Focusing for d$\mathcal{L}$
15-824 Logical Foundations of Cyber-Physical Systems
Fall 2018

Klaas Pruiksma

December 10, 2018

Goal

Develop a focused version of Differential Dynamic Logic(d$\mathcal{L}$) [3], with the intent that it serve as a basis for future work on the proof theory of d$\mathcal{L}$.

What is focusing?

Focused systems of proof, first described by Andreoli [1], restrict what proofs can be constructed. Each focused proof corresponds to a set of unfocused proofs.

Two major restrictions:

- Apply "invertible" proof rules when possible.
- When no invertible rules can be applied, "focus" on a formula and apply non-invertible rules to it until no longer possible.

d$\mathcal{L}$?

d$\mathcal{L}$, or Differential Dynamic Logic, is the system of logic we use to model the behaviour of hybrid systems and to prove properties of those models.

Approach

Followed (at a high level) the approach of Simmons [4]:

- Split the connectives of the logic into *synchronous* and *asynchronous* based on their behaviour when broken down by proof rules.
- Modify the sequent calculus to distinguish the two phases of proof construction.
- Prove logical properties of the resulting system (cut elimination, identity expansion)
- Derive soundness and completeness results from those properties

Results

- A sound (but not complete) focused system for d$\mathcal{L}$.
- Completeness fails (for this particular system) due to iteration $[\alpha^*]$.
- Iteration breaks both cut elimination and identity expansion in this focused setting for separate reasons.

What goes wrong with iteration?

Two separate issues:

- Cut elimination fails (or at least is difficult to prove) because of the *global rules* that break down iteration — the rules for loop invariants and variants.
- (The proof of) identity expansion fails because the rules for breaking down iterations do not reduce the formula to one that is structurally simpler.

Future Work

- Fix the issues with iteration to arrive at a sound and complete focused system.
- Investigate how such a system may be of use for normalizing proofs in a more general sense.

References I

[1] Jean-Marc Andreoli. Logic programming with focusing proofs in linear logic. *Journal of Logic and Computation*, 2(3): 297–347, 1992.

[2] Chuck Liang and Dale Miller. Focusing and polarization in linear, intuitionistic, and classical logics. *Theoretical Computer Science*, 410(46):4747–4768, 2009.

[3] André Platzer. Differential dynamic logic for hybrid systems. *J. Autom. Reas.*, 41(2):143–189, 2008. ISSN 0168-7433. doi: 10.1007/s10817-008-9103-8.

[4] Robert J Simmons. Structural focalization. *ACM Transactions on Computational Logic (TOCL)*, 15(3):21, 2014.

Questions?