BusyBees

Safe Controllers for Multi-Agent Swarms

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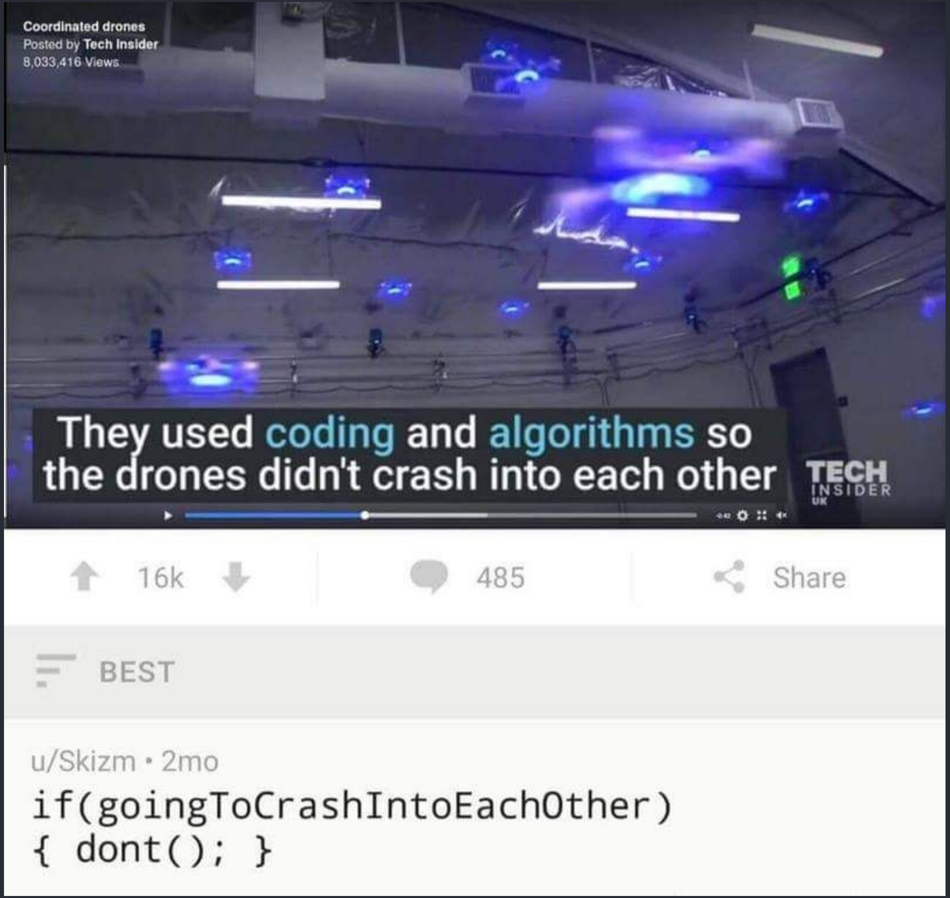
Overview

- Motivation
- Prior Work
- System Model
- 1D Case
- 2D Cases
- Applications

Robotics is Hard

https://spectrum.ieee.org/automaton/robotics/humanoids/darpa-robotics-challenge-robots-falling



https://imgur.com/gallery/qv1gQ

Biology Makes Swarms Look Easy

https://northfortynews.com/its-swarm-season-heres-what-to-do/

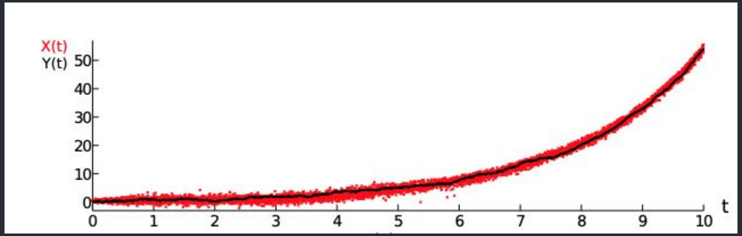
https://en.wikiversity.org/wiki/Algorithm_models/Grey_Wolf_Optimizer

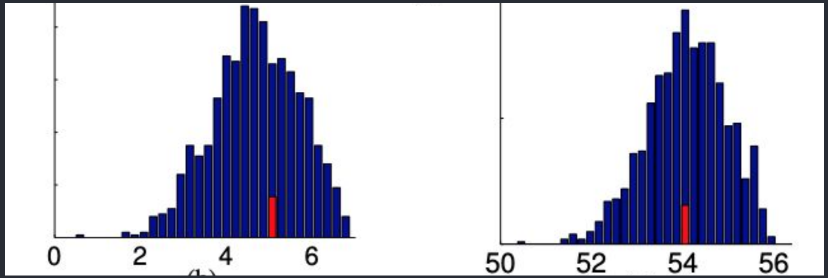
Goal

- What does it mean for a swarm to be “safe”?
- How do we design controllers for safe swarms?
- Applying kinematic principles to swarm controller design

Prior work

- Probabilistic models of swarms
  - Point masses with holonomic dynamics
  - Vector fields direct agents towards a clustering point
  - Good for describing large-scale dynamics
  - Poor at ensuring safety and collision-free behavior





Sartoretti, G., Hongler, M., and Filliger, R. (2014). The estimation problem and heterogeneous swarms of autonomous agents Stochastic Modeling Techniques and Data Analysis International Conference.

Prior work

- Barrier Certificates
  - Define safe set bounded by some barrier function
  - Correctly defined barrier function => always remain in safe set!
  - Provably safe collision-free controllers for n-agent swarms
  - Only physically close agents need to worry about collisions
  - Agents collaboratively brake and accelerate to avoid collision
  - Approximated differential dynamics as holonomic, not solid proof

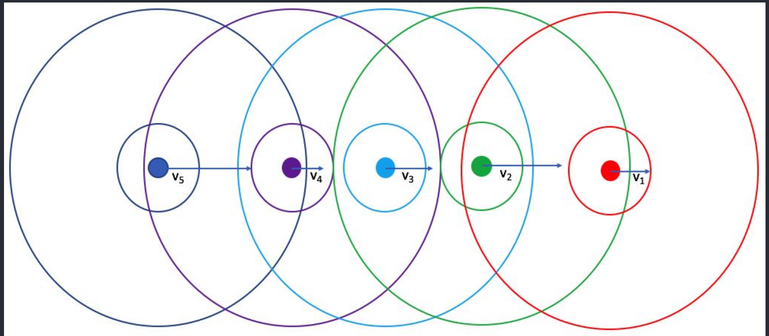
Model design

- n-agent system of differential drive agents
  - Maximum braking and acceleration [-B,A]
  - Non-negative velocity and maximum velocity
  - Minimum turning radius
- Time-triggered controller
  - All agents make coordinated decisions

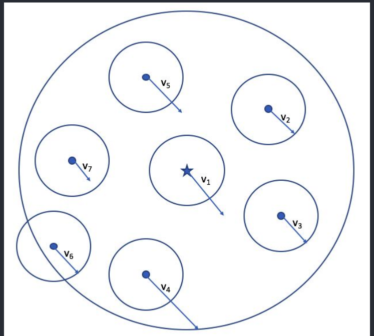
http://blog.ascens-ist.eu/2011/03/ensembles-and-mobile-robots-what-is-the-link/index.html

Model design

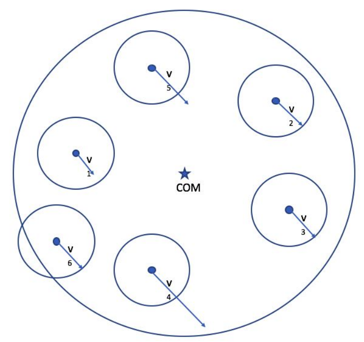
- Two safety constraints
  - Minimum distance - Can’t get too close to collide
  - Maximum distance - Can’t move too far apart or swarm disperses
- Maximum distance constraint depends on swarm structure



Train-like swarm



Heterogeneous Clustered swarm



Homogeneous Clustered swarm

Model Design

- Continuous Dynamics

$$\{x' = v \cdot d_x,\ y' = v \cdot d_y,\ v' = a,\ d_x' = -v \cdot d_y/r\ ,\ d_y' = v \cdot d_x/r,\ t' = 1$$
$$\&\ (v \geq 0 \land v \leq v_{max} \land t \leq T)\}$$

- Infinity norm rather than Euclidean for distance constraints

$$\max(abs(x_i - x_j), abs(y_i - y_j)) \geq r_{min}$$

$$\max(abs(x_i - x_j), abs(y_i - y_j)) \leq r_{max}$$

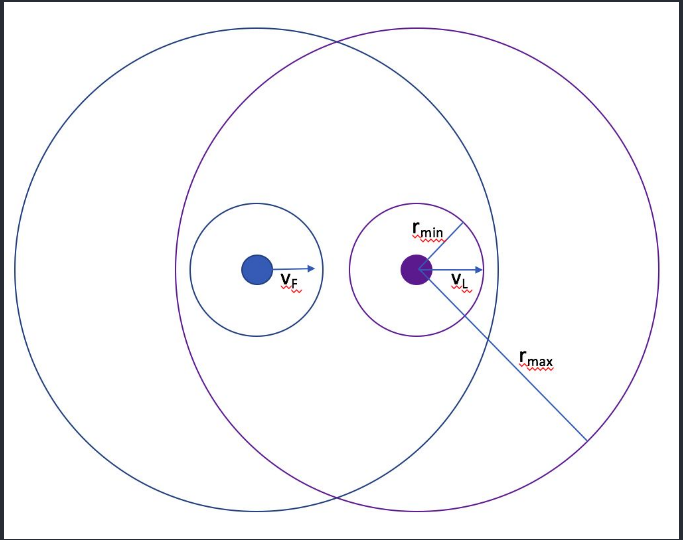
For safety of agents i and j, i != j

2-Agent Train on a Line

- Two agents on a line, want to satisfy both safety constraints
- Necessity of velocity constraints
- Agents do not collaborate in acceleration decisions
- ODE needs to be only for 1D case

$\{x_F' = v_F,\ x_L' = v_L,\ v_F' = a_F,\ v_L' = a_L,\ t' = 1$

$\&\ (v_F \geq 0 \land v_F \leq v_{max} \land v_L \geq 0 \land v_L \leq v_{max} \land t \leq T)\}$



2-Agent Train on a Line

- System Invariants

$$(x_L - x_F) + \frac{(v_L - v_F)v_L}{B} - \frac{(v_L - v_F)^2}{2B} \geq r_{min}$$

$$(x_L - x_F) + \frac{(v_L - v_F)(v_{max} - v_L)}{A} + \frac{(v_L - v_F)^2}{2A} \leq r_{max}$$

2-Agent Train on a Line

- Control Decisions for Follower

$$((x_L + v_L T + \frac{-BT^2}{2}) - (x_F + v_F T + \frac{a_F T^2}{2}))$$
$$+ \frac{((v_L - BT) - (v_F + a_F T))(v_L - BT)}{B} - \frac{((v_L - BT) - (v_F + a_F T))^2}{2B} \geq r_{min}$$

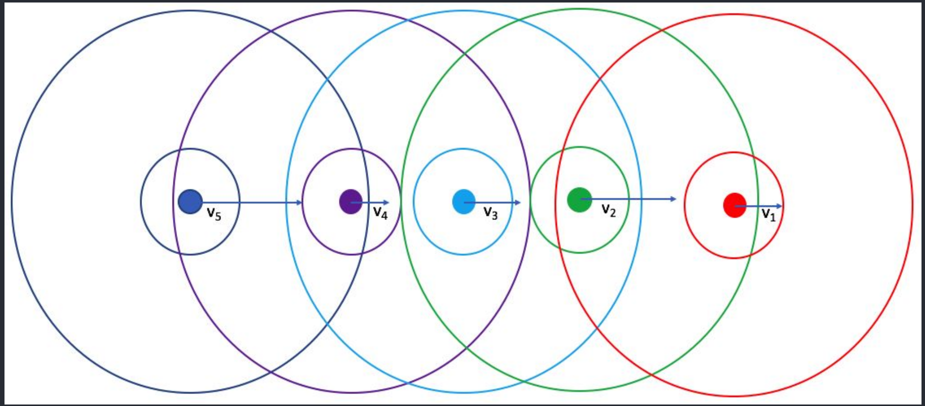
$$((x_L + v_L T + \frac{AT^2}{2}) - (x_F + v_F T + \frac{a_F T^2}{2}))$$
$$+ \frac{((v_L + AT) - (v_F + a_F T))(v_{max} - (v_L + AT))}{A}$$
$$+ \frac{((v_L + AT) - (v_F + a_F T))^2}{2A} \leq r_{max}$$

2-Agent Train on a Line

- Proof of Safety
  - Straightforward due to solvable ODE’s
  - Follower control decisions are derived from kinematics
  - Concern of vacuosity of control decisions

n-Agent Train on a Line

- Note the atomic nature of the 2-Agent controller
  - Leader agent does not base control decisions on the state of the Follower
  - Agent i makes control decisions based upon state of agent i-1
  - n-agent system is now n-1 2-Agent system
- We can’t model and prove an n-agent system with dL and KeYmaera X
- QdL and inductive arguments must suffice for now



n-Agent Train on a Line

- Convert the 2-Agent controller to QdL

$$ctrl \equiv \forall i : C a(i) := *; ?(a(i) \leq A \wedge a(i) \geq B$$

$$\wedge closeSafetyConstraint(i-1, i)$$

$$\wedge farSafetyConstraint(i-1, i))$$

$$evol \equiv t := 0; \forall i : C x(i)' = v(i), v(i)' = a(i),$$

$$t' = 1 \& v(i) \geq 0 \wedge v(i) \leq v_{max} \wedge t \leq T$$

n-Agent Train on a Line

- Proving safety of n-Agent System
  - Need to worry about transitive safety of the system
  - Gödel's Generalization Rule helps the proof become modular
  - Proof of 2-Agent system allows for application to the n-Agent case
  - Change control or dynamics in 2-Agent, generalizes to n-Agent

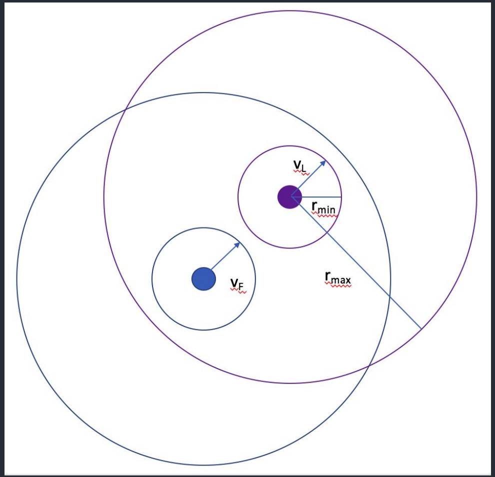
$$\frac{\phi \rightarrow \psi}{[\alpha]\phi \rightarrow [\alpha]\psi}$$

Agents on a Plane

- Moving from 1D to 2D with rotational dynamics is hard
  - Modified 1D controls should work for holonomic agents
  - Circular dynamics makes even the 2-agent case extremely challenging
  - Maximum distance safety constraint becomes the source of challenges
- Currently modeled system has all agents having the same controls
- Let’s look at the challenges and insights

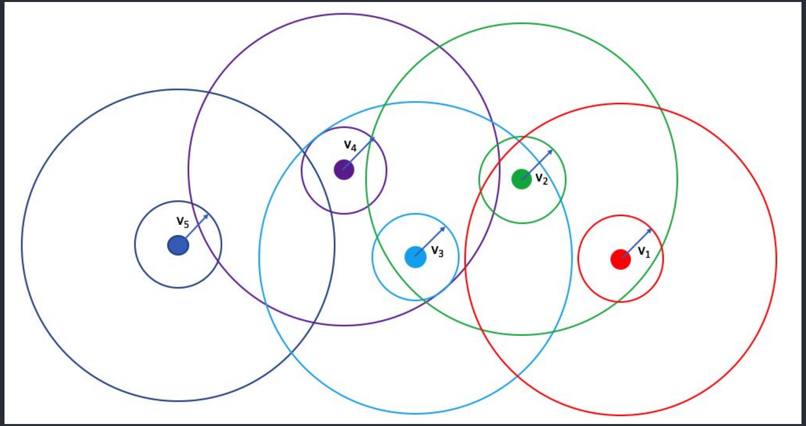
Agents on a Plane

- 2-Agent train in 2D
  - Control system must have coordinated actions between leader and follower
  - Large minimum turning radius forces collaborative actions
  - Velocity-dependent minimum turning radius may bring further insights



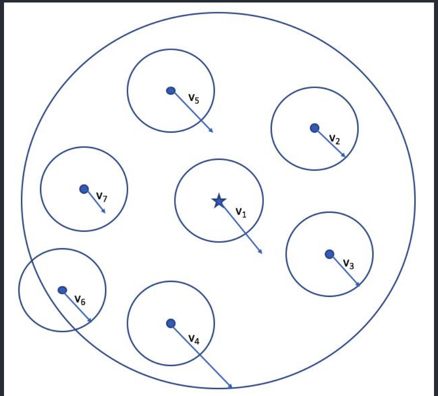
Agents on a Plane

- n-Agent Train in 2D
  - Can no longer atomically consider pairs of adjacent agents
  - Train crossing on itself can result in collisions, no longer modular
  - Can constrict motion to only one direction, prevent large changes in orientation



Agents on a Plane

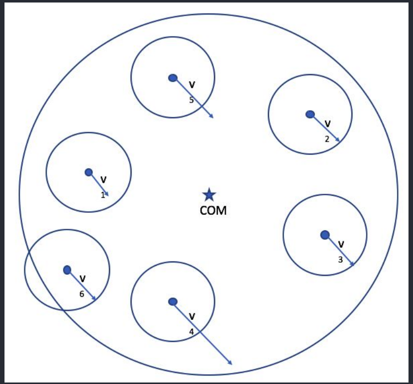
- n-Agent Heterogeneous Cluster
  - An advanced n-Agent train controller will likely be applicable
  - Quadratic increase in number of safety constraints
  - Need to identify when agents need to collaborate in control decisions



Agents on a Plane

- n-Agent Homogeneous Cluster
  - Center of Mass dynamics are very similar to dynamics of each agent
  - Differential invariants applied to agents can be applied to COM as well
  - COM constrained by dynamics of fastest-moving agent in swarm

$$M' = \frac{1}{\sum_{i=1}^{n} m_i} \sum_{i=1}^{n} m_i \mathbf{x_i}'$$



Applications

- n-Agent Heterogeneous Cluster
  - Agents moving within constrained factory environment
  - Use immobile “dummy” leader agent to model walls of factory
- n-Agent Train in 1D
  - Biomedical applications
  - Drug delivery robots in arteries
  - Robotic catheters for clearing blood clots

https://money.cnn.com/2014/05/22/technology/amazon-robots/index.html

Summary

- Safety of 2-Agents moving along line
- Modularity of 2-Agent Train control in 1D extends to n-Agent
- Analysis of challenges in the 2D case, future work