Modelling Safe Left Ventricular Assist Devices

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1 Abstract

This project uses formal logic to model and prove the safety of a Left Ventricular Assist Device (LVAD). LVAD's help patients with weak left ventricles pump sufficient blood to perform daily activities with ease. In this project, the LVAD is modelled as a control system, that modifies the speed of it's rotor to match the current blood flow demands. It uses the current heart rate and the velocity of the body to estimate the demanded heart rate of body. This device is extremely safety critical and the safety of this system is assured by ensuring that the

1. blood does not accumulate in the heart, more precisely the heart should not be expected to handle more than the maximum blood it can pump in each heart beat.

2. LVAD does not create a suction, more precisely the blood flow through the LVAD should not be more than the supplied blood flow.

2 Introduction

Medical Cyber Physical Systems(MCPS) are a class of Cyber Physical Systems(CPS) in which the safety of the patient is of prime importance. This project models such a safety critical Medical device, which is the Left Ventricular Assist Device or as it is more commonly called the LVAD. LVAD is used when the patients heart cannot pump sufficient amount of blood into the body. The LVAD augments the pumping capacity of the heart. This reduces the strain on the weak heart. In most cases a patient without an LVAD is subjected to reduced mobility and activity, as the heart cannot deal with even commonplace tasks which require increased blood flow. An LVAD is useful in such cases, as it greatly improves the quality of life of the patient [1].

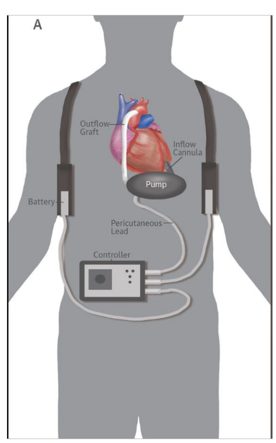

Figure 1: LVAD component diagram

Once a patient has been deemed fit as a candidate for use of LVAD, it is surgically inserted in the upper part of the abdomen. Figure 1 shows the main components of an LVAD. LVAD has a tube that pulls blood from the left ventricle into a pump. The pump then sends the blood into the aorta, from where it is distributed to the rest of the body. Another tube attached to the pump is brought out of the abdominal wall to the outside of the body and attached to the pump's battery and control system. [2] A detailed description of all the components of the LVAD as shown in Figure 1 is present in the appendix.

3 Related Work

In patients that have a weak heart, the blood flow through the heart is capped to the maximum it can handle and remaining blood is handled by the LVAD. The LVAD regulates the blood flow through the heart by taking a portion of the load. Cardiac output is the volume of blood the heart pumps through the circulatory system per minute ($\approx$ 5 L/min at rest) The relationship between heart rate the cardiac output is given by the equation below [3]:

$$CO = V_s \times hr \tag{1}$$

where,

CO	Cardiac Output
V_s	Stroke Volume
hr	Heart rate

Table 1: Term descriptions

Cardiac Output is referred to as Total Blood Flow (BFT) in the rest of this paper. Re-writing the terms:

$$BFT = V_s \times hr \tag{2}$$

To model the system with the blood flow, the pressure difference between the Left Ventricle and the Aorta needs to be measured [4]. This pressure difference is characterized by the following relationship:

$$LVP(t) - AoP(t) = R_i Q + L_i \frac{dQ}{dt} + R_0 Q + L_0 \frac{dQ}{dt} + R_p Q + L_P \frac{dQ}{dt} - H_p + R_k Q \tag{3}$$

$LVP(t)$	Left Venticular Pressure
$AoP(t)$	Aortic Pressure
Q	Blood Flow through the LVAD pump
R_i	Inlet Cannula Resistance
R_p	Pump Resistance
R_0	Outflow Cannula Resistance
L_i	Inlet Cannula Inertance
L_p	Pump Inertance
L_0	Outflow Cannula Inertance
R_k	non-linear time varying resistance, vascular resistance
H_p	Pressure gain across the pump

Table 2: Term Descriptions

Q is referenced as Blood Flow through LVAD (BFL) in the rest of paper. Rewriting the above equation.

$$LVP(t) - AoP(t) = R_i(BFL) + L_i \frac{d(BFL)}{dt} + R_0 BFL + L_0 \frac{d(BFL)}{dt} + R_p(BFL) + L_P \frac{d(BFL)}{dt} - H_p + R_k(BFL) \tag{4}$$

The relationship between BFH and BFL is defined as [4]

$$F_{min} = CO - Q \tag{5}$$

F_{min} models the blood flow through the heart (BFH) which needs to be minimized. Re-writing the above equation

$$BFH = BFT - BFL \tag{6}$$

The model in [4] requires the measurement of vascular resistances, that are not physically measurable. Heart rate on the other hand, is easy to measure. The relationship between heart rate and blood flow rate has already been established in equation 2. In this project, heart rate is used to model the blood flow.

Variations of stroke volume are less than order of $10 - 15\%$ [5]. These variations are seen when a person over-exerts themselves. In this model, it is assumed that a patient that uses an LVAD, will not be able to over-exert themselves (i.e. by running a marathon) therefore, to maintain the simplicity of the model, this project assumes that the stroke volume V_s to be constant value for a person.

The differential equation has been derived using multiple papers, the derivation is mentioned in detail in the ODE Modelling section below, the ODE that was thus arrived at modelled the heart rate, as an exponentially decaying or exponentially increasing function that asymptotically reaches the requested heart rate, known as the demanded heart rate (hrD). This is similar to the ordinary differential equation used in the [6], the controller and the safety conditions in this model are completely different. This model is time triggered, that allows the speed of the rotor to change in steps, while ensuring that the blood does not coagulate in the heart and that the LVAD does not draw more blood, as the demanded heart rate is met.

4 Modelling

The LVAD and the heart are modelled as two pipes in parallel. The flow through is given by BFT, which is the *Blood Flow Total*. The flow through the LVAD and the heart is given by BFL and BFH, which are *Blood flow through LVAD* and *Blood Flow through Heart* respectively. This approximation is shown in Figure 2. The sum of the flows through both the pipes gives us the total flow. This is shown in equation 7

$$BFT = BFL + BFT \tag{7}$$

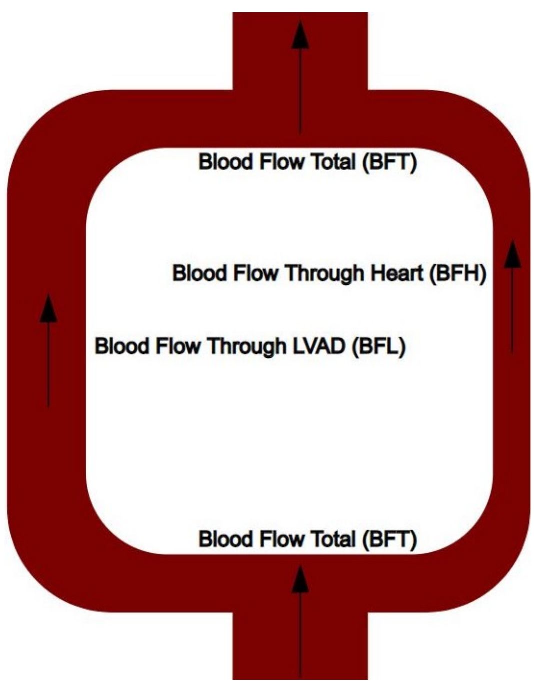

Figure 2: Modelling flow as pipes

Table 3 lists all the variables that are going to be used in the description of our model.

Variable Name	Description
V_S	Volume of blood pumped by heart in each stroke
C	LVAD Constant
BFH_{Max}	Maximum blood flow that the heart can handle
hr_{Rest}	Heart rate at rest
$hr_{Exercise}$	Heart rate at exercise
T	Maximum time T for which the ODE runs
α	Proportion of blood flowing through the heart
hr	Current heart rate
r	Rotor speed
hrD	Demanded heart rate

Table 3: Terms of the Model

4.1 Assumptions

To model something as complicated as the human heart accurately is daunting challenge. However, without being overly precise this model proves and models a number of interesting cases. In this section, all the assumption that have been made to arrive at the model have been captured and were used to prove the safe operation on the LVAD.

The first assumption this project makes is that the stroke volume, V_s is a constant. This assumption has already been discussed earlier in the Related Works section, but is stated here for completeness.

The second assumption is on the term C which is referred to as the LVAD constant in this paper. The constant term encapsulates all the physical specifications of the LVAD. This includes parameters like the diameter of the LVAD tube, current to rotor efficiency and current to rotor speed specification. Essentially the only moving part of the pump is the rotor, and flow of the blood through the pump is controlled solely by it. All the other terms are constants which will not vary. This is an overarching assumption, but it doesn't alter the basic validity of the model.

The heart rate at rest and the heart rate at exercise also have certain assumptions associated with them. Both these terms are constant values. The exercise heart rate comes with the assumption that the exercise does not mean the patient actively performs activities which would push his/her blood flow demand higher. The model assumes that exercise heart rate is below the lactate threshold. The lactate threshold gives us the upper bound at which the body starts producing lactate acid in the muscles and is usually an indicator of heavy activity. This is discussed further in the simplifications made while deriving the ODE. The patient will no be able to perform such activities, the LVAD will improve the quality of life and will allow the patient to perform their daily activities but it will not transform to be able to perform over-reaching tasks. The upper bound in this project does take care of situations which arrive in day to day functioning of a regular person. This distinction is generally made clear to the patient before they choose to implant an LVAD [7]. The only assumption that has been placed on the heart rate at rest is that it is greater than zero.

One abstraction that has been used in the current model is that the rotor speed is being controlled. In actuality, the electric current that is supplied to the rotor is the control variable. However the two are directly proportional and abstraction does not alter our model greatly.

The constant BFH_{Max} gives the maximum allowable flow through the heart, which the heart can handle. Several assumptions are made about this constant. The heart cannot handle the complete flow required on the body independently, even at rest. If it could, then there would be no need of an LVAD. Second assumption that is made is that this constant can be clinically determined based on the how weak the heart is and the heart rate without the LVAD.

4.2 Pre-Conditions

The current model has a number of preconditions. Some are a result of the assumptions made earlier. Most importantly, the terms of the safety condition are also a part of the precondition. This comes from the fact that the model needs to be safe initially, so that it will be safe after running the hybrid program. The precondition used in the final model are listed below.

Pre-Conditions

```
0 < a & a < 1
r > 0
Vs > 0
C > 0
hr > 0
hr > hrRest
hr < hrExercise
hrRest < hrExercise
hrRest > 0
BFHmax > 0
BFHmax < Vs*hrExercise
BFHmax > a*Vs*hr
BFHmax < Vs*hr
(1-a)*Vs*hr >= r*C
T > 0
```

The pre-condition $0 < a \,\&\, a < 1$, conveys that α is bounded by 0 to 1, and more importantly is not inclusive of either 0 or 1. If α value was 0, it would imply that the portion of blood flowing through the heart is 0. All the work would be done by the LVAD. This is not the case and the reasoning behind this is explained in the section pertaining to the safety conditions more clearly. The other extreme, when the value of α is 1 corresponds to the heart taking the entire flow through it. If the heart was capable of doing this, then there would have been no need of an LVAD.

The rotor speed needs to be greater than zero. This ensures that the rotor never stops running. The stroke volume is the amount of blood that the heart pumps out per beat and cannot be a negative. The current heart rate is positive and is further bounded by the heart rate at rest and the heart rate at exercise. These are constants, whose values have been discussed in the previous section. The condition that the heart rate at rest is greater than 0 and that the heart rate at rest is greater than the heart rate at exercise have also been indicated here.

The flow of blood through the heart is given by $a \cdot Vs \cdot hr$, and this term is always lesser than BFH_{Max}. This is also part of safety condition for this model along with $(1-a) \cdot Vs \cdot hr \geq r \cdot C$. These two terms are discussed in detail in the section pertaining to the safety condition. Of note in the precondition are the terms, $BFH_{Max} < Vs \cdot hr_{Exercise}$ and $BFH_{Max} < Vs \cdot hr$. These terms reinforce the bounds on the maximum flow through the heart.

4.3 ODE

The ordinary differential equation for this project, models the exponential decay or growth of the heart rate according to the demanded heart rate. It is a time triggered model.

The ODE for heart rate was has been summarized in this section.

Heart rate kinetics is modelled as [8]:

$$hr'(v,t) = A(hr(v,t) - hrMin)^B (hrMax - hr(v,t))^C (D(v,t) - hr(v,t))^E \tag{8}$$

A patient fitted with an LVAD will not be able to perform extremely strenuous exercise, the motive of the LVAD is to provide the patient with an ability to perform daily tasks without any extra exertion. Hence, it is simple enough assumption that the patient will never break the lactate threshold (running a marathon) due the fragile state of their heart.

Below the lactate threshold, heart rate decays or grows asymptotically towards the demanded heart rate ($D(v,t)$). This can be achieved by setting $B = C = 0$. Rewriting equation 8 with $B = C = 0$

$$hr'(v,t) = A(D(v,t) - hr(v,t))^E \tag{9}$$

E cannot be an even number and has to be $A > 0$.For the simplicity of the model, this project assumes that both E and A as 1. The simplified ODE that is used for this mode is

$$hr'(v,t) = D(v,t) - hr(v,t) \tag{10}$$

hrD is the demanded heart rate and can be estimated by measuring the current velocity of movement. For this project, this measurement is assumed to be done by embedding an acclerometer in the LVAD controller. Heart rate demanded is directly proportional to velocity [9]. M is the constant of proportionality

$$hrD = M \cdot v \tag{11}$$

For a person with an LVAD, it can be assumed that they will be in zone 1 [9]

$$hr' = hrD - hr \tag{12}$$

BFT through a healthy heart is given by the product of the stroke volume and the heart rate. The stroke volume is the amount of blood that the heart can pump in a single beat. The heart rate is the number of beats per minute. Thus, their product gives us the volume of blood that the heart can pump out in a minute, which is the blood flow rate. This is called the *Blood Flow Total* or BFT.

$$BFT = V_s \cdot hr \tag{13}$$

The model is time triggered, the rotor of the LVAD pump is an actuator with a response time and hence it is not practical to continuously change the rotor speed. Instead it is being changed in steps to match the new flow rate estimated as explained in the controller section.

Complete ODE for the model:

ODE

```
{hr' = hrD - hr, t' = 1 & t <= T}
```

T is the maximum time the ODE can be run for before the rotor speed has to be adjusted.

4.4 Controller

Controller

```
{
    hrD:= *; ?(hrD >= hrRest & hrD <= hrExercise);
}
if(hrD > hr)
{
        tVal := min(hrD, (hrD - (1-T+((T^2)/2) -((T^3)/6) )* (hrD - hr)));
        a:=*;?(a>0 & a < BFHmax/(Vs*tVal) & a < 1);
        r :=*; ?(r>0 & r <= ((1-a)*Vs*hr)/C);
}
else
{
        tVal := max(hr, (hrD - (1-T+((T^2)/2)-((T^3)/6))* (hrD - hr)));
        r :=*; ?(r>0 & r <= ((1-a)*Vs*tVal)/C);
}
```

The controller needs to ensure that safety conditions are not violated even after time T. Solution to the ODE:

$$hr = hrD - ((hrD - hr_0) \cdot e^{-t}) \tag{14}$$

Taylor Series Expansion of the solution (truncated at 4 terms):

$$hr = hrD - (hrD - hr_0)(1 - t + \frac{t^2}{2} - \frac{t^3}{6}) \tag{15}$$

The heart rate demand hrD is known after time T, this accounts for the response time of the acclerometer to update the model of the new value. The value of demanded heart rate can be any value within the bounds of $hrRest$ and $hrExercise$

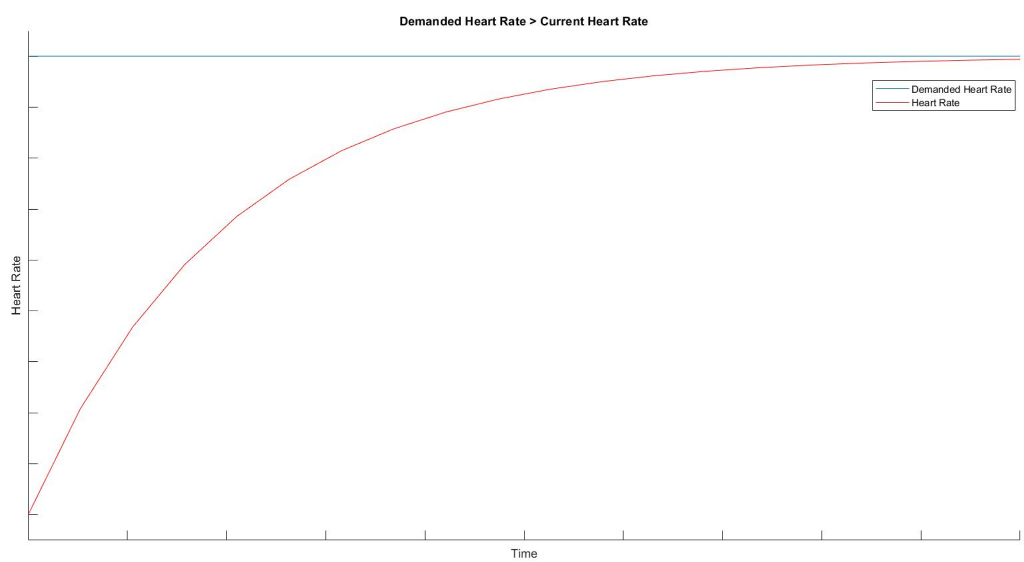

Figure 3: $hrD > hr$

The controller checks if the demanded heart rate hrD is greater than the current heart rate hr, and tries to match the demand by following the curve in Figure 3. In the case where the heart rate is increasing. The value of α, that indicates the proportion of the blood flow through the heart should be adjusted to a value such that after the increased blood flow the amount of blood flowing through the heart indicated by

$$BFH = V_s \cdot hr_T \tag{16}$$

where hr_T is the heart rate after time T and BFH should still be lesser than the maximum blood flow that the heart can handle BFH_{max}. Therefore,

$$BFH_{max} > Vs \cdot hr_T \tag{17}$$

In order to ensure this, the value of hr_T can be estimated by using the Taylor series expansion. In this project, 4 terms of the Taylor series expansion were used to estimate the value of hr at time, T.

$$hr_T = hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr) \tag{18}$$

If the value of T is large then hr_T might overshoot the value of hrD since this is not the expectation as per 3. The minimum of the estimated and the demand is assigned to the value hr_T Equation 18, is modified:

$$hr_T = min(hrD, hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr)) \tag{19}$$

and now α is allowed to take any value of the in the range of $(0 - min(1, \frac{BFH_{max}}{V_s \cdot hr_T}))$

To ensure that the LVAD is not drawing too much blood, the pump rotor speed, r also needs to be adjusted. This is done by controlling the pump current. The constant to map from pump current to rotor speed is dependent on the specification of the rotor.The volumetric flow of blood through the LVAD, BFL is modelled as shown in the equation 20

$$BFL = r \cdot C \tag{20}$$

C is a constant that is dependent on the pump configuration- current draw, efficiency and pump diameter. The total blood flow, BFT, is modelled as shown in the equation 13

$$BFT = V_s \cdot hr \tag{21}$$

$$BFT \geq BFH + BFL \tag{22}$$

$$V_s \cdot hr \geq \alpha \cdot V_s \cdot hr + r \cdot C \tag{23}$$

On solving it the equation 24 is obtained

$$(1 - \alpha) \cdot V_s \cdot hr \geq r \cdot C \tag{24}$$

using equation 24, the value of r can be adjust to be in the range of $(0, \frac{((1 - \alpha) \cdot Vs \cdot hr)}{C}]$

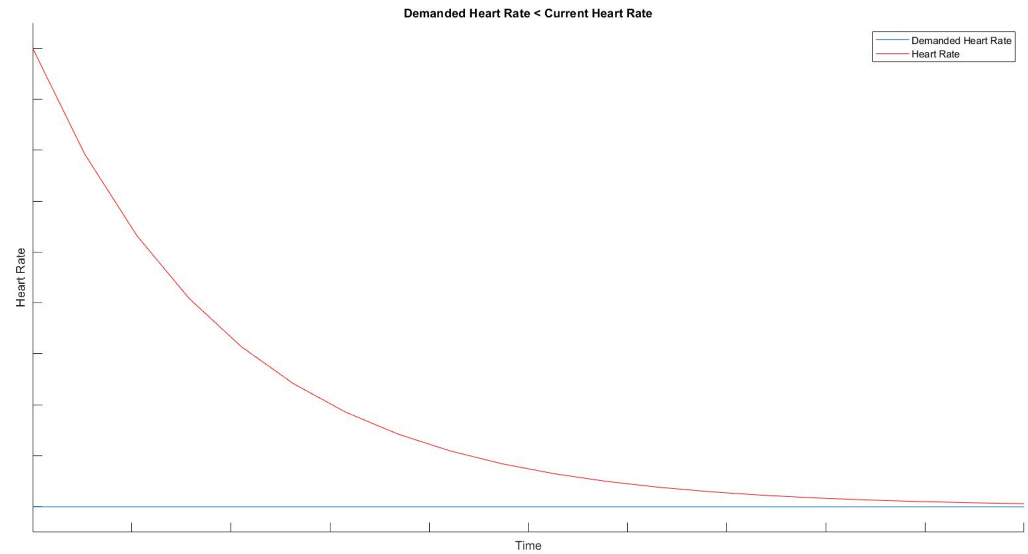

Figure 4: $hrD < hr$

The controller checks if the demanded heart rate hrD is lesser than or equal to than the current heart rate hr, and tries to match the demand by following the curve in Figure 4.

In the case where the heart rate is decreasing. The value of α, can be retained as $hrT < hr$ and if equation 25 is currently satisfied then it will be satisfied for a lower value as well.

$$BFH_{max} > Vs \cdot hr \tag{25}$$

In this case, the rotor value has to be adjusted, to ensure that the equation 22 will still be satisfied after time T.

$$BFT \geq BFH + BFL \tag{26}$$

or more prescisely equation 24 should be satisfied.

$$(1 - \alpha) \cdot V_s \cdot hr_T \geq r \cdot C \tag{27}$$

In order to ensure this, the value can hr_T can be estimated by using the Taylor series expansion. In this project, 4 terms of the Taylor series expansion were used to estimate the value of hr at time, T. Again, if the value of T is large enough there is a change the estimated value of hr_T will be lesser than the demanded heart rate hrD. Therefore,

$$hr_T = max(hrD, hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr)) \tag{28}$$

and now r is allowed to take any value of the in the range of $(0 - \frac{(1-\alpha) \cdot V_s * hr_T}{C}]$

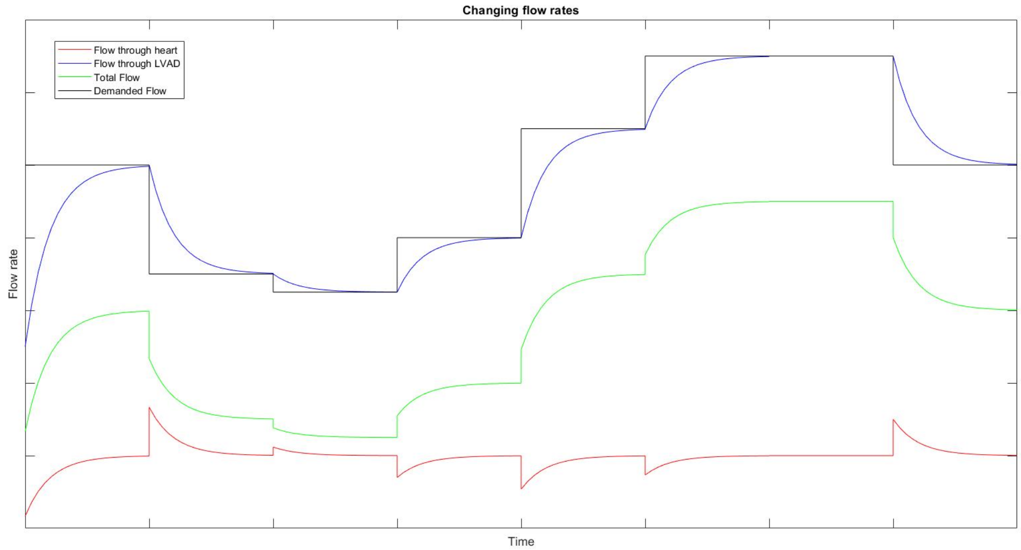

Figure 5: Flow rates over different time steps

Figure 5 plots the varying flow through the heart and the LVAD. It also gives us the total combined flow of the two. In this model the demanded flow is fixed for a given time step.

4.5 Safety

A malfunctioning LVAD can damage the heart. Thus, the safety conditions need to be extremely strong and robust. Two edge cases are considered which when violated will adversely affect the heart. These two edge cases correspond to the two unsafe states that the LVAD can operate in. The first case occurs when the LVAD pulls too much blood away from the heart. The second case deals with the LVAD not drawing enough blood away from the heart.

In the first case of concern is with the LVAD drawing too much blood away from the heart. If this happens then the heart collapses due to a lack of blood to pump. This condition, is called suction and damages the heart tissue [10]. Thus a bound is placed on the maximum value that the LVAD rotor can take. More precisely the flow through the LVAD is limited. The total flow through the LVAD has to be lesser than the difference of the total flow and the flow through the heart. The safety condition can be arrived upon as shown below,

$$\begin{aligned} BFT &> BFH + BFL \\ V_s \cdot hr &\geq \alpha \cdot V_s \cdot hr + C \cdot r \\ (1-\alpha) \cdot V_s \cdot hr &\geq r \cdot C \end{aligned}$$

This equation places an upped bound on the speed of the rotor. Note, that the rotor speed is actually controlled by the current being supplied to the rotor.

The second safety condition has to deal with the other extreme of blood accumulating in the heart. This case occurs when the LVAD is not drawing at least a certain minimum amount of blood away from the heart. When this happens, it leads to a host of other complications as the heart tries to cope up with the increased demand [11]. The weakened heart can handle only up to BFH_{Max}. This quantity is a constant based on the state of heart and can be determined clinically based on how weak the heart is. In this model, the assumption is that heart cannot operate independently. It always needs a LVAD. Equation 29 states that the blood flow through the heart must always be less than the maximum blood flow that the heart can handle. As long as this condition is met, blood will not accumulate in the heart and safe operation of the system continues. The maximum blood flow is always greater than or equal to this value.

$$BFH_{Max} \geq \alpha \cdot V_s \cdot hr \tag{29}$$

4.6 Invariant

The invariant of this model, keeps track of all the variables of the model which are changing. The bounds placed in these variables are fixed and our never violated. Thus the conditions on the heart rate, the proportional flow through heart and the rotor speed checked.

The first two terms of the invariant, are the safety condition and should be satisfied at all times. Thus they naturally are a part of the invariant.

Using the invariant it is ensured that the value of heart rate, never violates the bounds that are set on it. The heart rate is always between the heart rate at rest and the heart rate at exercise.

Invariant

```
( (1-a)*Vs*hr >=r*C )
( BFHmax >= a*Vs*hr )
( hr >= hrRest )
( hr <= hrExercise )
( 0 < a & a < 1 )
( r > 0 )
```

Further the basic two assumptions which imply that the LVAD is always running ($r > 0$) and that the flow through the heart is always constrained by, ($0 < \alpha < 1$) are also part of the invariant. These terms are non-deterministically assigned in the controller. The bounds in the invariant make sure that they do not end up out of the permissible range for any run of the ODE. From a modelling perspective, it is known that the α is non-zero positive. V_s is a constant. hr can take any value between hr_{Rest} and $hr_{Exercise}$. The limiting case, which gives the lower bound value is when the heart rate is the heart rate at rest. This means that even when the patient is at complete rest, the heart is not strong enough to pump blood independently.

5 Proof Strategies

KeYmaera X was used to prove the model. The model was proved in stages, and was slowly built up to the model in 6_LVAD_taylor_4_max_min. This section details the proof of this model, which encompasses all the portions that were proved separately.

The cases $hrD > hr$ and $hrD <= hr$ were separated. In $hrD > hr$ case, it is known the hr will increment towards hrD but will never actually reach hrD to maintain symmetry with the other case, it ODE is dC [12] with $hrD \geq hr$, on differentiating this

$$hrD - hr >= 0 \tag{30}$$

$$-hr' >= 0 \tag{31}$$

$$0 >= hr' \tag{32}$$

$$0 >= hrD - hr \tag{33}$$

It was found these were of the form of Darboux inequality and $p' \geq gp$ and $g = -1$. This is expected as the solution of the ODE is $hr = hrD - ((hrD - hr_0) \cdot e^{-t})$

The ODE was proved using the dbx lemma. To indicate the value of hr is increasing. A dC with $hr >= hr_0$ was performed, hr_0 being the initial value, this was provable by dI as on differentiating it reaches $hrD - hr >= 0$ which is already in the domain constraint.

The min of hr_T was expanded and the case where $hr_T = hrD$ proves by weaken, as the conditions that are required are already in the domain constraint.

The case where $hr_T = hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$, was dC with $hr \leq hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$ on differentiating 4 times reaches $hr \geq hr_0$, which already exists in the domain constraint, 3 dC each with the differential of the previous post condition were made.

And the remaining case was differentialy weakened and was proved by propositional logic, since the current value of $hr \leq hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$ has already been proved the value of the post conditions that were derived will also be true.

In $hrD <= hr$ case, it is known the hr will decrement towards hrD but will never actually reach hrD but to take care of the case the hrD value chosen is the current hr value ODE is dC with $hrD \leq hr$, on differentiating this

$$hrD - hr >= 0 \tag{34}$$

$$-hr' <= 0 \tag{35}$$

$$0 <= hr' \tag{36}$$

$$0 <= hrD - hr \tag{37}$$

This is also of the darboux inequality form with $g = -1$. The ODE was proved using the dbx lemma. To indicate the value of hr is decreasing a dC with $hr \leq hr_0$ was performed, hr_0 being the initial value, this was provable by dI as on differentiating it reaches $hrD - hr <= 0$ which is already in the domain constraint.

The max of hr_T was expanded and the case where $hr_T = hrD$ proves by weaken, as the conditions that are required are already in the domain constraint, that $hrD - hr <= 0$.

The case where $hr_T = hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$, was dC with $hr \geq hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$ differentiating 4 times reaches $hr \leq hr_0$, which already exists in the domain constraint, 3 dC each with the differential of the previous post condition were made.

And the remaining case was differentialy weakened and was proved by propositional logic, since the current value of $hr \geq hrD - (1 - T + \frac{T^2}{2} - \frac{T^3}{6})(hrD - hr_0)$ has already been proved the value of the post conditions that were derived will also be true.

6 Results and Observations

1. Safety of the LVAD can be assured by just varying the rotor speed of the LVAD pump. As the change in the rotor value indirectly controls α, if the rotor speed increase then the flow through the heart is reduced, to match the total flow.

2. This projects has a generalized approach, wherein it does not assign any specific values, this allows for lot of flexibility and can be used to fit any LVAD pump specifications.

3. Taylor Series results in 1 less of its term on differentiating and this was exploited in the proof on each differentiation the the sign to hr flips in the Taylor Series. This does not allow this model to be proved for odd number of terms as it counter intuitive to set the reverse the bounds on hr

7 Deliverables

As discussed earlier our proofs were incrementally developed. In this section a brief description of each of the models used is presented. These models consists of our deliverable that were submitted along with the project.

Model Name: $1\_LVAD\_all\_det$
In this model, the demanded heart rate is chosen non-deterministically between the two edge cases, namely heart rate at rest and heart rate at exercise. To begin with assign 'a' and 'r' values deterministically. This represented the most simplistic case.

Model Name: $2\_LVAD\_all\_non\_det$
In this model, the demanded heart rate (hrD), the proportion of flow through the heart(α) and the rotor speed (r) are all non-deterministically assigned within the appropriate range of value that they can take. For hrD valid ranges of values are the ones shown in the Controller block in section 4.4. The value of α and r lie between the range as shown below.

Non-Det Assignment

```
a:=*;?(a>0 & a < BFHmax/(Vs*hrD) & a < 1);
r :=*; ?(r>0 & r <= ((1-a)*Vs*hr)/C);
```

Model Name: $3\_LVAD\_taylor\_2\_terms$
This model built on the last case, and the use of the Taylor series expansion for the exponential function to approximation the value of the heart rate after time T. The current model was made into a time triggered model and more closely resembles the one that has been discussed in the paper. Here the linear case is considered, that is, only the first two terms of Taylor series expansion

Model Name: $4\_LVAD\_taylor2\_max\_min$
In this model, the heart rate value at time T and the demanded heart rate value are compared one of them is picked. When the demanded heart rate is greater than the current heart rate, take the minimum case is taken. That is the lower of either the approximated Taylor series value of the heart rate or the demanded heart rate is taken to calculate the value of α. Conversely, for the case when the demanded heart rate is lesser than the current heart rate, then the maximum of the two terms is taken to evaluate the value of r.

Model Name: $5\_LVAD\_taylor\_4terms$
In this model the Taylor series expansion of two terms is taken and extend that to the four terms. In this case, the min-max bounds are not used.

Model Name: $6\_LVAD\_taylor\_4\_max\_min$
This is the model, which have presented in the paper.

8 Contributions

Equal work was performed by both project members

9 Future Scope

1. The variation in the stroke volume, can be added to the model to make it more precise.
2. The constants A and E that have been approximated to 1 can also be modified to better represent the model. Though this will change the decay and growth rate, it will not have a large impact on the model as a whole. More differential cuts will be required to get to the darboux inequality form.

3. The current controller is not efficient as it allows α to take on a range of values. This is important from a safety stand point, but is not very efficient.

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Appendix

Description of LVAD components

The primary components of a LVAD, like the one shown in Figure 1 are the Inflow Cannula, Pump, Percutanious Lead, Controller and Batteries. The Inflow Cannula is placed in the left ventricle by making a small incision in the heart. It draws blood from the left ventricle into the pump. It provides an alternate path for the blood flow. The pump is located in the upper abdomen, next to the left ventricle. It contains a a friction less rotor which is magnetically levitated. This reduces the heat generated due to friction. Some rotors have axial rotational speeds of up to 10,000 rotations per minute. An important safety criteria is to ensure that the rotor always operates within a fixed bound, so that the rotor does not draw more blood than can be supplied. The outflow graft connects the rotor output to the aorta. The Percutanious Lead connects the controller to the rotor. The controller is responsible for regulating the rotor speed of the LVAD, by changing the current supplied to the rotor. The controller as an embedded device performs basic functions like; setting alarms, monitoring LVAD performance and allowing data to be stored for analysis. The rechargeable lithium ion batteries, power the LVAD. These need to be carried separately as shown in Figure 1.