

Slalom

Modelling obstacle avoidance during skiing

Abstract

dL is a powerful logical language for proving properties of hybrid systems. Using dL , I design various models of downhill skiing. Taking advantage of sequent calculus rules for loop invariants and solution-independent differential equation reasoning, I prove that an event-triggered controller guides a skier down a course containing multiple obstacles while maintaining many safety properties. These include maintaining a “downhill” velocity, staying within horizontal constraints, and avoiding two-dimensional obstacles. The results generalize to many systems involving circular motion, and provide a fundamental starting point for proving the safety of steering-based obstacle avoidance.

Introduction

Skiing is a prime example of a system that relies on steering control for safety. While travelling downhill, it may not be feasible to suddenly decelerate in order to avoid hitting an obstacle. As a result, skier’s use awareness of the approaching terrain to choose a path that isn’t too difficult to ski and avoids any obstacles. This approach is useful in many moving systems, since braking is usually inefficient. For example, being able to avoid obstacles on the fly would allow a delivery robot to reach its goal with fewer stops and in less time. Even for systems that rely on an emergency braking system, having a secondary controller that uses steering-based obstacle avoidance would be useful in less dangerous situations (this secondary controller could then hand off control to the braking system if it reached an overly dangerous scenario). My project demonstrates the power of this steering-based obstacle avoidance in a simple model based on circular motion. By modelling the movement of a skier using differential equations for a circle, I present a controller capable of choosing paths that avoid obstacles while also continuously moving towards the goal, respecting bounds on the turning radius, and staying within horizontal constraints.

Related work

There are many similar modelling projects focused on safe 2-d motion and obstacle avoidance that lend insight to this project. For example, the Coaster X project¹ models a similar 2-dimensional motion of a roller coaster and aims to prove safety in the level of vertical acceleration. The project even went so far as to generate automatic proofs for new roller coasters based on user input. The tool demonstrates the novel technique of CDPA, in which modular proofs pertaining to segments of the bigger model are eventually combined to prove a property about the larger system.

Another project² leverages this hybrid systems “gestalt” to analyze the Next-Generation Airborne Collision Avoidance System (ACAS), software used by the FAA to advise pilots in avoid collisions with other planes. While this model is more complex than that of the Coaster X project (and significantly more so compared to this project), it tackles this complexity by breaking the advisory system into different “safe regions” ie certain configurations in which the advisories of the ACAS are understood to be safe. The project extends this idea by also proving regions in which advisories may not be safe on their own but are correctable by subsequent advisories, which provides insight into how the ACAS can correct existing flaws.

My project uses a much simpler model than either of the above, and as a result, doesn't rely on region-based reasoning about the system. Instead, I leverage symmetrical, periodic steering to make my proofs powerfully inductive. I am able to show that my invariants for a given ski-turn will hold throughout the next turn due to the symmetry of its motion. This approach works well for simple, uniform trajectories, but is unlikely to generalize to asymmetric situations.

Basic modelling assumptions

¹Bohrer, B., Luo, A., Chuang, X. A., & Platzer, A. (2018). CoasterX: A Case Study in Component-Driven Hybrid Systems Proof Automation. *IFAC-PapersOnLine*, 51(16), 55-60. doi:10.1016/j.ifacol.2018.08.010

² Jeannin, J., Ghorbal, K., Kouskoulas, Y., Schmidt, A., Gardner, R., Mitsch, S., & Platzer, A. (2016). A formally verified hybrid system for safe advisories in the next-generation airborne collision avoidance system. *International Journal on Software Tools for Technology Transfer*, 19(6), 717-741. doi:10.1007/s10009-016-0434-1

I model the skier as a point moving through the xy -plane and assume that the skier undergoes circular motion at a constant speed. The goal of the skier, while on the course, will be:

- 1) Maintain a turning radius within a given range
- 2) Stay between the left and right boundaries
- 3) Avoid designated obstacles, represented by points or regions in the plane
- 4) Always ski downhill (maintain a non-negative y -velocity)
- 5) Respect the laws of circular motion

The goal of this physical description is to capture the trade off that occurs between steering and speed. If a skier attempts to take a direct line on a course, he/she may build up an unsafe amount of speed. Conversely, an overly roundabout trajectory may lead to technical sharp turns that could result in a “wipe-out” or loss of control. Rather than model the more complicated interplay between steering and speed by designing a safety condition that incorporates speed, I opted for the more elegant approach of keeping the speed constant while bounding the turning radius, forcing the skier to safely adhere to steering requirements. Admittedly, this oversimplifies the physics of downhill speed and slightly conflates safety with physical impossibility. However, this modelling simplicity allows for the proof of more complex properties while mostly respecting the true nature of skiing. Since we are primarily interested in the ability to steer, and we impose constraints on this ability, our model accurately captures the technicality of real-world skiing.

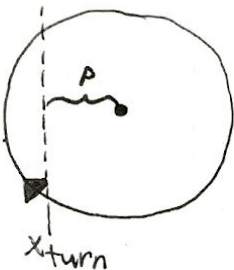
Event-triggered control

In our model we opt for an event-triggered controller as opposed to time-triggered. While time-triggered controllers are usually favored because they adhere more closely to reality, a time-triggered controller wouldn't accurately capture the approach of steering-reliant obstacle avoidance. The idea behind time-triggered controllers is that the controller imposes discrete changes to the system at least every so often. However, in the case of skiing, being able to frequently adjust one's turning doesn't help much compared to being able to choose *when* to change your turn. Skiers usually hold a turn

for a fairly long time but are able to change this turn almost exactly when they choose. Without this ability, it becomes very difficult to choose paths that resemble your desired path. As an example, consider the path consisting of symmetric, alternating semi-circles, depicted below.

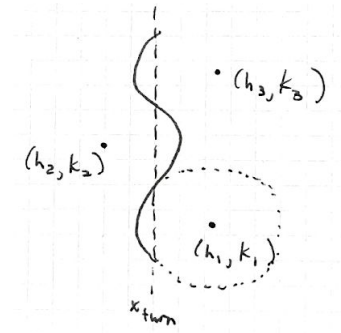


While this maneuver is feasible for an intermediate, real-world skier, a time-triggered controller would make this impossible. The controller would be unable to calculate the time it takes to complete one semi-circular turn based on its velocity, because the calculation would involve π which is transcendental. This would force the skier to take a path that isn't assuredly straight. Instead I propose a controller that chooses its x_{turn} , namely the x-coordinate at which it will change its turn. This can be thought of intuitively as a skier deciding at what point on the slope they will shift their feet next. In order to ski in alternating semi-circles like the above, the controller would maintain a constant value for x_{turn} , keep the same turning radius, and simply alternate



its rotational direction. While this approach parallels the approach of real skiing, it has the loophole of allowing skiers to repeatedly choose an x_{turn} that would cause them to take infinitely narrow turns, as shown to the left.

As the skier increases the distance denoted p , the “width” of the turn decreases. This can be done to obtain an arbitrarily narrow turn. However, this loophole is not a fundamental issue with the model since skilled skiers should have control of the width of their turn. This skill could be bounded by placing a restriction on the width of the turn (this can be achieved, for example, by placing a lower bound on p). In this sense, being able to choose an x_{turn} with certain limitations captures the notion of advanced skiing. While advanced skiing is not the focus of this project, an unproven model is provided that demonstrates how a controller could implement narrow turning to stay in bounds on a tight course. Doing so involves



pushing the center of motion away from x_{turn} in an alternating fashion (see figure below).

Methodology of safety conditions

I enforce safety conditions by imposing restrictions on the skier's position, turning radius, and y-velocity for all runs of the model. For example, $[\alpha](0 < x < 5 \wedge (x, y) \neq (1, 3))$ could represent a course that is 5 units wide and has a zero dimensional obstacle at position (1, 3). It is important to note that the box modality $[\alpha]P$, covers all intermediary states of the system. Since differential equations run for a nondeterministic time period and the number of repetitions of the loop is also nondeterministic, there is a run of the hybrid program for every point in time. So the box modality allows us to assert that a given condition holds throughout the entire model, including the very beginning. I use the formula $[\alpha]y_{vel} \geq 0$ to enforce that the skier never skis up hill. I ultimately choose to represent this constraint on y-velocity as a safety condition rather than a domain constraint because real-life skiers are able to cut back up the hill for a short time, but it would likely result in a crash (the preliminary model does not make this distinction as we will see). This is often used to come to a stop in real-world skiing, but our simplified model doesn't account for that, since we largely abstract away velocity in order to focus on steering control.

Preliminary model

Before discussing the full model including obstacle avoidance and steering control using the x_{turn} event, I highlight basic properties and techniques about a preliminary version, shown below.

```
/* Input constants */
rLo    Skier's minimum turning radius
rHi    Skier's maximum turning radius
xLo    Left bound of the course
xHi    Right bound of the course
yLo    Starting line of the course
yHi    Finish line of the course
tHi    Maximum allotted time to cross the finish
v      Linear velocity of robot
```

Naveen Pai (15424)
nvpai@andrew.cmu.edu

```
/* Controlled fields */
x      Position of robot in x direction
y      Position of robot in y direction
t      Elapsed time of execution (does not reset)
h      x coordinate of center of circular track
k      y coordinate of center of circular track
trackr Robot track radius (- means clockwise)
mult   Factor by which controller changes track

/* Initial conditions */
(
  rLo > 0 &
  rHi > rLo &
  xHi > xLo &
  yHi >= yLo &
  tHi > 0 &
  v > 0 &
  xLo <= x &
  xHi >= x &
  y = yLo &
  t = 0 &
  2*rLo <= (xHi-xLo) &
  h = (xHi+xLo) / 2 &
  k = yLo &
  abs(trackr) >= rLo &
  abs(trackr) <= rHi &
  (x-h)^2 + (y-k)^2 = trackr^2 &
  (x >= (xHi+xLo)/2 <-> trackr >= 0)
)
->
[ {
  /* CONTROL */
  {
    {
      ? (v/trackr)*(x - h) = 0;
      mult := -1;
    }
    ++
    {
      mult := 1;
    }
  }
  trackr := mult*trackr;
}

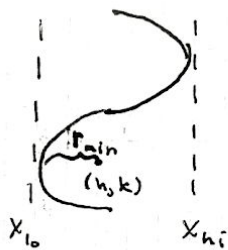
/* PLANT */
{
  h := (h-x)*mult + x;
  k := (k-y)*mult + y;
  {
    x' = -(v/trackr)*(y - k),
    y' = (v/trackr)*(x - h),
    t' = 1
  }
}
```

```

    & (v/trackr)*(x - h) >= 0
  }
}
}* @invariant(
  (x-h)^2 + (y-k)^2 = trackr^2 &
  h = (xHi+xLo)/2 &
  rLo <= abs(trackr) &
  rHi >= abs(trackr) &
  abs(h-xLo) >= abs(trackr) &
  abs(h-xHi) >= abs(trackr) &
  (v/trackr)*(x - h) >= 0
) ]
(
  (y <= yHi -> (
    rLo <= abs(trackr) &
    rHi >= abs(trackr) &
    xLo <= x &
    xHi >= x &
    (v/trackr)*(x - h) >= 0 &
    (x-h)^2 + (y-k)^2 = trackr^2
  ))
)

```

The model describes the approach of skiing in alternating semi-circles down a course with horizontal boundaries $x_{lo} < x_{hi}$. I refer to this as intermediate skiing, because the skier is not skilled enough for narrow turns. As such, the skier can only ski down a course where the minimum turn radius does not exceed half the width of the course (see figure below). Note, we use the convention that a negative *trackr* corresponds to



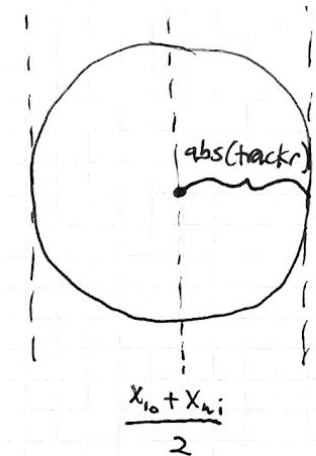
clockwise movement about the center of motion. So at each turn we flip the sign of *trackr*. We also consider the positive y-direction to be “downhill”, meaning that the y-coordinate increases throughout the progression of the model. Note that when the turning radius is changed by manipulating *mult*, the new center of motion (h, k) is calculated so that the path of the next turn is tangent to the path

previous turn (ie the skier’s motion is continuous). However, this preliminary model uses the domain constraint “ $(v/trackr)*(x - h) >= 0$ ” which allows the skier to automatically begin the next turn when his y-velocity is zero (ie he is at the end of the semi-circle). This is not realistic, because it essentially says that physics is doing the work for the skier, who doesn’t need to think about when to turn. However, using this

simplification, I provide the basics of skiing down an inputted course while staying in bounds.

Preliminary proof sketch

- The inductive invariant “ $rLo \leq \text{abs}(\text{trackr})$ & $rHi \geq \text{abs}(\text{trackr})$ ” follow trivially, since $\text{abs}(\text{trackr})$ remains constant throughout. We only change the direction of our turn, not the sharpness.
- The invariant “ $h = (xHi+xLo)/2$ ” holds arithmetically from the following idea: when we change our turn, $(v/\text{trackr})*(x - h)$ so we know $x=h$. So “ $h := (h-x)*\text{mult} + x$ ” does not change h regardless of the value of mult .
- The invariant “ $xLo \leq x$ & $xHi \geq x$ ” follows from the other invariant that “ $h = (xHi+xLo)/2$ ”. Since we required that “ $xLo \leq x$ & $xHi \geq x$ ” hold initially and we know that “ $(x-h)^2 + (y-k)^2 = \text{trackr}^2$ ” throughout, it must be the case that “ $\text{abs}(\text{trackr}) \leq (xHi-xLo)/2$ ” (see figure below). So the entire circle of motion is within the course, making it impossible for the skier to stray from the course.



- The invariant “ $(v/\text{trackr})*(x - h) \geq 0$ ” follows from the domain constraint. In other words, we get this assumption for free, since physics ensures that this remains true.
- “ $(x-h)^2 + (y-k)^2 = \text{trackr}^2$ ” follows from a direct application of the differential invariant rule, since in our ODE, $2x' + 2y' = 0$. So we are assured that circular motion is respected throughout.

Adding steering control

The previous model can be updated to control skiing using x_{turn} as described previously.

This involves changing the ODE to match the following:

```
{
  x' = -(v/trackr)*(y - k),
  y' = (v/trackr)*(x - h),
  t' = 1
  & x <= xTurn
}
++
{
  x' = -(v/trackr)*(y - k),
  y' = (v/trackr)*(x - h),
  t' = 1
  & x >= xTurn
}
```

We now allow physics to progress regardless of the value of x , however when x reaches x_{turn} , we allow the controller to adjust the turn of the skier. I update the controller to guide the skier along the middle of the course by maintaining $x_{turn} = \frac{x_{hi} + x_{lo}}{2}$. However, more sophisticated reasoning is now required to ensure that y -velocity remains non-negative. I introduce the following invariant: when the skier is on the left side of the course ($x < \frac{x_{hi} + x_{lo}}{2}$) he is turning towards the right side (ie $h = \frac{x_{hi} + x_{lo}}{2} \wedge trackr < 0$). When the skier is on the right side of the course, he is turning towards the left side of the course. This can be summarized with the following:

$$(x > (x_{hi} + x_{lo})/2 \rightarrow trackr > 0) \ \& \ (x < (x_{hi} + x_{lo})/2 \rightarrow trackr < 0)$$

To ensure this invariant is respected, I update the controller to check the direction of the x -velocity when the skier reaches x_{turn} . If the skier is travelling rightward at x_{turn} , the controller makes sure the skier begins turning towards the left. Similarly, if the skier is travelling leftward at x_{turn} , the controller makes sure the skier begins turning towards the left. However, we still only change the direction of turn; the magnitude of the turning radius stays constant (ie $mult = \pm 1$). So we still model only intermediate skiing.

Proof of steering control

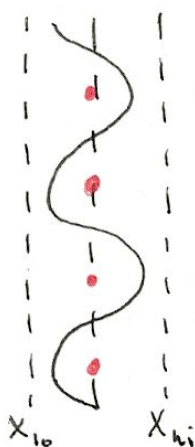
- The proof of invariant “ $h = (x_{Hi}+x_{Lo})/2$ ” now changes. If we assume $x_{turn} = h = \frac{x_{hi}+x_{lo}}{2}$ held in the previous iteration, and we know that $x = x_{turn}$ when the center of motion changes, we can conclude arithmetically that $(h-x)*mult + x = (x_{Hi}+x_{Lo})/2$.
- The proof of the invariant “ $(v/trackr)*(x - h) \geq 0$ ” now becomes non trivial. We break the proof into the following cases, based on the structure of the controller’s choice of *mult*:
 - The skier is initially travelling left using the ODE that describes the right side of the course (ie domain constraint $x \geq x_{turn}$). Note in this case it must be true that $x = h = x_{turn}$ initially, in order for the domain constraint to be satisfied initially. In this case, the ODE must run for 0 time, otherwise the domain constraint will be violated (ie the skier will enter the the left side of the course which is not described by the current ODE). So at the end of the run $x = h = x_{turn}$ proving “ $(v/trackr)*(x - h) \geq 0$ ”. We reason about this proof using differential cuts to show that the x and y velocities do not increase, setting up the differential invariant that x will not increase (and must therefore stay the same).
 - The skier is initially travelling right using the ODE that describes the right side of the course. Here we rely on the invariant that “ $x > (x_{Hi}+x_{Lo})/2 \rightarrow trackr > 0$ ”. This allows us to assume “ $trackr > 0$ ”. In this case, the controller guarantees that $mult = -1$. So we can prove “ $(v/(-1*trackr))*(x - h) \geq 0$ ” using the GV proof rule, the inferred sign of “*trackr*” and the knowledge that $x \geq h$.
 - The remainder of the cases are symmetric to one of the two above cases, and involve the same techniques.
- The proof of the invariant “ $(x > (x_{Hi}+x_{Lo})/2 \rightarrow trackr > 0) \ \& \ (x < (x_{Hi}+x_{Lo})/2 \rightarrow trackr < 0)$ ” is also complex. For simplicity, I will only describe the leftward symmetric case: “ $(x < (x_{Hi}+x_{Lo})/2 \rightarrow trackr < 0)$ ”. The other

case is symmetric and uses the same technique. We prove this invariant holds inductively using the following cases:

- $trackr < 0$: In this case, we can use the GV proof rule, since $trackr$ remains constant throughout the run of the ODE.
- $trackr \geq 0$: Here the controller guarantees that, if there is a non-zero run of the ODE (meaning that the y-velocity doesn't cause the domain constraint to be immediately violated), $mult = -1$. So we can show $-1 * trackr < 0$, using the GV proof rule as above (since the value of $trackr$ changes sign for the next turn).
- There are many trivial cases where the domain constraint $x \geq x_{turn}$ trivializes the invariant " $(x < (x_{Hi} + x_{Lo}) / 2 \rightarrow trackr < 0)$ ". This is because " $x_{Turn} = (x_{Hi} + x_{Lo}) / 2$ ", so we know that " $x \geq (x_{Hi} + x_{Lo}) / 2$ ". This trivializes the conclusion that $(x < (x_{Hi} + x_{Lo}) / 2 \rightarrow trackr < 0)$, since we know the condition of the implication does not hold.
- There are also trivial cases in which we show there are no non-zero runs of the ODE. In these cases, we can show that " $x \geq (x_{Hi} + x_{Lo}) / 2$ " after any runs of the ODE, trivializing the invariant as we did above.

Avoiding zero-dimensional obstacles

Having showed that skiing in alternating semi-circles is safe on an empty course, I can show that it also avoids vertically aligned, zero-dimensional obstacles (see figure below).



I call the uniform distance between each obstacle "gate", reminiscent of the slalom gates in certain ski races. While intermediate skiing (ie symmetric, alternating semi-circles) can be used to avoid an arbitrary number of evenly-spaced, vertically aligned obstacles, in my model I simplify the number of obstacles to 3. This is necessary in order to express that the skier never runs into obstacles in dL . If we let there be an arbitrary number of obstacles, n , we would be forced to use the safety condition $(\exists n \in \mathbb{N}(y = y_{first} + n \cdot gate)) \rightarrow x \neq (\frac{x_{hi} + x_{lo}}{2})$. However, since first order logic of the integers is undecidable, this would not be

provable with dL . Instead I fix $n = 3$, but choose an approach that generalizes inductively. Since the skier navigates the third obstacle in the same exact way it navigates the first, it is clear that the skier will be able to handle a fourth obstacle, and even a fifth, and so forth. However, proving this rigorously would require a new dL formula and proof tactic for each number of obstacles. My safety condition for 3 obstacles is then, as follows:

$$(y = y_{first} \mid y = y_{first} + gate \mid y = y_{first} + 2 * gate) \rightarrow x \neq (x_{hi} + x_{lo}) / 2$$

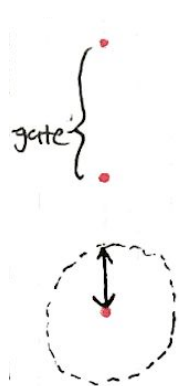
This essentially expresses that the position of the skier (x, y) never matches the position of an obstacle $(\frac{x_{hi} + x_{lo}}{2}, y_{first} + n \cdot gate)$, $n \in \{0, 1, 2\}$. Note that y_{first} is an input constant representing the y-coordinate of the first obstacle.

Proof of zero-dimensional obstacle avoidance

I add the following invariant to prove the new safety condition:

$$y \leq y_{first} + 2 * gate \rightarrow (k = y_{first} \mid k = y_{first} + gate \mid k = y_{first} + 2 * gate)$$

This expresses that, while the skier has not yet passed every obstacle, its center of motion is one of the obstacles. Since we have already assumed that the skier's turning radius is non-zero, this ensures that the skier doesn't hit the obstacle it is currently turning around. When we add the additional assumption that the turning radius has magnitude $\frac{gate}{2}$, we can conclude that the obstacle does not hit any of the other obstacles (see figure below). Note that if $y > y_{first} + 2 * gate$, it can't possibly overlap with any



obstacles, because it has already passed the last one. I prove this invariant using the arithmetic of calculating new centers. Using the assignment " $h := (h - x) * mult + x$ ", I show that when $(h, k) = (\frac{x_{hi} + x_{lo}}{2}, y_{first})$, the next center of motion will be $(\frac{x_{hi} + x_{lo}}{2}, y_{first} + gate)$, since the turning radius is assumed to have magnitude $\frac{gate}{2}$. I can apply the same reasoning for $(h, k) = (\frac{x_{hi} + x_{lo}}{2}, y_{first} + gate)$ changing to $(\frac{x_{hi} + x_{lo}}{2}, y_{first} + 2 * gate)$. For the last case where $(h, k) = (\frac{x_{hi} + x_{lo}}{2}, y_{first} + 2 * gate)$. I can show that either

$y > y_{first} + 2 * gate$, which already satisfies the invariant, or the center of motion does not

change. So we still have $(h, k) = (\frac{x_{hi}+x_{lo}}{2}, y_{first} + 2gate)$ which satisfies the invariant. All of these strategies follow easily using the QE tool in Keymaera X.

Avoiding two-dimensional obstacles

Having shown the skier avoids certain points on the course, I can generalize the model to prove that the skier avoids circular regions surrounding those points. In other words, the skier avoids circular, two-dimensional obstacles instead of zero-dimensional ones. I update the obstacle avoidance safety condition to the following:

$$\begin{aligned} &(x - ((x_{hi} + x_{lo}) / 2))^2 + (y - (y_{first}))^2 > thick^2 \ \& \\ &(x - ((x_{hi} + x_{lo}) / 2))^2 + (y - (y_{first} + gate))^2 > thick^2 \ \& \\ &(x - ((x_{hi} + x_{lo}) / 2))^2 + (y - (y_{first} + 2 * gate))^2 > thick^2 \end{aligned}$$

This expresses that the distance from the obstacle to every other point exceeds the radius of the obstacle (denoted “thick”).

Proof of two-dimensional obstacle avoidance

We can prove this property by only slightly modifying our previous invariant:

$$y < y_{first} + 2 * gate + gate / 2 \ \rightarrow \ (k = y_{first} \ | \ k = y_{first} + gate \ | \ k = y_{first} + 2 * gate)$$

This expresses that, before the skier has fully “moved on” from each of the three obstacles, it’s center of motion is one of the obstacles. Here I using “moving on” to mean that the obstacle to be at least $\frac{gate}{2}$ units past the last obstacle, along the y-axis. This essentially expands the stated range at which the skier is asserted to be turning around some obstacle. Doing so allows us to conclude the safety condition, as long as we ensure that $0 < thick < \frac{gate}{2}$. We can conclude this using our invariant “ $(x-h)^2 + (y-k)^2 = trackr^2$ ” and the assertion that “ $abs(trackr) = gate / 2$ ”. In fact, the change to this invariant is so slight that it requires no changes to the inductive tactic used in the previous task. So my approach generalizes well.

Conclusion

In this paper I present a sound and generalizable model for intermediate skiing in the presence of two-dimensional obstacles and horizontal boundaries. This model demonstrates the feasibility of no-braking strategies for avoidance of obstacles that are

uniform and predictable. As such, this model provides a starting point for secondary obstacle avoidance systems that seek to preserve efficiency before resorting to an emergency braking protocol.

While this model accomplishes most of my proposed objectives, there are still numerous opportunities for expansion. For instance, this model does not prove efficiency guarantees involving crossing a finish line, or further, crossing a finish line by a designated time. I did not prove this property because the desired condition would have to involve an existential quantifier, and it is difficult to deal with these using basic dL rules.

Furthermore, the model is not suitable for obstacles scattered non-uniformly across the plane. This is because the model is limited to an intermediate skiing strategy in which the skier completes an entire semi-circle each turn. A possible augmentation would be to update the controller to make advanced, narrow turns. This would be possible by allowing the x-coordinate of the center of motion to move left and right each turn. However, this control strategy introduces complex reasoning about differential equations. While I created a model for advanced skiing on an empty course, I was unable to verify it, since the differential equations became very complex to reason about with so few assumptions. However, advanced skiing, potentially paired with region-based control strategies, could enable a skier take on much more complex, non-uniform obstacle patterns.

Lastly, this model does not handle multiple skiers attempting a course together. The safety of such a system would likely rely on hybrid games. A key difficulty with this model, however, would be expressing mutual safety of the skiers using opposing objectives. Since no skier would set out trying to crash into another, it seems contrived to set up a hybrid game in which skier A tries to evade skier B who is conversely aiming to crash into skier B.

Despite these loose ends, the presented model generalizes to many new challenges and applications, while proving many powerful properties.

Deliverables

Naveen Pai (15424)
nvpai@andrew.cmu.edu

I have created the following proven archives of the models described above:

- “preliminary_0.kya”: preliminary model
- “event_triggered_1.kya”: steering control using the x_{turn} strategy
- “multi_obstacles_2.kya”: zero-dimensional obstacle avoidance
- “2d_obstacles_3.kya”: two-dimensional obstacle avoidance

I also provide an unproven model “advanced_skier_4.kyx” which offers an approach for narrowing the turns of the skier to stay in bounds on a tighter course.