Slalom

modelling obstacle avoidance in skiing

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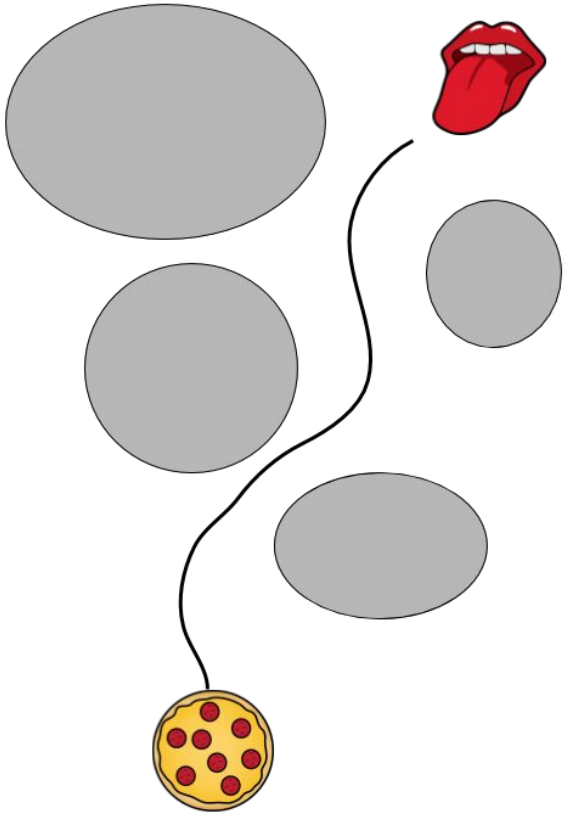
Motivation

Properties of skiing

- Can’t rely on braking
- Path consists entirely of turns
- Navigate anticipated obstacles

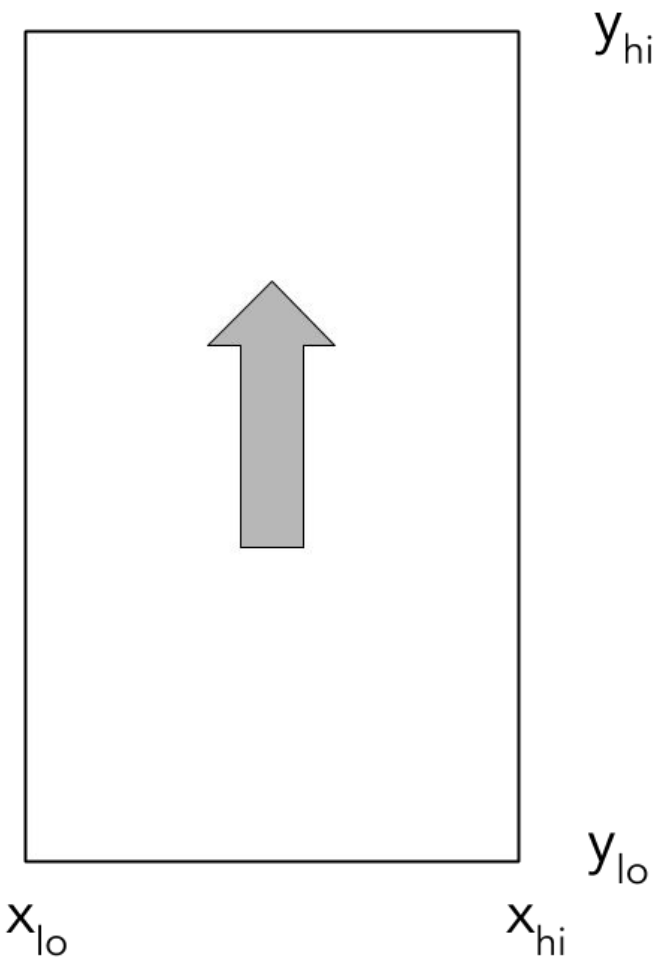
Applications

brake failure!



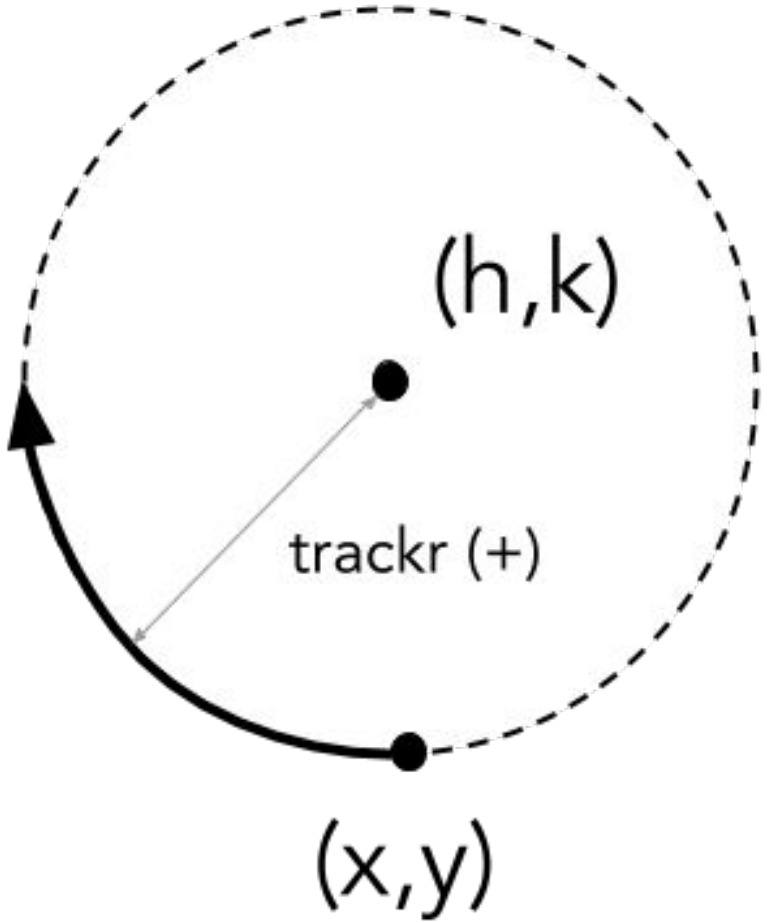
The Model...

The Course



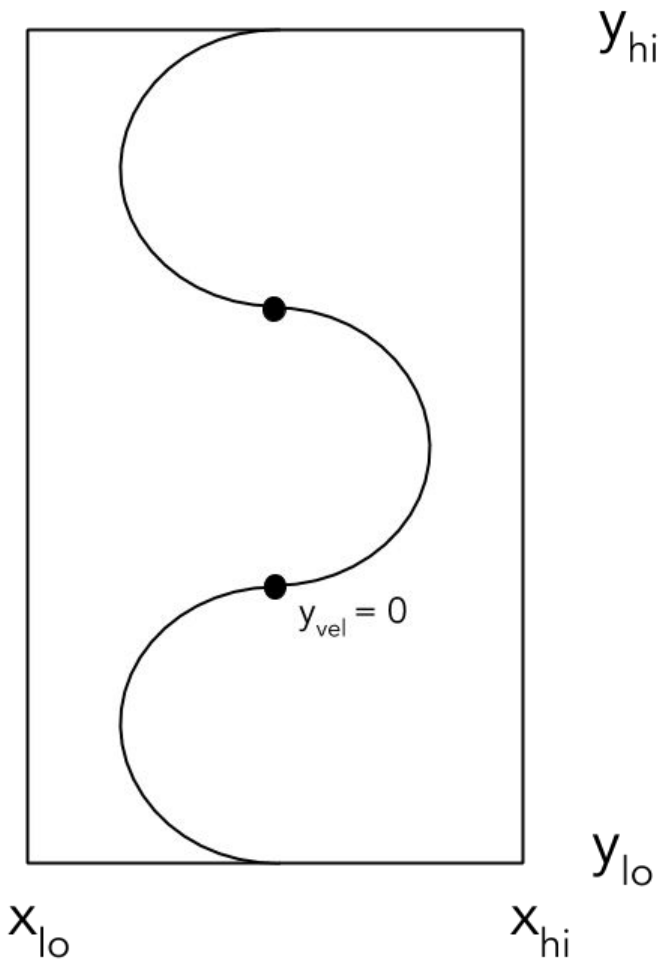
Skier’s motion

- circular motion
- rotational direction
- control turning radius
- constant velocity



First proof

- skier stays in bounds
- skier maintains safe turning radius
- circular motion is respected



Controller is too lazy...

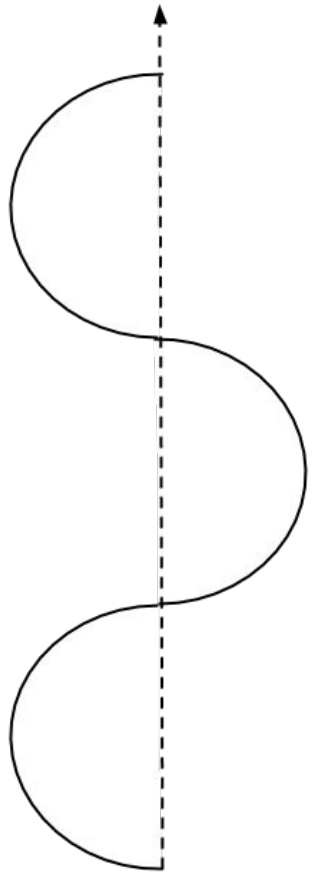
How about time triggered control?

IDEA

- controller wakes up at least every T seconds

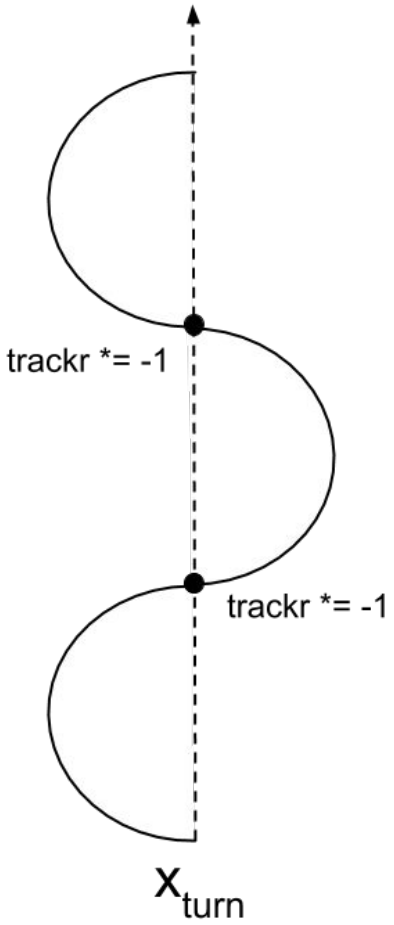
PROBLEMS

- frequent control adjustment isn't helpful
- loses control of when to turn
- deciding path relies on π, can't ski in a straight line □



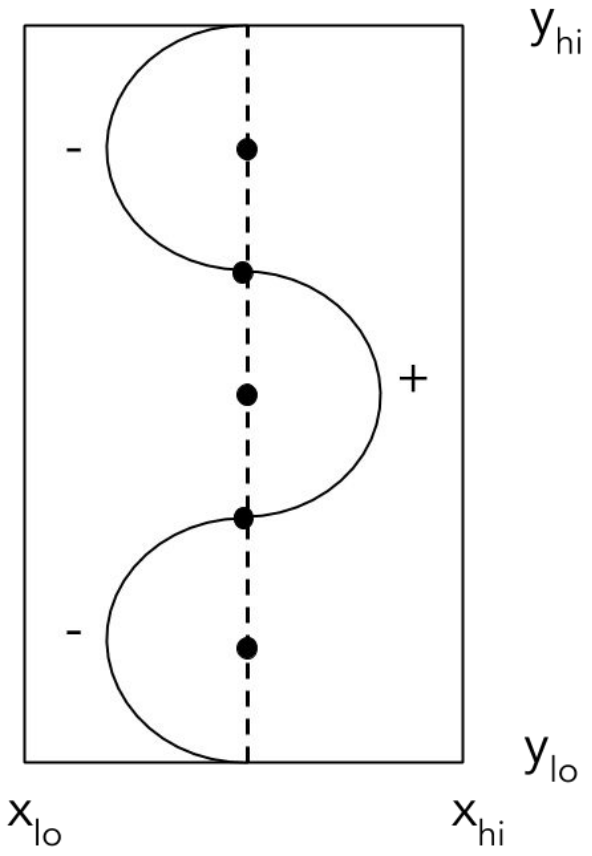
Event triggered control (reloaded)

- controller chooses x-coordinate, x_{turn}
- sleeps until x_{turn} is reached
- skier looks down at slope, decides to shift feet at a certain point
- downside: could lead to unrealistically small turns



Second proof

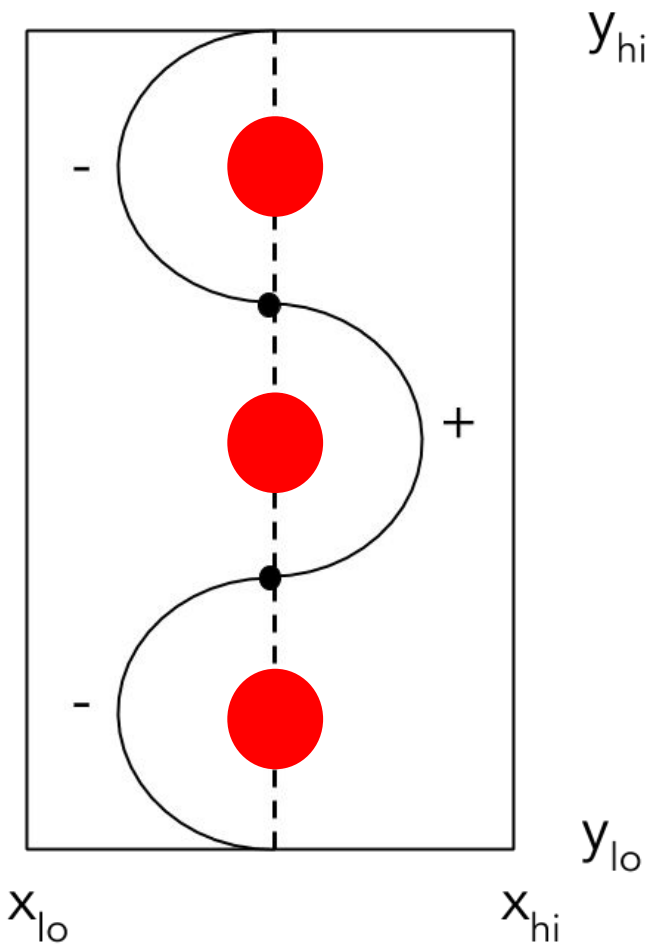
- most difficult step in project
- must reason that y-velocity stays non-negative
  - introduce invariant that involves rotational direction
  - show that ODE's respect this invariant



Obstacle avoidance approach

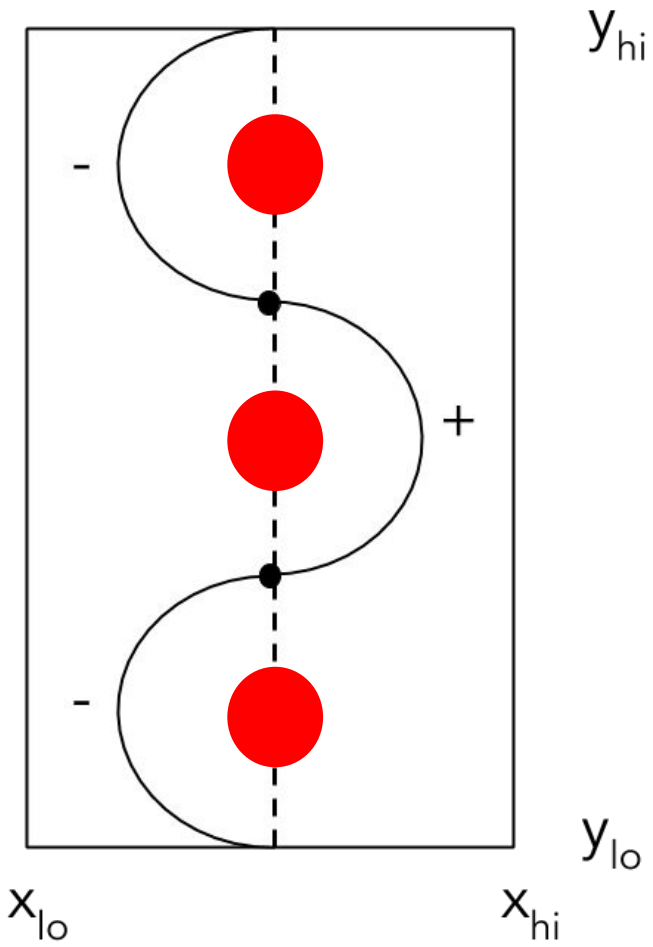
- include 3 obstacles in formula
  - difficult to reason about natural numbers in dL
  - however, controller's approach generalizes
  - proof technique also generalizes

- vertically aligned, evenly spaced



Obstacle avoidance proof

- Restrict obstacle spacing based on course width
- Introduce invariant about center of motion
  - obstacles act as the center of motion
  - turning radius is half the distance between obstacles



Summary: proven properties

1. skier stays in bounds on an inputted course
2. skier never strays uphill
3. skier maintains safe, bounded circular motion
4. skier avoids evenly-spaced, circular obstacles