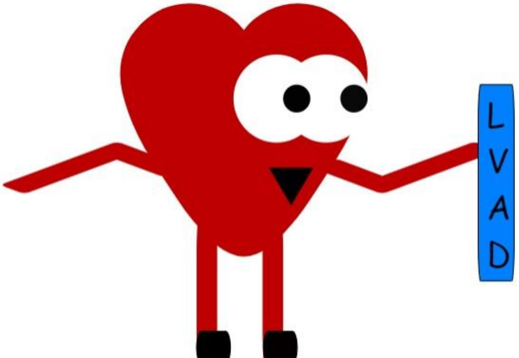
Modelling Safe Left Ventricular Assist Devices



Rishabh Brajabasi and Naina Checka

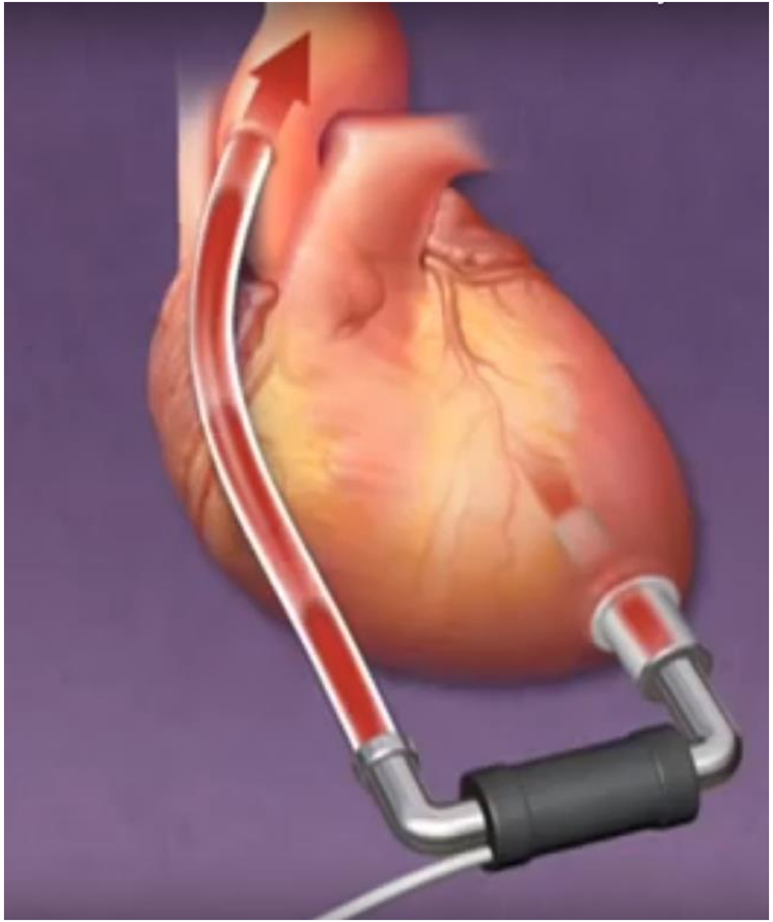
How does an LVAD work ?

Blood enters the heart in the left ventricle.

Exits heart from the aorta.

Heart is too weak to contract completely.

The LVAD provide an alternate path for the blood to flow.



Pipe diagram

Rotational Speed of the pump : r

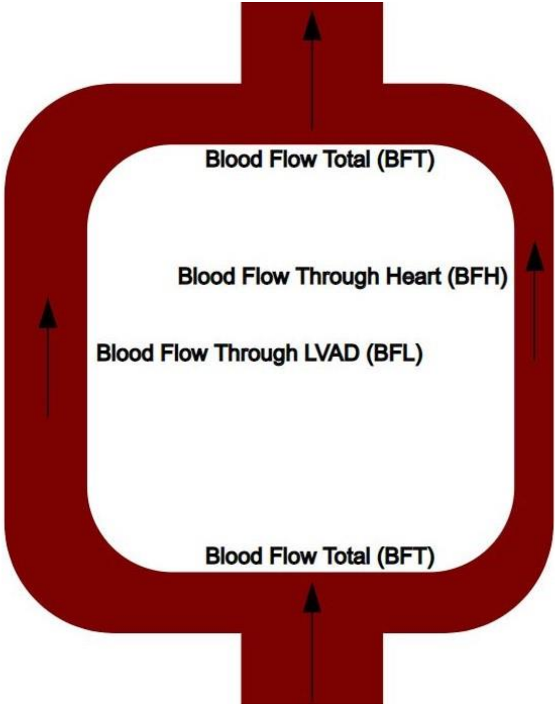
r is dependent on the pump current.

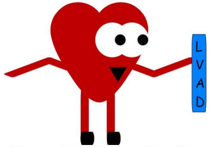
$$r \propto \sqrt{I}$$

Proportion of total blood flow that flows through the heart :

$$a = \frac{BFH}{BFT}$$

$$BFT = V_s \cdot hr$$
$$BFH = \alpha \cdot V_s \cdot hr$$
$$BFL = r \cdot C$$





Safety LVAD

Suction:

LVAD draws too much blood away from the heart.

Set upper limit on rotor speed.

$$BFT \geq BFH + BFL$$

$$V_s \cdot hr \geq \alpha \cdot V_s \cdot hr + C \cdot r$$

$$(1 - \alpha) \cdot V_s \cdot hr \geq r \cdot C$$

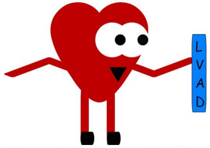
Accumulation:

LVAD **does not** draw too much blood away from the heart.

Set limit on flow allowed through heart.

$$BFH_{Max} \geq \alpha \cdot V_S \cdot hr$$

ODE heart rate



hrD is the demanded heart rate which depends on the activity being performed by the patient.

Demanded heart rate is bound by ***Heart rate at Rest*** and ***Heart rate at Exercise***.

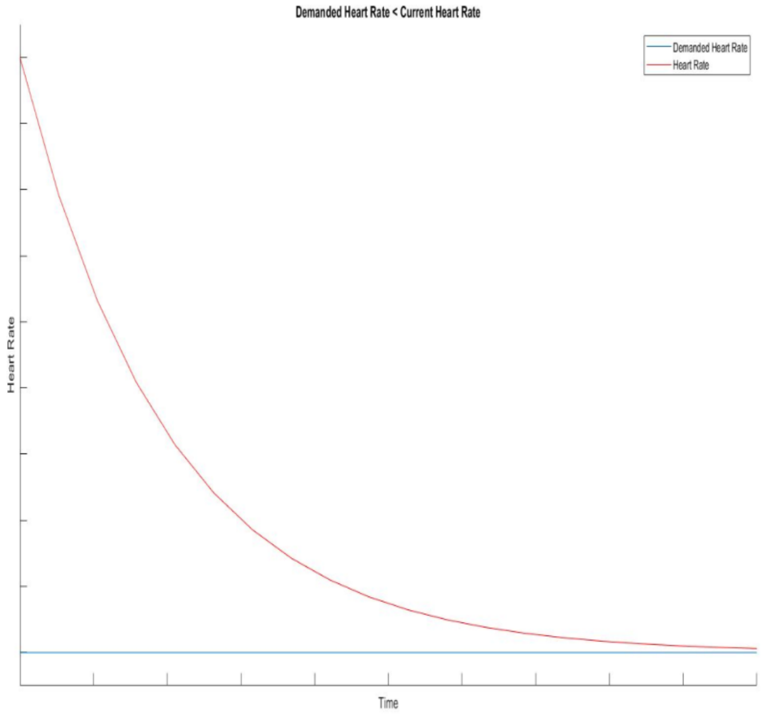
$$hr' = hrD - hr, t' = 1 \,\&\, t <= T$$

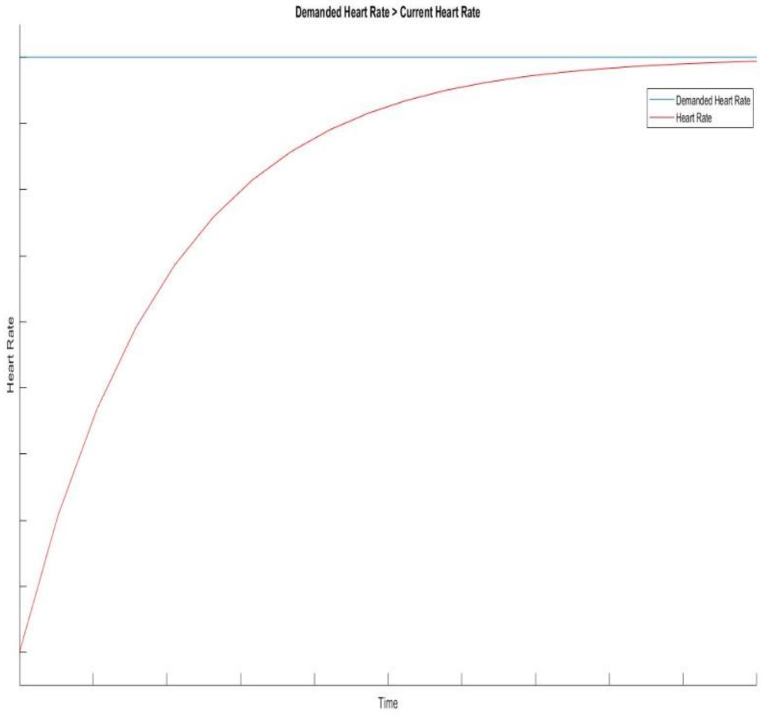
Solution to the ODE

The solution to the ODE is used to estimate the value of hr after time T in the controller

$$hr = hrD - ((hrD - hr_0) \cdot e^{-t})$$

Change in heart rate





CONTROLLER

$$hrD := * \; ; \; ?\,(hrD \geq hr_{Rest} \;\; \& \;\; hrD \leq hr_{Exercise})\,;$$

if (hrD > hr)

{

$$tVal := \min\left(hrD,\; \left(hrD - \left(1 - T + \frac{T^2}{2} - \frac{T^3}{6}\right) * \left(hrD - hr\right)\right)\right)$$

$$a := *;\; ?\left(a > 0 \;\&\; a < \frac{BFHmax}{(Vs * tVal)} \;\&\; a < 1\right)$$

$$r := *;?\left(r > 0 \;\&\; r <= \frac{((1-a) * Vs * hr)}{C}\right)$$

}

else

{

$$tVal := \max\left(hr,\; \left(hrD - \left(1 - T + \frac{T^2}{2} - \frac{T^3}{6}\right) * \left(hrD - hr\right)\right)\right)$$

$$r := *;?\left(r > 0 \;\&\; r <= \frac{(1-a) * Vs * tVal}{C}\right)$$

}

Set demanded heart rate non-deterministically.

Taylor series expansion upto 4 terms.

Finding the safe values of ‘a’ or ‘r’

CONTROLLER

if (hrD > hr)

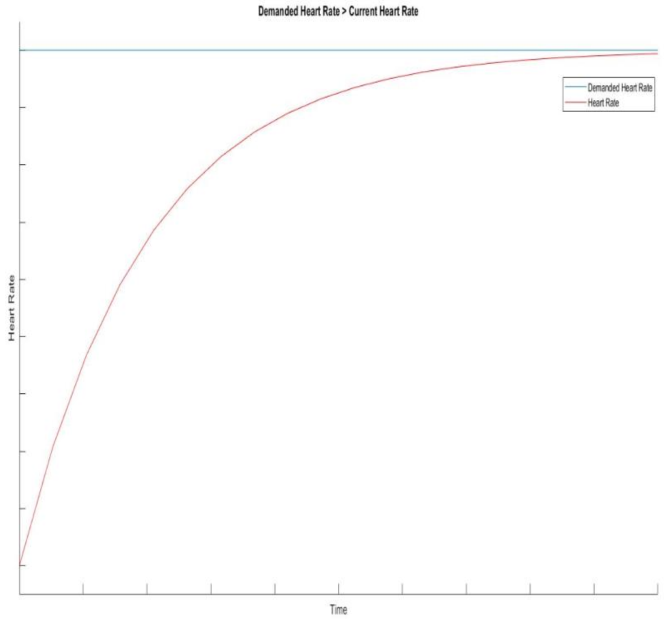
{

$$\text{tVal} := \min\left(\text{hrD}, \left(\text{hrD - }\left(\text{1-T+ }\frac{T^2}{2} - \frac{T^3}{6}\right) * \left(hrD - hr\right)\right)\right)$$

$$a := *;\ ?\left(a > 0\ \&\ a < \frac{BFHmax}{(Vs * tVal)}\ \&\ a < 1\right)$$

$$r := *; ?\left(r > 0\ \&\ r <= \frac{((1-a) * Vs * hr)}{C}\right)$$

}



CONTROLLER

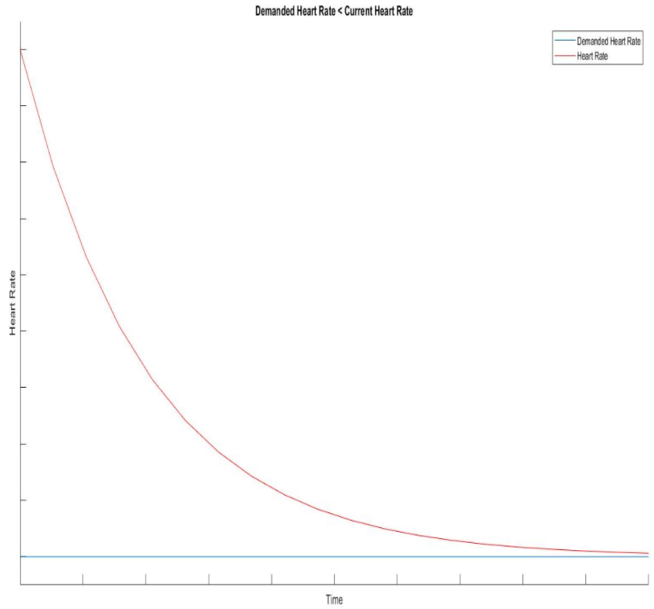
else

{

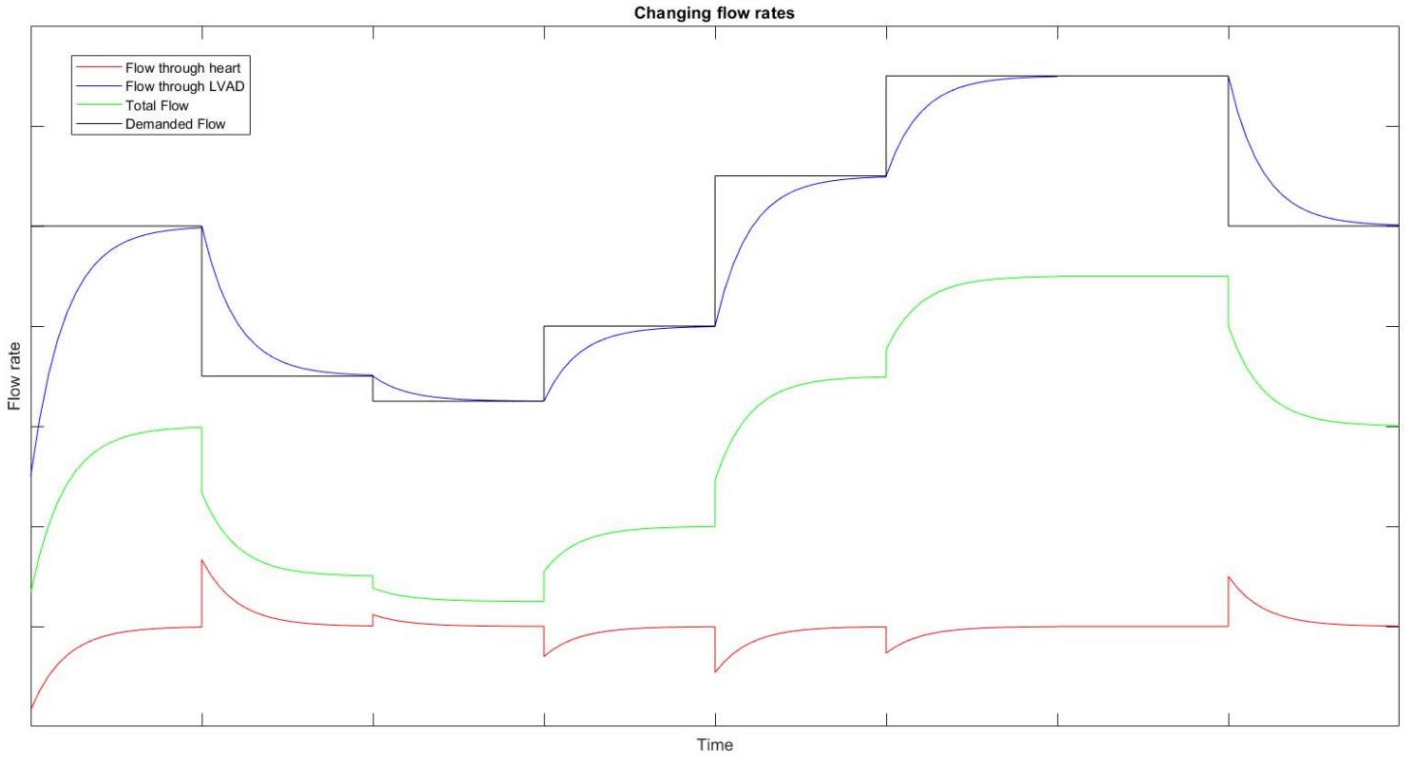
$$tVal := \max\left(hr,\ \left(hrD - \left(1\text{-}T + \frac{T^2}{2} - \frac{T^3}{6}\right) * \left(hrD - hr\right)\right)\right)$$

$$r := *;?\left(r > 0 \,\&\, r <= \frac{(1-a) * Vs * tVal}{C}\right)$$

}



Blood flow adapting to change in demand

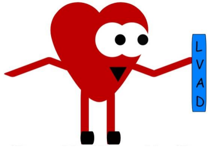


Proof Strategy

```
unfold ; loop({`(1-a)*Vs()*hr>=r*C()&BFHmax()>=a*Vs()*hr&hr>=hrRest()&hr<=hrExercise()&(0 < a&a <
1)&r>0`}, 1) ; <(
  QE,
  QE,
  notGreater(1.0.1.0.1.0.0) ; unfold ; <(
    dC({`hrD>=hr`}, 'R) ; <(
      dC({`hr>=old(hr)`}, 'R) ; <(
        edit({`a < BFHmax()/(Vs()*expand(min((hrD,hrD-(1-T()+T()^2/2-T()^3/6)*(hrD-hr_0)))))`},
        -19) ; orL(-25) ; <(
          dW('R) ; QE,
          dC({`hrD-(1-t+t^2/2-t^3/6)*(hrD-old(hr))-hr>=0`}, 'R) ; <(
            dW('R) ; QE,
            dC({`(1-t+t^2/2)*(hrD-old(hr))-hrD+hr>=0`}, 'R) ; <(
              dI('R),
              dC({`(t-1)*(hrD-old(hr))+hrD-hr>=0`}, 'R) ; <(
                dI('R),
                dI('R)
                )
              )
            )
          ),
        dI('R)
        ),
      dbx(1)
      ),
```

Proof Strategy

```
dC({`hrD<=hr`}, 'R) ; <(
  dC({`hr<=old(hr)`}, 'R) ; <(
    edit({`r<=(1-a)*Vs()*expand(max((hrD,hrD-(1-T()+T()^2/2-T()^3/6)*(hrD-hr_0))))/C()`}, -19)
    ; orL(-22) ; <(
      dW('R) ; QE,
      dC({`hrD-(1-t+t^2/2-t^3/6)*(hrD-old(hr))-hr<=0`}, 'R) ; <(
        dW('R) ; QE,
        dC({`(1-t+t^2/2)*(hrD-old(hr))-hrD+hr<=0`}, 'R) ; <(
          dI('R),
          dC({`(t-1)*(hrD-old(hr))+hrD-hr<=0`}, 'R) ; <(
            dI('R),
            dI('R)
            )
          )
        )
      ),
    dI('R)
    ),
  dbx(1)
  )
 )
)
```



Conclusion

Proved that the safety of the system is always ensured.

Rotor speed is continuously changing to match the changing demand.