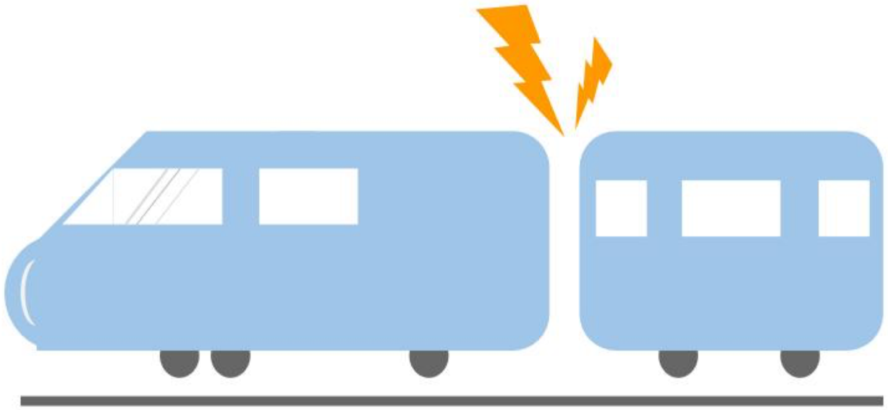
Lots & Lots of Trains

Efficient Transit with Distributed Hybrid Systems



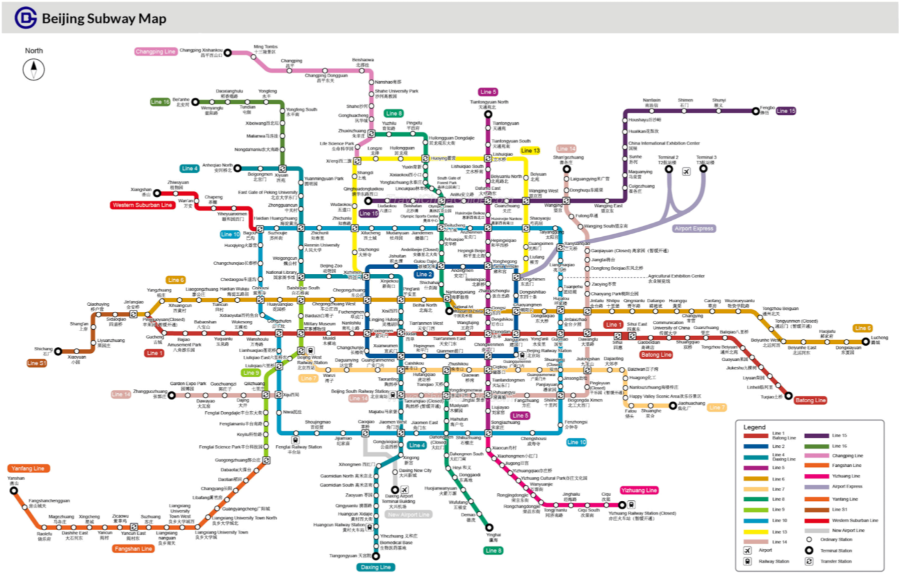
Josh Kalapos

Relevancy: Train Safety & Efficiency

Busiest subway is in Beijing, with 10 million daily riders.

New York subway fleet consists of 6,418 vehicles across 424 unique stations.

Recent subway delays in New York cost $300+ million in estimated lost work time over the course of a year*



*https://www.nytimes.com/2017/10/12/nyregion/subway-delays-lost-work-time-cost-new-york.html

Relevancy: Train Simulations

Simulations	Provided Results
OPEN TRACK	Show safety.
trenolab	Simulate random scenarios.
	Show infrastructure compliance with train types and control systems.
	Draft timetables.

*NH04

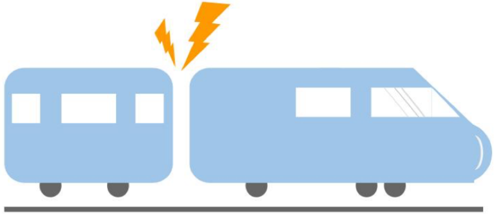
Goals

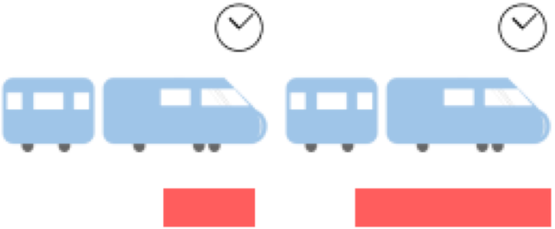
Track Safety

Prove trains will never run into one another on the tracks, no matter their choice.

Bound Efficiency

Justify a measure of efficiency, and an upper bound for that value.



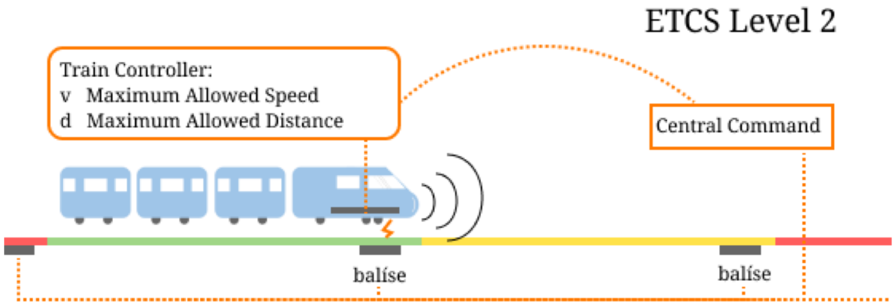


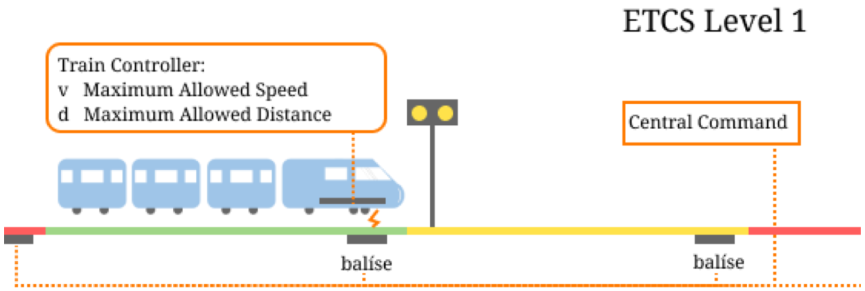
Background: Train Systems

European Train Control System:

Central Command grants a “movement authority” to each train containing:

1. Maximum Allowed Speed
2. Maximum Allowed Distance

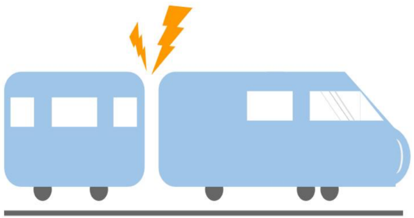




Background: Track Safety

Collisions

Occurs when a following train hits a leading train.



Maximum Velocity

Velocity of train must be under the limit at all times.

Background: Train Efficiency

Throughput

Maximum number of trains to travel a given route in a given time period.

Time of Travel

Time spent traveling from one station to another given no delays.

Delay

Sum of delays of all trains and at all stations in a given period of time.

Train Delay

Given a set of stations $\mathbf{S}$ in a system, and a set of trains in the system $\mathbf{T}$, the total Train Delay is the sum of all delays for train $t \in \mathbf{T}$, whose actual arrival time at station s is $t_{s,actual}$ and expected arrival time is $t_{s,expected}$:

$$\sum_{t \in \mathbf{T}, s \in \mathbf{S}} t_{s,actual} - t_{s,expected} \text{ if } t_{s,expected} < t_{s,actual}$$

Assumptions

Physical

No track friction, wind resistance

Train length is zero

Trains are identical (Maximum acceleration and braking)

Trains in the system start out safe initially

Control

ETCS-Level 2: Continuous access to position of other trains, and can be given decisions based on their maximum velocity

Train Safety: Lead/Follower Induction

If every train can safely follow its leader, then our system can be safe without modeling more than two trains simultaneously.

One train is safe on a given track.

Assume k trains are safe on a track.

Show k+1 trains are safe on a track.

Basic Train Motion Model: Time Trigger

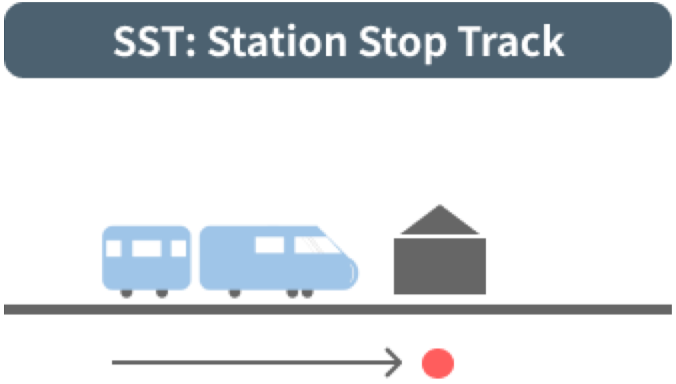
Acceleration Safety Test

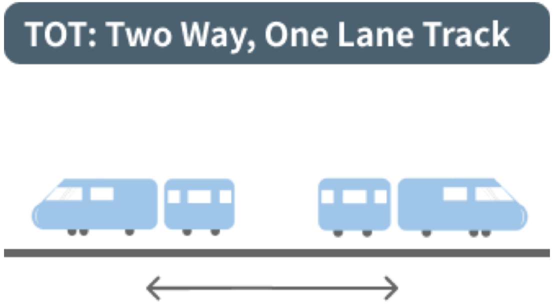
$$\text{posF} + \frac{1}{2}AT^2 + \text{velF} + \frac{(velF+AT)^2}{2B} \leq \text{posL} - \text{buffer}$$

ODE: Physics of Two Trains

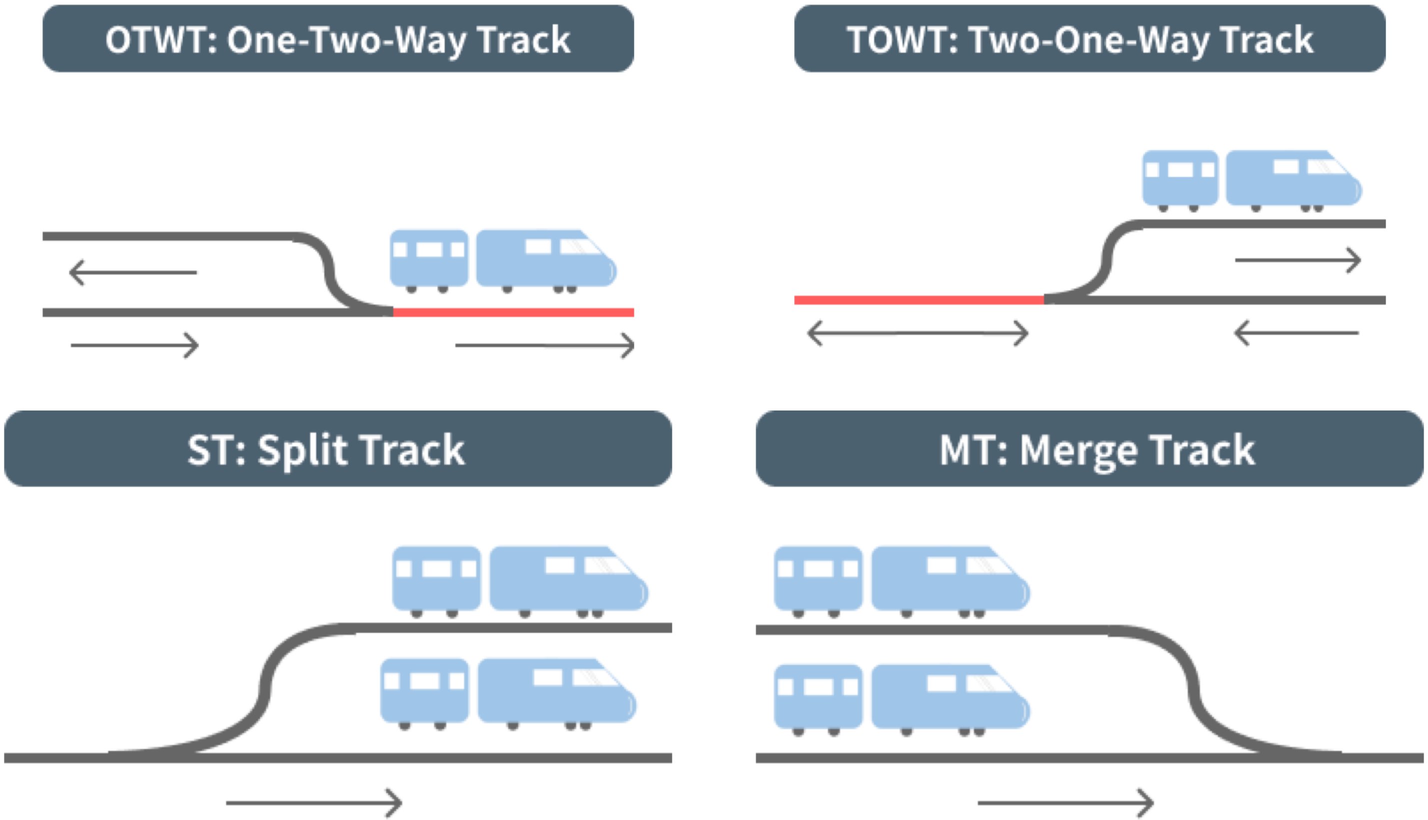
$$\{\text{posF}' = \text{velF},\ \text{velF}' = \text{accF},\ \text{posL}' = \text{velL},\ \text{velL}' = \text{accL},\ \text{t}'{=}1\ \&\ \text{t} \leq \text{T}\ \&\ \text{velA} \geq 0\ \&\ \text{velB} \geq 0\ \}$$

Track Types : Simple Segments





Track Types : Transitions and Merging



Track Composition

Attaching multiple simple segments together creates a much more complex railway

Event Triggered Models

Alternative to the time triggered models that we have previously been using. Opportunity to deliver discrete signalling or prove invariants for multiple stations.

Decisions made when posF reaches a position of interest.

Example: Consider multiple balises by assigning balise := balise + D

ODE: Physics of Two Trains

$$\{\text{posF}' = \text{velF},\ \text{velF}' = \text{accF},\ \text{posL}' = \text{velL},\ \text{velL}' = \text{accL},\ \text{t}'{=}1\ \&\ \text{posF} \geq \text{balise}\ \} \cup$$
$$\{\text{posF}' = \text{velF},\ \text{velF}' = \text{accF},\ \text{posL}' = \text{velL},\ \text{velL}' = \text{accL},\ \text{t}'{=}1\ \&\ \text{posF} \leq \text{balise}\ \}$$

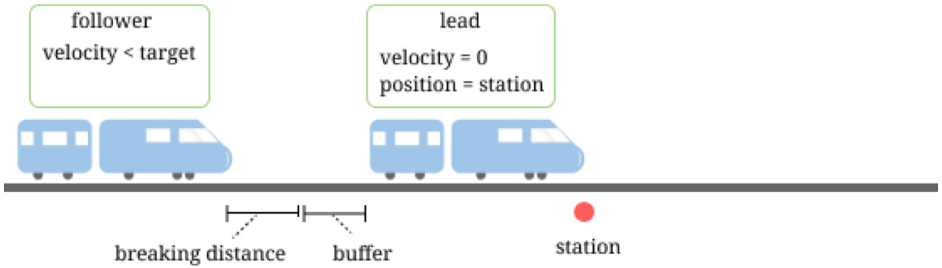
Delay Analysis

Given Assumption: Delay for a given station s for any train is less than maxDelay.

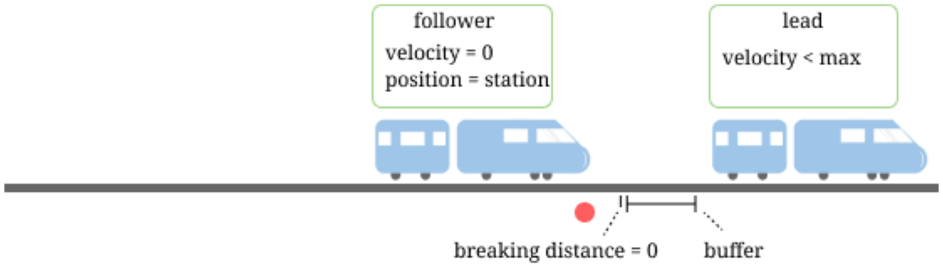
Minimize Total Delay: Minimum possible delay = maxDelay * numTrains * numStations

Delay Propagation Elimination

Before Delay



After Delay



Delay Propagation Elimination

Preserving Delay Invariant

$$\text{Delay Invariant} \rightarrow [\textit{Delay HP}]((\text{Delay Invariant} \mid \text{posL} = \text{station}) \,\&\, \text{posF} \leq \text{posL})$$

Constant Invariants

$$\text{buffer} \geq \text{maxDelay} * \text{targetVel}$$

$$\frac{\text{buffer} + \text{targetVel}^2}{(2 * B)} \geq (1/2) * A * \left(\frac{B}{(\text{targetVel})^2}\right)$$

$$\text{maxVel} \geq \frac{(\text{A} * \text{targetVel})}{B}$$

Proofs

Track types proven to be safe, minus merging.

Merging tracks requires more assumptions to be safe (signal for safe to merge in), as there is no single train to follow.

Delay modeled but not proved.

Discussion

Simple models can actually be made to describe large train systems.

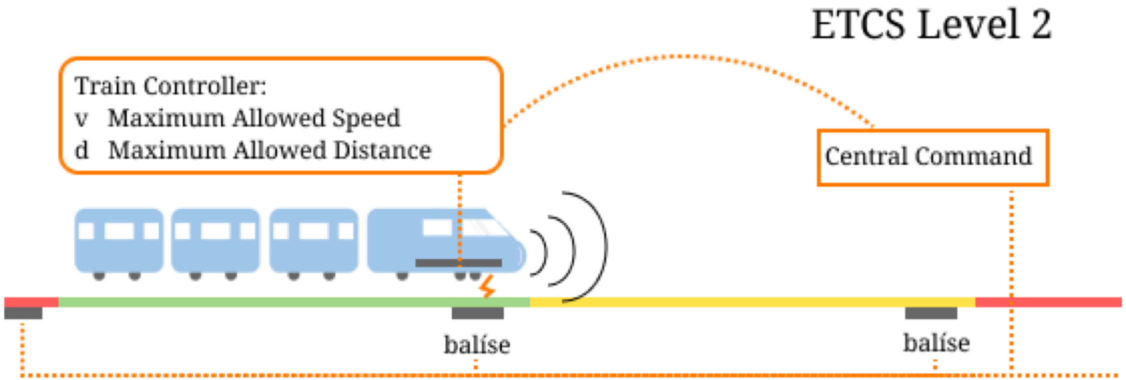
If a train can identify a lead train to follow, life is easy. Though sometimes, only knowing the lead train's position is not enough.

> If not, more effort required to determine which actions are safe (traffic signals, more detailed movement authority).
>
> Options for more information to be given to trains so that they can determine their own safety, or that we must change to event triggered modeling/hybrid to deliver information differently.

Conclusions

Information for Track Safety: ETCS Level-2 control closely aligns with the behavior of necessary controllers in efficient hybrid programs of trains.

Delay Recovery: The assumptions required for delay recovery negatively impact other efficiency measures; throughput and time of travel



Future Improvements

Additional track types.

Proving efficiency of ETCS-2 (Delay propagation elimination).

Applying inductive reasoning by incrementing stations using event triggers.

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Proving Liveness.

Thank You!

References

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