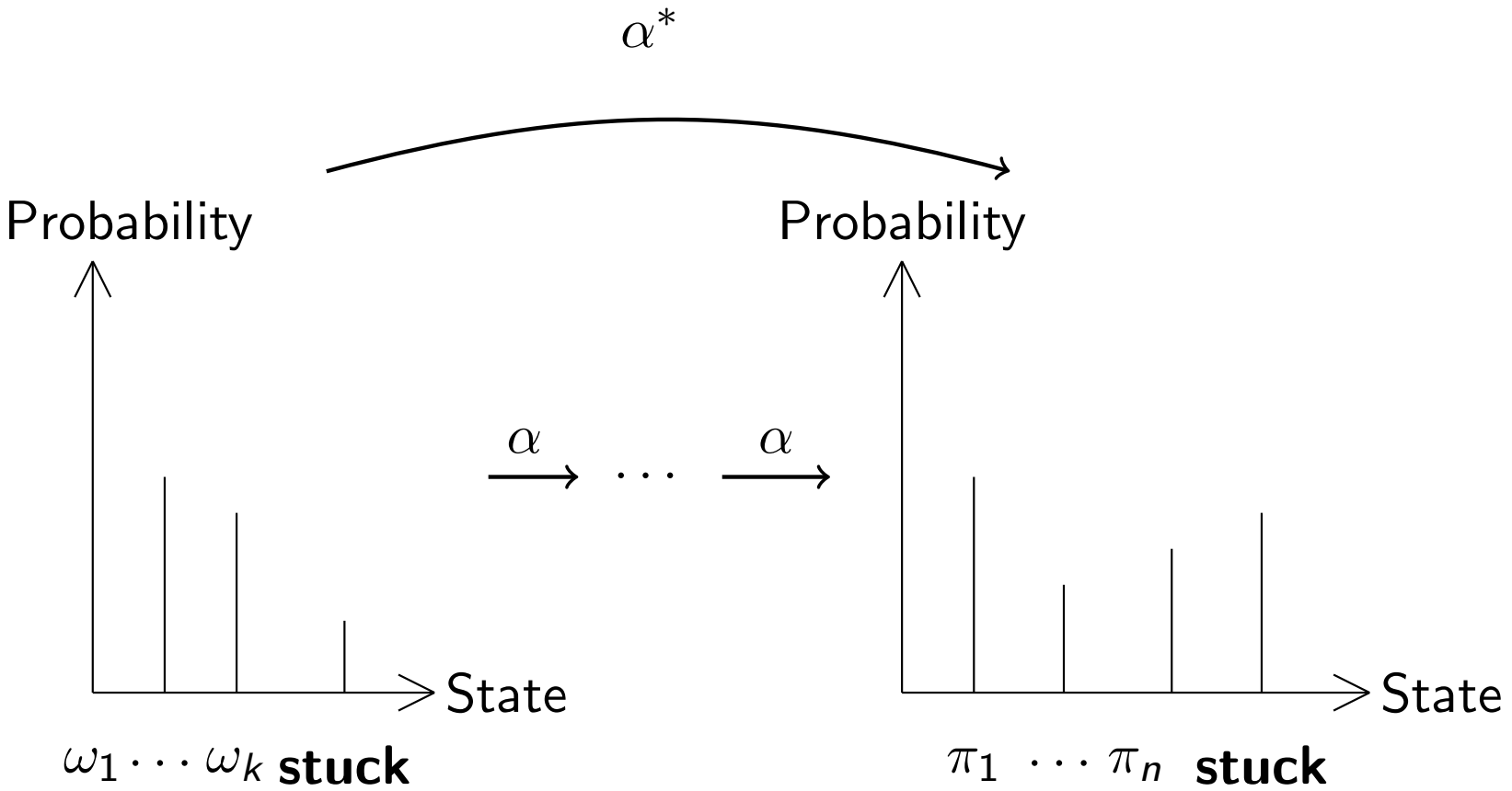
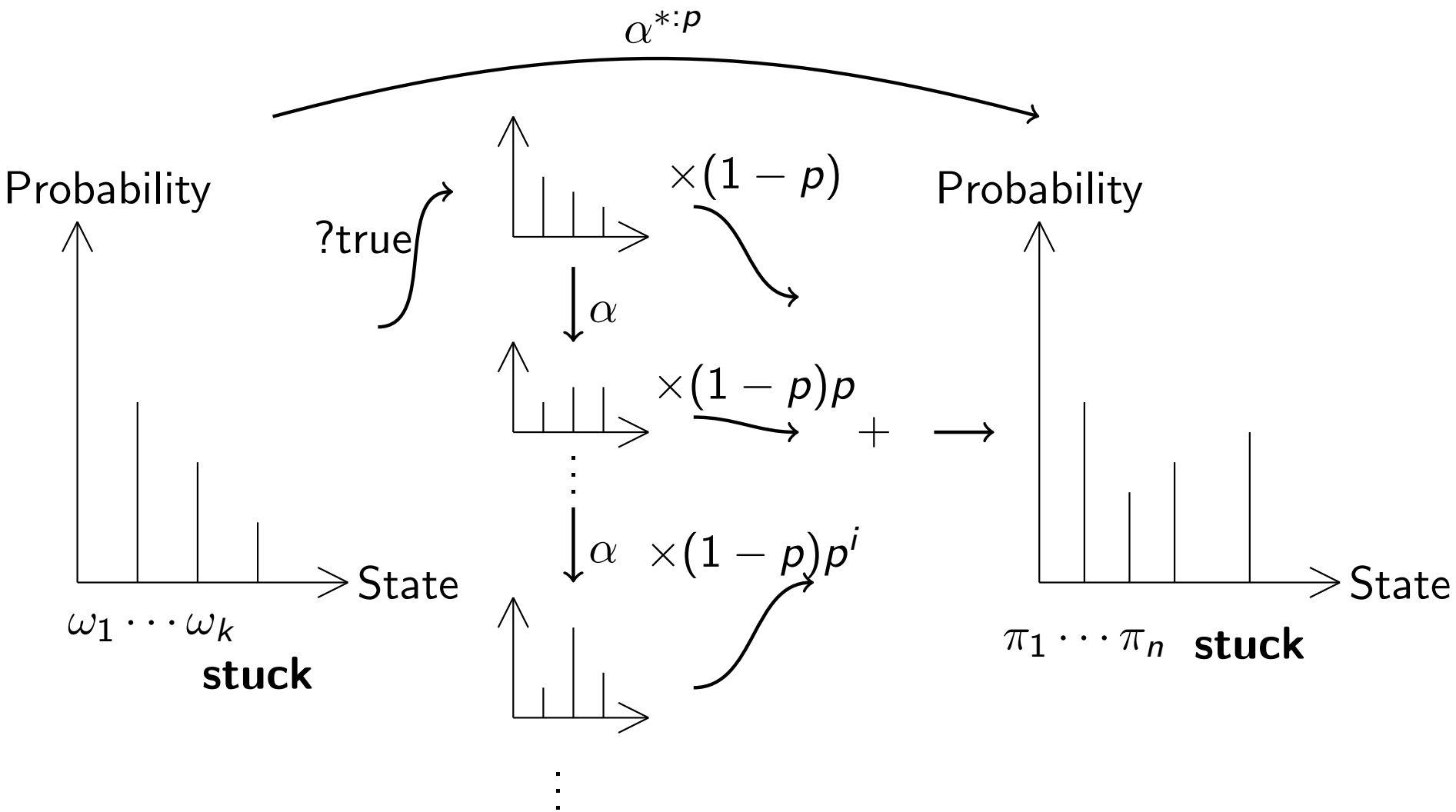


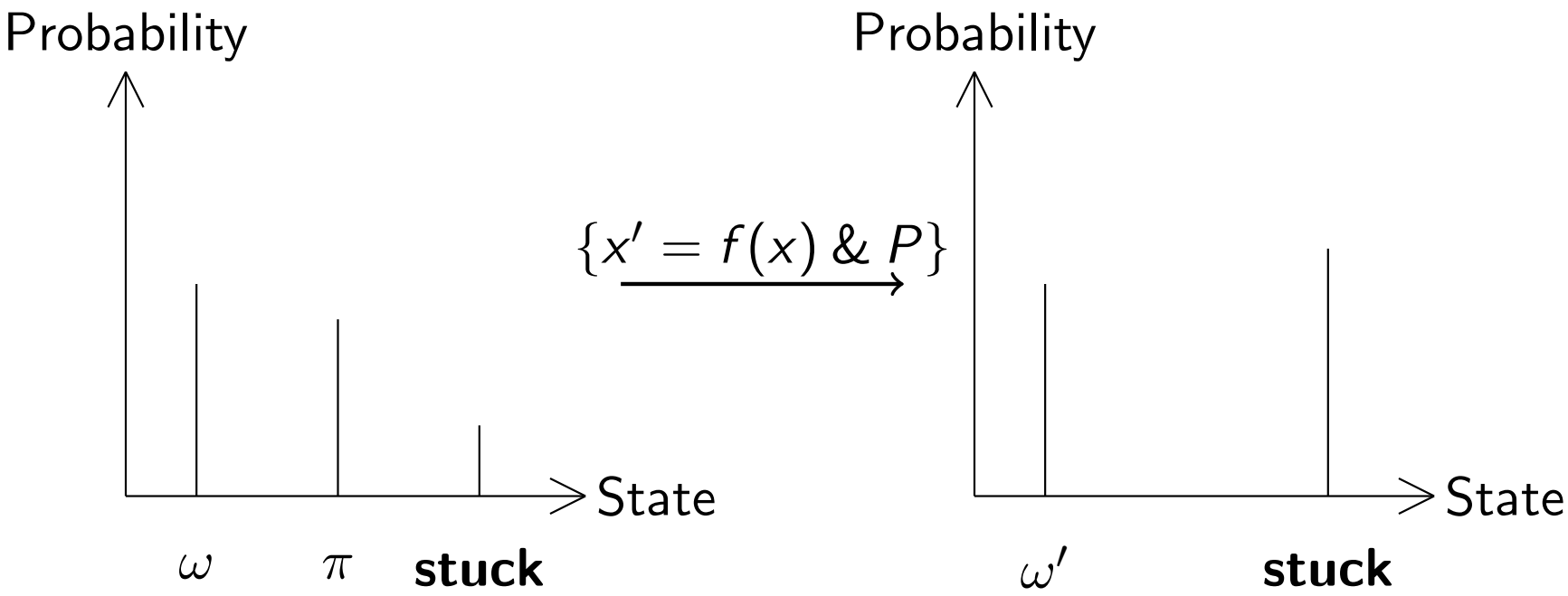
Program Semantics: Nondeterministic Loop



Program Semantics: Probabilistic Loop



Program Semantics: ODE



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$$\omega \in [\![\langle\!|\alpha|\!\rangle P \leq p]\!] \text{ iff } \omega \in [\![\sigma(\rho(\alpha, P), p)]\!]$$

Probability Bound

$$\sigma(\rho(\alpha, P), p)$$

- $\mathcal{I}$: indicator

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- σ: finalize ρ

$$\sigma((\{P_1, \ldots, P_n\}, e), p) \equiv \left(\bigwedge_{i=1}^{n} P_i \right) \rightarrow (e \leq p)$$

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Loop Probability Bounds

$$\rho(\beta; \alpha^*, P) = \left(\bigcup_{i=0}^{\infty} X_i, \sup_i e_i \right) \text{ where } (X_i, e_i) = \rho(\beta; \alpha^i, P) \text{ for } i \in \mathbb{N}$$

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Theorem (Probabilistic loop unrolling)

$$[\![\alpha^{*:p}]\!] = [\![(1-p)\{?\mathit{true}\} \oplus p\{\alpha; \alpha^{*:p}\}]\!]$$

Translation to d$\mathcal{L}$: Formulas

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Theorems

Theorem (Soundness of Translation)

Let $P \overset{\mathrm{d}\mathcal{L}}{\rightsquigarrow} \overline{P}$. *$P$ is valid in NDS*$\mathrm{d}\mathcal{L}$ *iff* $\overline{P}$ *is valid in* $\mathrm{d}\mathcal{L}$.

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If $\omega \vDash \langle\!\langle \alpha \rangle\!\rangle P \leq p$ *then* $\forall (Det(\omega), \nu) \in [\![\alpha]\!]$, $\mathbf{P}\left[\pi \vDash P\right] \leq p$ *for* $\pi \sim \nu$.

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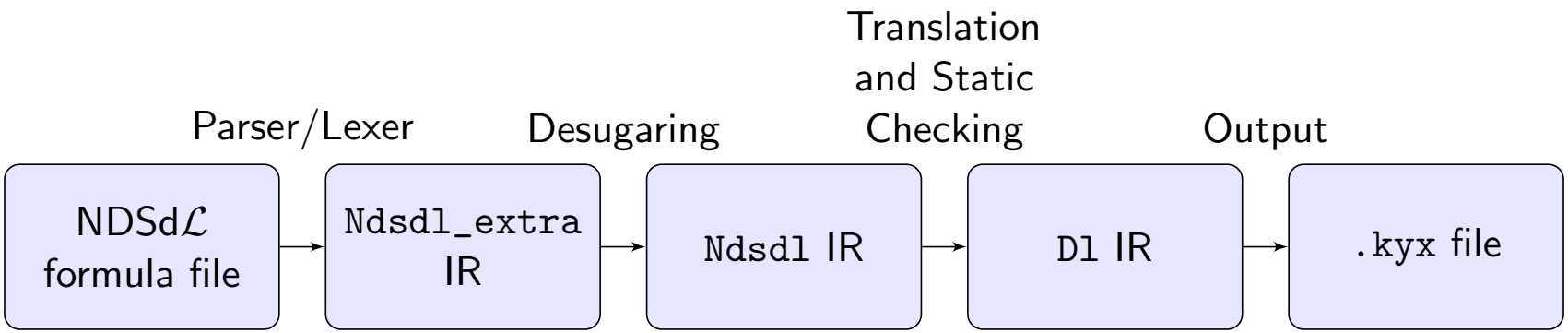
Theorem (Conservative Extension)

Every valid $\mathrm{d}\mathcal{L}$ *formula is also a valid NDS*$\mathrm{d}\mathcal{L}$ *formula.*

Compiler

- ∼850 lines of OCaml



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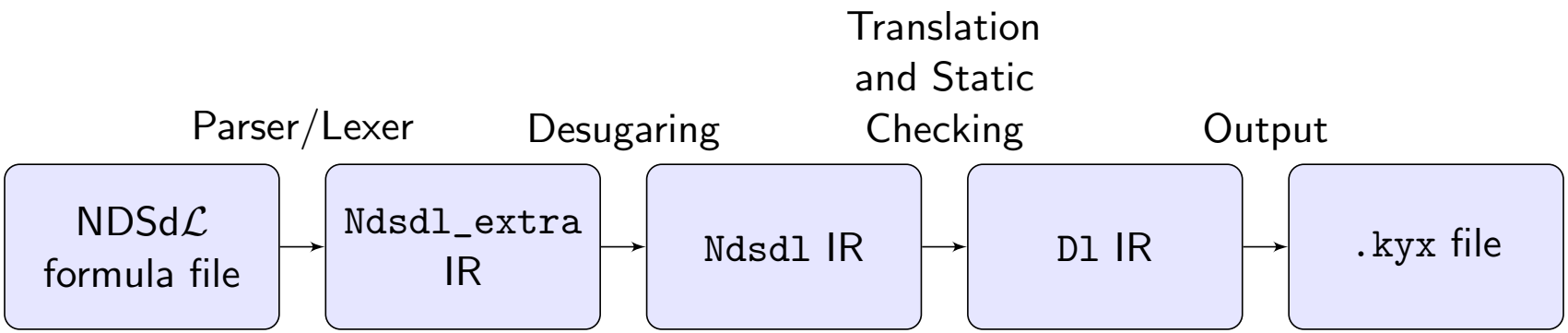

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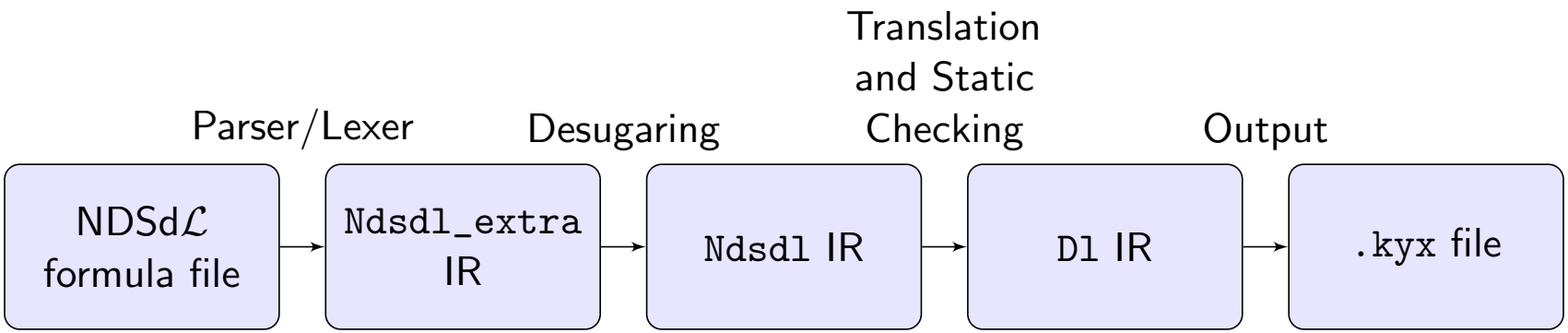

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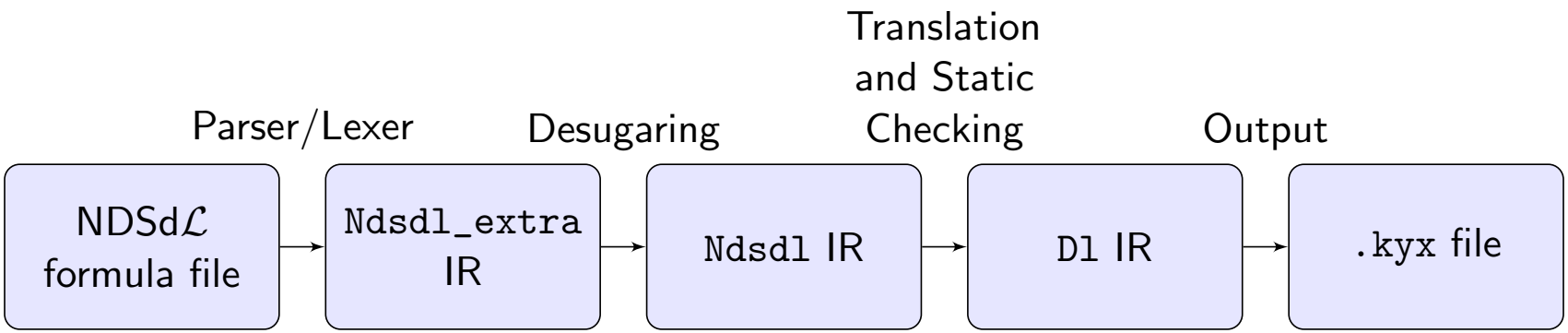
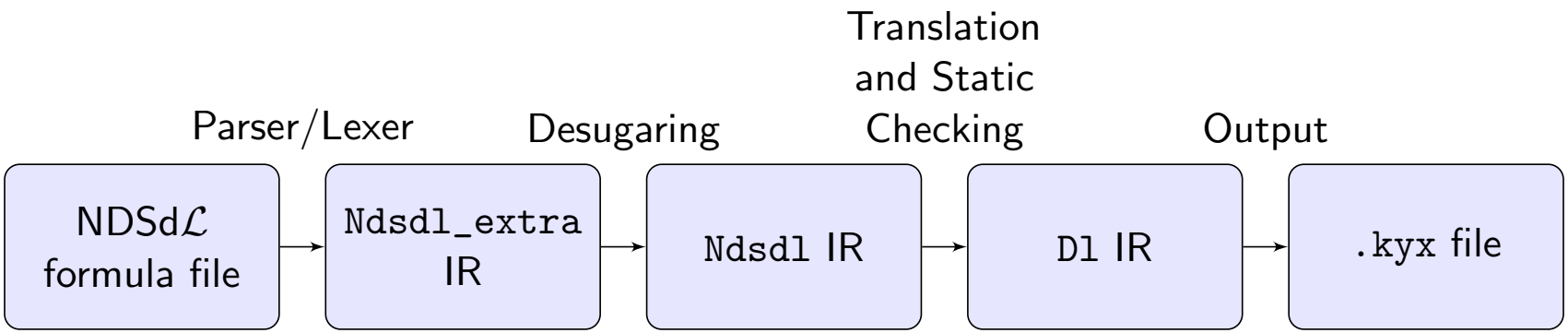
  - `x := Bernoulli(p)` $\equiv$ `x := {p:1, (1-p):0}`
  - `x := Geometric(p)` $\equiv$ `x := 1; {x := x+1}*:(1-p)`



Compiler

- ~850 lines of OCaml
- Syntactic sugar:
  - `x := Bernoulli(p)` $\equiv$ `x := {p:1, (1-p):0}`
  - `x := Geometric(p)` $\equiv$ `x := 1; {x := x+1}*:(1-p)`
  - `{a}*@unroll(n)`, `{a}*:p@unroll(n)`, `x := Geometric(p)@unroll(n)`



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Not valid!

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- Want to prove: probability of ≥ 3 trials is bounded by $1/16$

```
<| x := Geometric(3/4) |>(x>=3) <= 1/16
```

Not valid!

```
<| x := Geometric(3/4)@unroll(2) |>(x>=3) <= 1/16
```

Example (translated)

```
ProgramVariables
  Real tt_0;
  Real tt_1;
  Real tt_2;
  Real x;
End.

Problem
  ((((true) & (((<x := 1;>(<?true;>((x) >= (3)))) & ((tt_0) = (1))) | ((!(<x :=
       1;>(<?true;>((x) >= (3))))) & ((tt_0) = (0))))) & (((<x := 1;>(<x := (x)
       +(1);>(<?true;>((x) >= (3))))) & ((tt_1) = (1))) | ((!(<x := 1;>(<x := (x)
       +(1);>(<?true;>((x) >= (3)))))) & ((tt_1) = (0))))) & (((<x := 1;>(<x := (
       x)+(1);>(<x := (x)+(1);>(<{x := (x)+(1);}*>((x) >= (3)))))) & ((tt_2) =
       (1))) | ((!(<x := 1;>(<x := (x)+(1);>(<x := (x)+(1);>(<{x := (x)+(1);}*>((
       x) >= (3)))))))) & ((tt_2) = (0))))) -> ((((0)+(((1)-((1)-((3)/(4))))*(tt_0
       )))+(((1)-((3)/(4)))*(((0)+(((1)-((1)-((3)/(4))))*(tt_1)))+(((1)-((3)/(4))
       )*(tt_2))))) <= ((1)/(16)))
End.
```

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- Soundness theorems
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- Future work: continuous probability distributions

Questions?

$$\frac{\sigma(\rho(\alpha, P), p) \overset{\mathrm{d}\mathcal{L}}{\rightsquigarrow} Q}{\langle\!|\alpha|\!\rangle P \leq p \overset{\mathrm{d}\mathcal{L}}{\rightsquigarrow} Q} \ (\mathrm{d}\mathcal{L}\text{-}\langle\!|\,|\!\rangle)$$