Lots & Lots of Trains

Efficient Transit with Distributed Hybrid Systems
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Relevancy: Train Safety & Efficiency

Busiest subway is in Beijing, with 10 million daily riders.

New York subway fleet consists of 6,418 vehicles across 424 unique stations.

Recent subway delays in New York cost $300+ million in estimated lost work time over the course of a year*

Relevancy: Train Simulations

Simulations

- Show safety.
- Simulate random scenarios.
- Show infrastructure compliance with train types and control systems.
- Draft timetables.

Provided Results

*NH04
Goals

Track Safety
Prove trains will never run into one another on the tracks, no matter their choice.

Bound Efficiency
Justify a measure of efficiency, and an upper bound for that value.
Background: Train Systems

European Train Control System:

Central Command grants a “movement authority” to each train containing:

1. Maximum Allowed Speed
2. Maximum Allowed Distance
Background: Track Safety

Collisions
Occurs when a following train hits a leading train.

Maximum Velocity
Velocity of train must be under the limit at all times.
Background: Train Efficiency

**Throughput**
Maximum number of trains to travel a given route in a given time period.

**Time of Travel**
Time spent traveling from one station to another given no delays.

**Delay**
Sum of delays of all trains and at all stations in a given period of time.

**Train Delay**
Given a set of stations $S$ in a system, and a set of trains in the system $T$, the total Train Delay is the sum of all delays for train $t \in T$, whose actual arrival time at station $s$ is $t_{s,actual}$ and expected arrival time is $t_{s,expected}$:

$$\sum_{t \in T, s \in S} t_{s,actual} - t_{s,expected} \text{ if } t_{s,expected} < t_{s,actual}$$
## Assumptions

### Physical
- No track friction, wind resistance
- Train length is zero
- Trains are identical (Maximum acceleration and braking)
- Trains in the system start out safe initially

### Control
- ETCS-Level 2: Continuous access to position of other trains, and can be given decisions based on their maximum velocity
Train Safety: Lead/Follower Induction

If every train can safely follow its leader, then our system can be safe without modeling more than two trains simultaneously.

One train is safe on a given track.

Assume k trains are safe on a track.

Show k+1 trains are safe on a track.
Basic Train Motion Model: Time Trigger

**Acceleration Safety Test**

\[ \text{posF} + \frac{1}{2}AT^2 + \text{velF} + \left(\frac{\text{velF} + AT}{2B}\right)^2 \leq \text{posL} - \text{buffer} \]

**ODE: Physics of Two Trains**

\{ \text{posF}' = \text{velF}, \text{velF}' = \text{accF}, \text{posL}' = \text{velL}, \text{velL}' = \text{accL}, \ t' = 1 \ & \ t \leq T \ & \ \text{velA} \geq 0 \ & \ \text{velB} \geq 0 \ \}
Track Types: Simple Segments

- **OOT: One Way, One Lane Track**
  ![Diagram of OOT: One Way, One Lane Track]

- **SST: Station Stop Track**
  ![Diagram of SST: Station Stop Track]

- **TOT: Two Way, One Lane Track**
  ![Diagram of TOT: Two Way, One Lane Track]
Track Types: Transitions and Merging

OTWT: One-Two-Way Track

TOWT: Two-One-Way Track

ST: Split Track

MT: Merge Track
Track Composition

Attaching multiple simple segments together creates a much more complex railway
Event Triggered Models

Alternative to the time triggered models that we have previously been using. Opportunity to deliver discrete signalling or prove invariants for multiple stations.

Decisions made when \( \text{posF} \) reaches a position of interest.

Example: Consider multiple balises by assigning \( \text{balise} := \text{balise} + D \)

\[
\{ \text{posF}' = \text{velF}, \text{velF}' = \text{accF}, \text{posL}' = \text{velL}, \text{velL}' = \text{accL}, t' = 1 \& \text{posF} \geq \text{balise} \} \cup \\
\{ \text{posF}' = \text{velF}, \text{velF}' = \text{accF}, \text{posL}' = \text{velL}, \text{velL}' = \text{accL}, t' = 1 \& \text{posF} \leq \text{balise} \}
\]
Delay Analysis

**Given Assumption:** Delay for a given station $s$ for any train is less than $\text{maxDelay}$.

**Minimize Total Delay:** Minimum possible delay $= \text{maxDelay} \times \text{numTrains} \times \text{numStations}$
Delay Propagation Elimination

Before Delay

After Delay
Delay Propagation Elimination

Preserving Delay Invariant

Delay Invariant $\rightarrow [\text{Delay HP}](\text{Delay Invariant} | \text{posL} = \text{station}) \land \text{posF} \leq \text{posL})$

Constant Invariants

\[
\text{buffer} \geq \text{maxDelay} \times \text{targetVel} \\
\frac{\text{buffer} + \text{targetVel}^2}{(2 \times B)} \geq \frac{1}{2} \times A \times \left(\frac{B}{(\text{targetVel})^2}\right) \\
\text{maxVel} \geq \left(\frac{A \times \text{targetVel}}{B}\right)
\]
Proofs

Track types proven to be safe, minus merging.

Merging tracks requires more assumptions to be safe (signal for safe to merge in), as there is no single train to follow.

Delay modeled but not proved.
Discussion

Simple models can actually be made to describe large train systems.

If a train can identify a lead train to follow, life is easy. Though sometimes, only knowing the lead train’s position is not enough.

If not, more effort required to determine which actions are safe (traffic signals, more detailed movement authority).

Options for more information to be given to trains so that they can determine their own safety, or that we must change to event triggered modeling/hybrid to deliver information differently.
Conclusions

**Information for Track Safety:** ETCS Level-2 control closely aligns with the behavior of necessary controllers in efficient hybrid programs of trains.

**Delay Recovery:** The assumptions required for delay recovery negatively impact other efficiency measures; throughput and time of travel.
Future Improvements

Additional track types.

Proving efficiency of ETCS-2 (Delay propagation elimination).

Applying inductive reasoning by incrementing stations using event triggers.

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Proving Liveness.
Thank You!
References


