

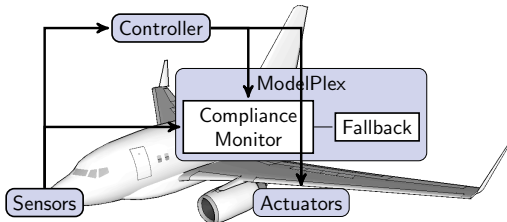
14: Verified Models & Verified Runtime Validation

15-424: Foundations of Cyber-Physical Systems

Stefan Mitsch André Platzer

Computer Science Department
Carnegie Mellon University, Pittsburgh, PA

Simplex for Hybrid System Models (FMSD'16)



- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

Correctness Questions in Complex System Design

Safety The system must be safe under all circumstances

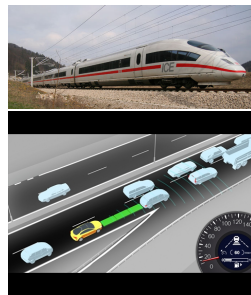
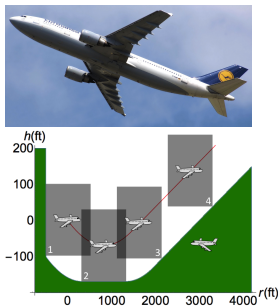
Liveness The system must reach a given goal

How do we make cyber-physical systems safe?

Extensive testing?

Code reviews?

When are we done? How many test cases are enough? Did we cover all relevant tests?



Benefits of Logical Foundations for CPS V & V

Proofs

LICS'12, JAR'16

Safety Formalize system properties: What is “Safe”? “Reach goal”?

Models Formalize system models, clarify behavior

Assumptions Make assumptions explicit rather than silently

Predictions Safety analysis predicts behavior for infinitely many cases

Constraints Reveal invariants, switching conditions, operating conditions

Design Invariants/proofs guide safe controller design

Byproducts

Analysis Determine design trade-offs & feasibility early

arXiv

Synthesis Turn models into code & safety monitors

ModelPlex

Certificate Proofs as evidence for certification

CPP'16

Tools

KeYmaera X aXiomatic Tactical Theorem Prover for CPS

CADE'15

An aXiomatic Tactical Theorem Prover for CPS

<http://keymaeraX.org/>

KeYmaera X Dashboard Models Proofs 2 Help ▾

Hybrid Car ▶ Auto ✎ Normalize ⏮ Step back

Propositional ▾ Quantifiers ▾ Hybrid Programs ▾ Differential Equations ▾ Closing ▾

ImPLYR(1) & loop("v >= 0")(1) & on(("Induction Step", composeb(1) & choiceb(1) & assignb(1, 0::Nil) & choiceb(1, 1::Nil) & assignb(1, 1::0::Nil)), ("Base Case", QE), ("Use Ce" Execute ▾

Base Case 4 Use Case 5 Induction Step 11

Base Case 4: $\vdash -1: v \geq 0 \wedge B > 0 \wedge A \geq 0$

Use Case 5: $\vdash 1: [\{x'=v, v'=A \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0) \wedge [\{x'=v, v'=0 \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0) \wedge [a := -B] [\{x'=v, v'=a \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0)$

Induction Step 11: $\vdash [\{x'=v, v'=A \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0) \wedge [a := 0 \vee a := -B] [\{x'=v, v'=a \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0)$

Proof Step

$[a := A] [\{x'=v, v'=a \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0) \wedge [a := 0 \vee a := -B] [\{x'=v, v'=a \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0)$

$[a := A \vee a := 0 \vee a := -B] [\{x'=v, v'=a \wedge v \geq 0\}] (v \geq 0 \wedge B > 0 \wedge A \geq 0)$

$[a := A \vee a := 0 \vee a := -B]; \{x'=v, v'=a \wedge v \geq 0\} (v \geq 0 \wedge B > 0 \wedge A \geq 0)$

$[a := A \vee a := 0 \vee a := -B]; \{x'=v, v'=a \wedge v \geq 0\}^* v \geq 0$

$v \geq 0 \wedge A > 0 \wedge B > 0 \rightarrow [\{a := A \vee a := 0 \vee a := -B\}; \{x'=v, v'=a \wedge v \geq 0\}^*] v \geq 0$

Proof Step

$[a := c] p(x) \leftrightarrow p(c)$

$G \square$

$\frac{\Gamma \vdash [a] a = -B, \Delta \quad \frac{a = -B \quad \Gamma \vdash P}{\Gamma \vdash [a] P, \Delta}}{\Gamma \vdash [a] P, \Delta}$

An aXiomatic Tactical Theorem Prover for CPS

<http://keymaeraX.org/>

- Small Core** Increases trust, modularity, enables experimentation (1677)
- Tactics** Bridging between small core and (Hilbert)
powerful reasoning steps (Sequent++)
- Separation** Tactics can make courageous inferences
Core establishes soundness
- Search&Do** Search-based tactics that follow proof search strategies
Constructive tactics that directly build a proof
- Interaction** Interactive proofs mixed with tactical proofs and proof search
- Extensible** Flexible for new algorithms, new tactics, new logics, new
proof rules, new axioms, ...
- Customize** Modular user interface, API

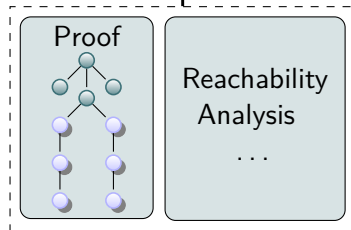
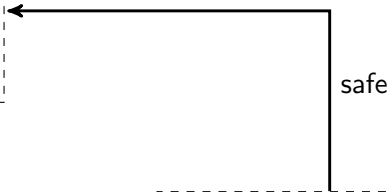
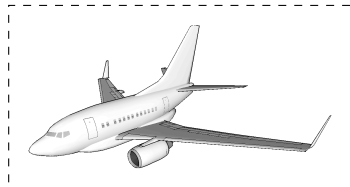
KeYmaera X Microkernel for Soundness

| \approx LOC | | |
|---------------|-----------------------------|-------------------|
| KeYmaera X | 1 652 | } hybrid prover |
| KeYmaera | 65 989 | |
| KeY | 51 328 | } Java |
| Nuprl | 15 000 + 50 000 | |
| MetaPRL | 8 196 | } general math |
| Isabelle/Pure | 8 913 | |
| Coq | 16 538 | |
| HOL Light | 396 | |
| PHAVer | 30 000 | } hybrid verifier |
| HSolver | 20 000 | |
| SpaceEx | 100 000 | |
| Flow* | 25 000 | |
| dReal | 50 000 + millions | |
| HyCreate2 | 6 081 + user model analysis | |

Disclaimer: Self-reported estimates of the soundness-critical lines of code + rules

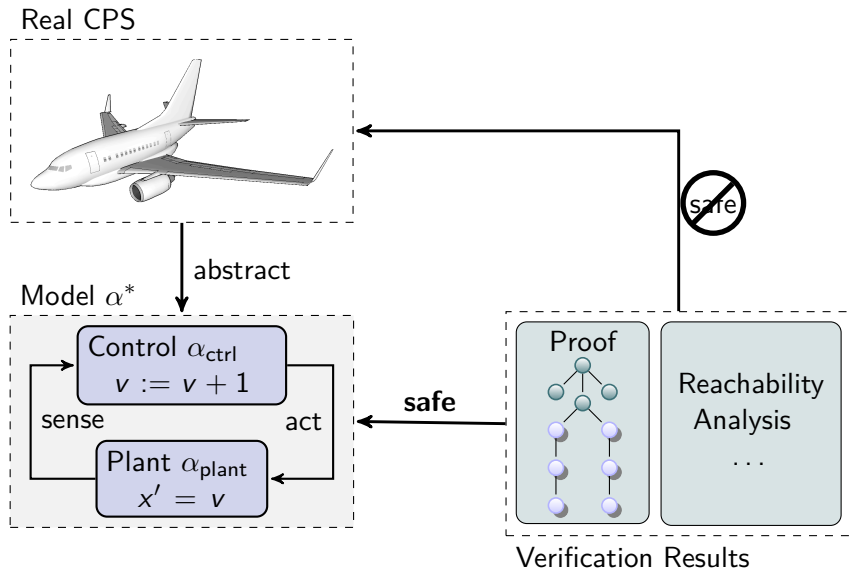
Formal Verification in CPS Development

Real CPS

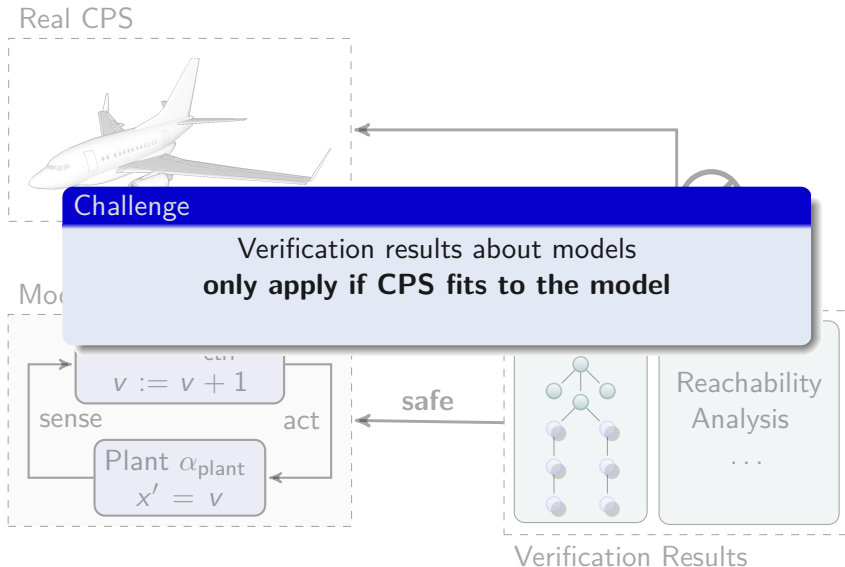


Verification Results

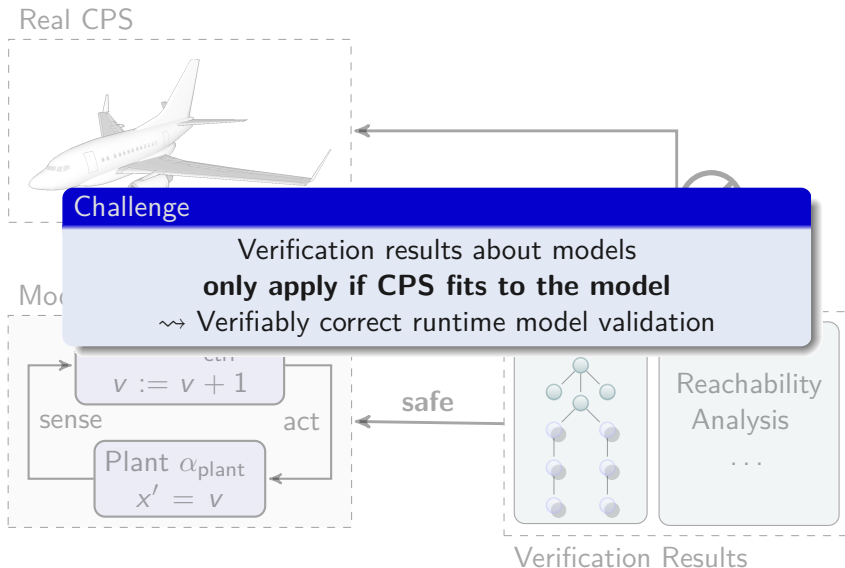
Formal Verification in CPS Development



Formal Verification in CPS Development



Formal Verification in CPS Development

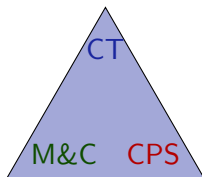


- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

Learning Objectives

Verified Models & Verified Runtime Validation

proof in a model vs. truth in reality
tracing assumptions
turning provers upside down
correct-by-construction
dynamic contracts
proofs for CPS implementations



models vs. reality
inevitable differences
model compliance
architectural design

tame CPS complexity
prediction vs. run
runtime validation
online monitor

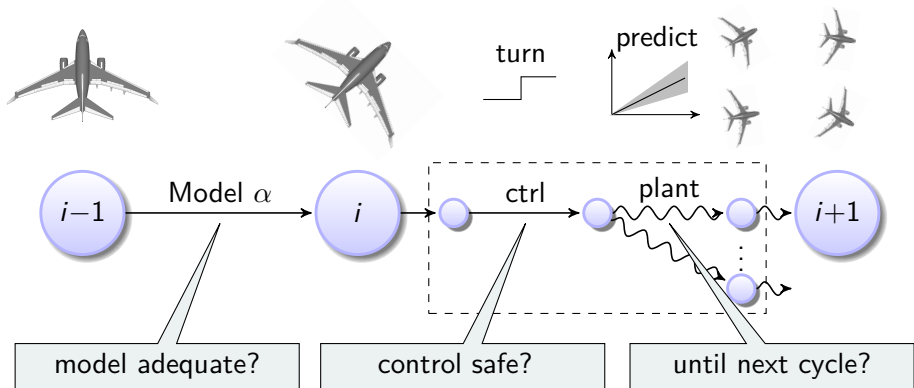
ModelPlex ensures that verification results about models
apply to CPS implementations

Outline

- 1 Motivation
- 2 Learning Objectives
- 3 **ModelPlex Runtime**
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

ModelPlex Runtime Model Validation

ModelPlex **ensures** that **verification results** about models
apply to CPS implementations



ModelPlex Runtime Model Validation

ModelPlex **ensures that verification results** about models
apply to CPS implementations

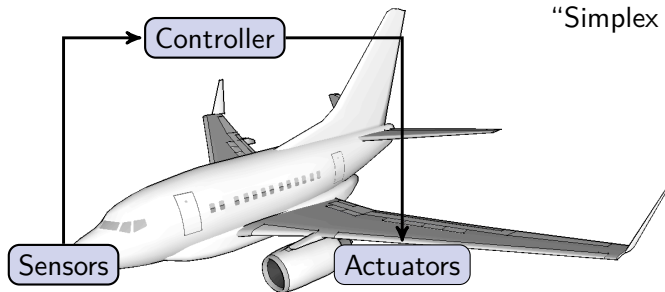
Contributions

- Verification results about models transfer to CPS when validating model compliance
- Compliance with model is characterizable in logic
- Compliance formula transformed by proof to executable monitor
- Correct-by-construction provably correct runtime model validation

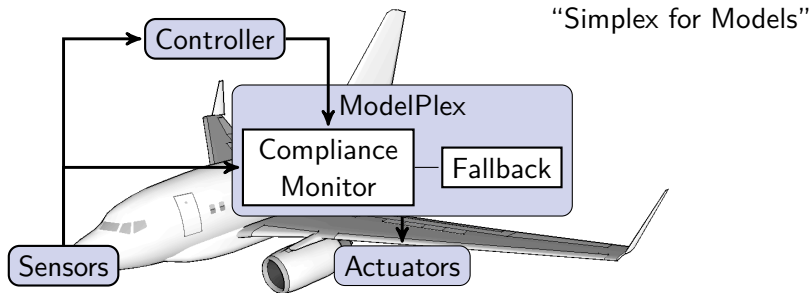
model adequate?

control safe?

until next cycle?



“Simplex for Models”



Compliance Monitor Checks CPS for compliance with model at runtime

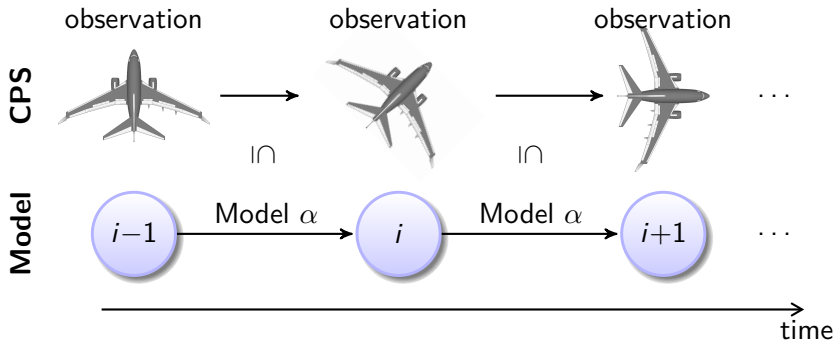
- Model Monitor: model adequate?
- Controller Monitor: control safe?
- Prediction Monitor: until next cycle?

Fallback Safe action, executed when monitor is not satisfied (veto)

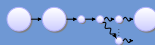
Challenge What conditions do the monitors need to check to be safe?

Is current CPS behavior included in the behavior of the model?

- CPS observed through sensors
- Model describes behavior of CPS between states

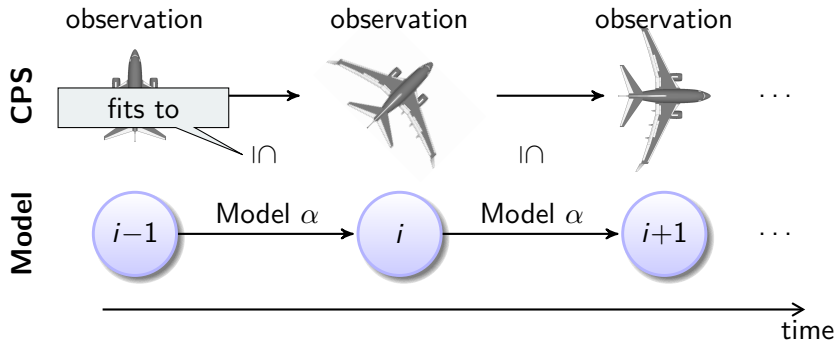


Detect non-compliance ASAP to initiate fallback actions while still safe

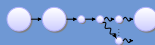


Is current CPS behavior included in the behavior of the model?

- CPS observed through sensors
- Model describes behavior of CPS between states



Detect non-compliance ASAP to initiate fallback actions while still safe



Is current CPS behavior included in the behavior of the model?

- CPS observed through sensors
- Model describes behavior of CPS between states

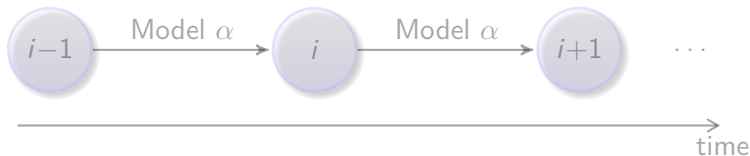
Challenge

CPS

Model describes behavior,
but at runtime we get sampled observations

~> **Transform model into observation-monitor**

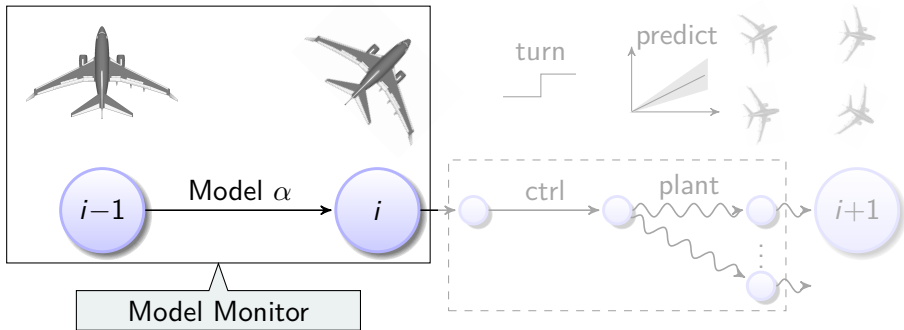
Model



Detect non-compliance ASAP to initiate fallback actions while still safe

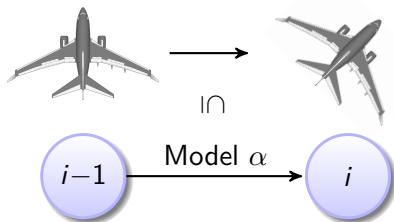
- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

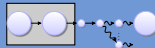
Outline



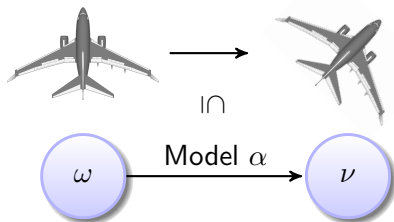


When are two states linked through a run of model α ?



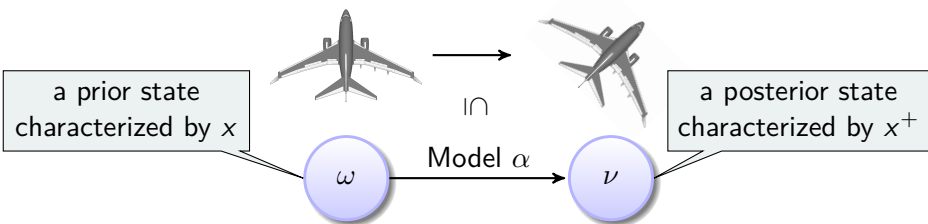


When are two states linked through a run of model α ?





When are two states linked through a run of model α ?



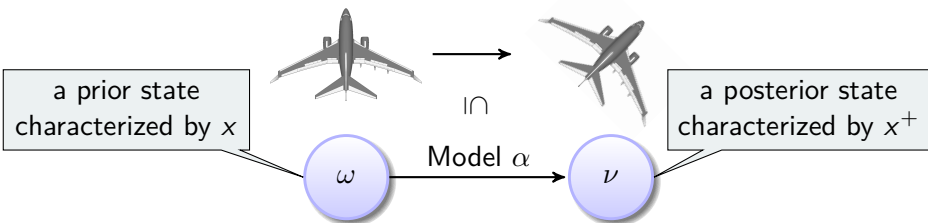
Semantical:

$$(\omega, \nu) \in \llbracket \alpha \rrbracket$$

reachability relation of α



When are two states linked through a run of model α ?



Offline

Semantical:

$$(\omega, \nu) \in \llbracket \alpha \rrbracket$$

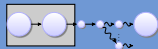


Lemma

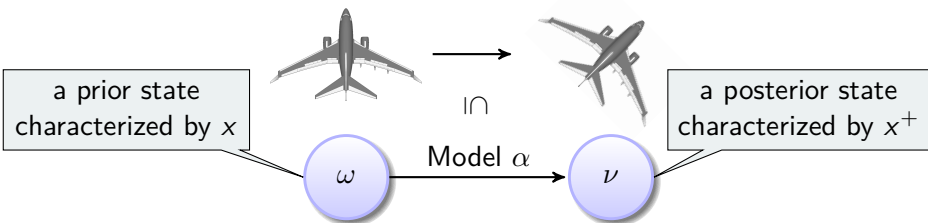
Logical dL:

$$(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$$

exists a run of α to a state where $x = x^+$



When are two states linked through a run of model α ?



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

\Leftrightarrow Lemma

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

\Leftrightarrow d \mathcal{L} proof

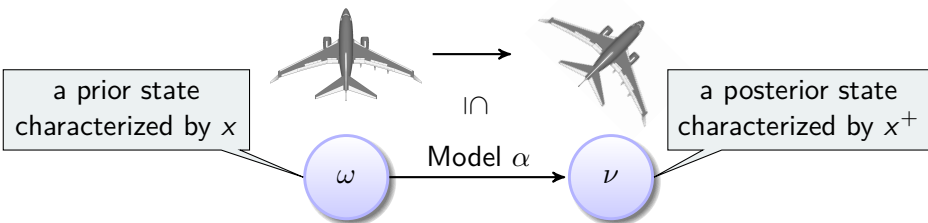
Arithmetical: $(\omega, \nu) \models F(x, x^+)$

exists a run of α to a state where $x = x^+$

check at runtime (efficient)



When are two states linked through a run of model α ?



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

\Downarrow Lemma

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

\Uparrow d \mathcal{L} proof

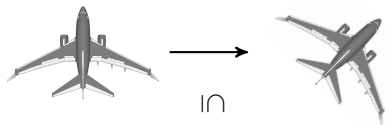
Arithmetical: $(\omega, \nu) \models F(x, x^+)$

exists a run of α to a state where $x = x^+$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

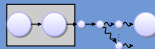
\Downarrow Lemma

Logical dL: $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

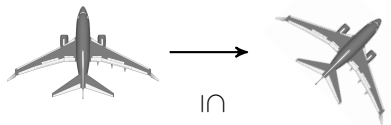
\Uparrow dL proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Offline

Init $\omega \models A$

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

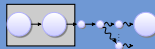
\Downarrow Lemma

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

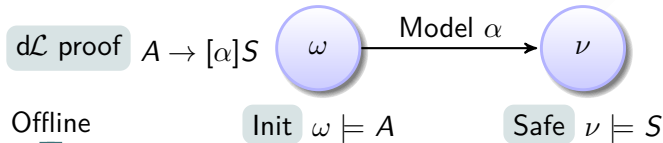
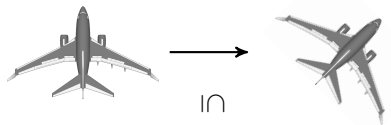
\Uparrow d \mathcal{L} proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

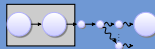
\Downarrow **Lemma**

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

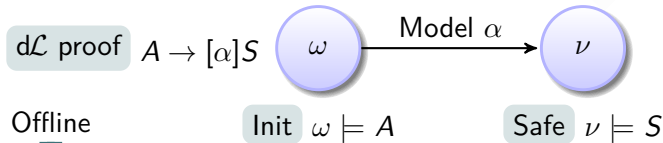
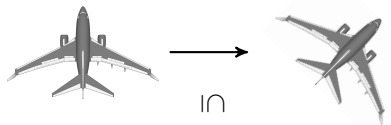
\Uparrow **d \mathcal{L} proof**

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

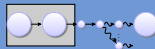
\Downarrow Lemma

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

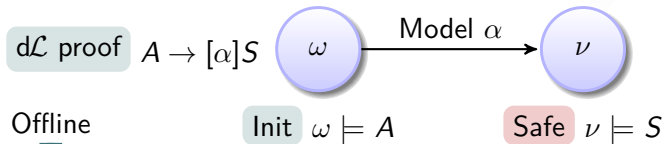
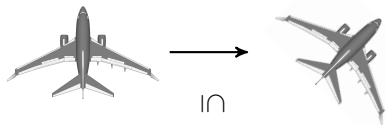
\Uparrow d \mathcal{L} proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces **CPS safety** to runtime monitor with offline proof



Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

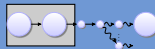
\Downarrow **Lemma**

Logical dL: $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

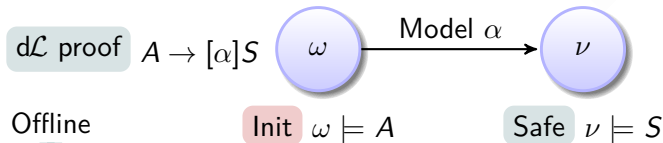
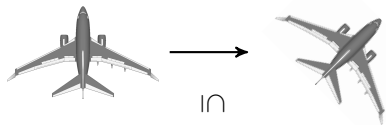
\Uparrow **dL proof**

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to **runtime** monitor with offline proof



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

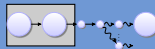
\Downarrow **Lemma**

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

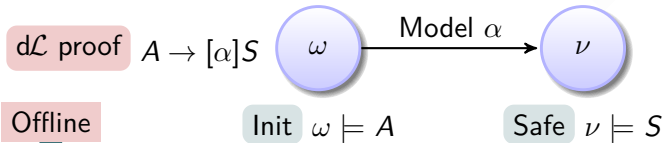
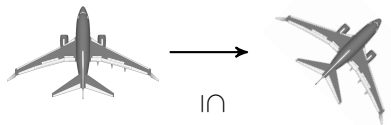
\Uparrow **d \mathcal{L} proof**

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with **offline** proof



Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

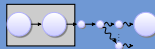
\Downarrow **Lemma**

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

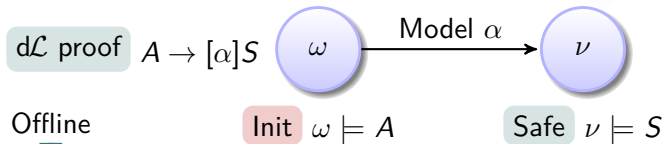
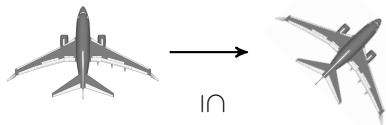
\Uparrow **d \mathcal{L} proof**

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to **runtime** monitor with offline proof



Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

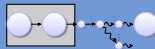
\Downarrow **Lemma**

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

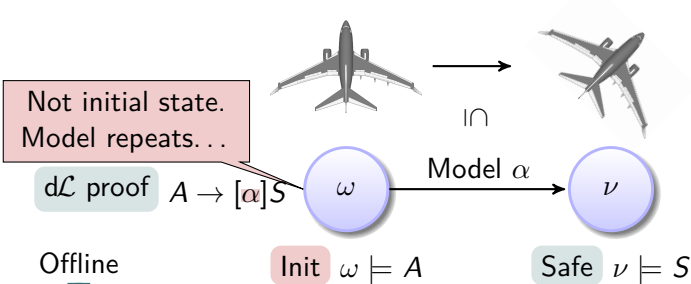
\Uparrow **d \mathcal{L} proof**

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Offline

Semantical: $(\omega, \nu) \in \llbracket \alpha \rrbracket$

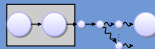
\Downarrow Lemma

Logical $d\mathcal{L}$: $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

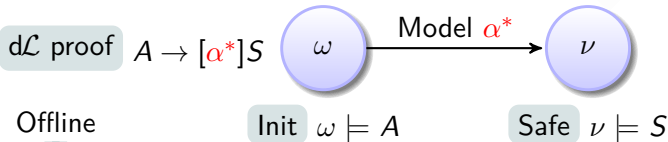
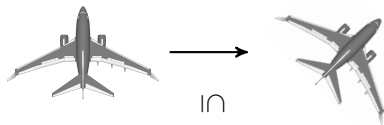
\Uparrow $d\mathcal{L}$ proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



Logic reduces CPS safety to runtime monitor with offline proof



Semantical: $(\omega, \nu) \in \llbracket \alpha^* \rrbracket$

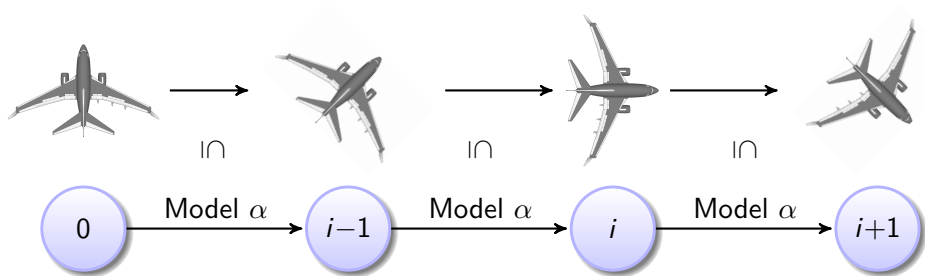
\Downarrow Lemma

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \alpha^* \rangle (x = x^+)$

\Uparrow d \mathcal{L} proof

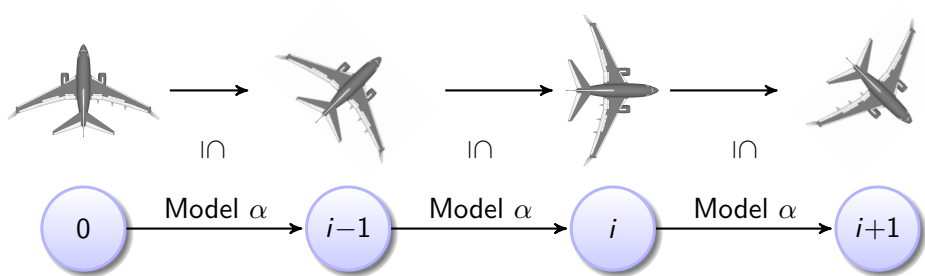
Arithmetical: $(\omega, \nu) \models F(x, x^+)$

check at runtime (efficient)



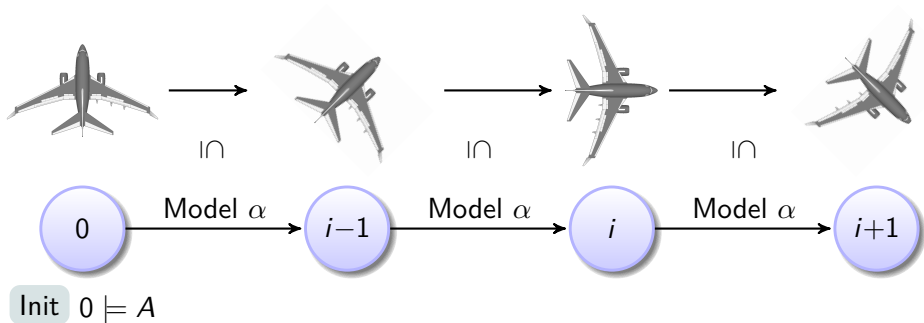


$d\mathcal{L}$ proof $A \rightarrow [\alpha^*]S$



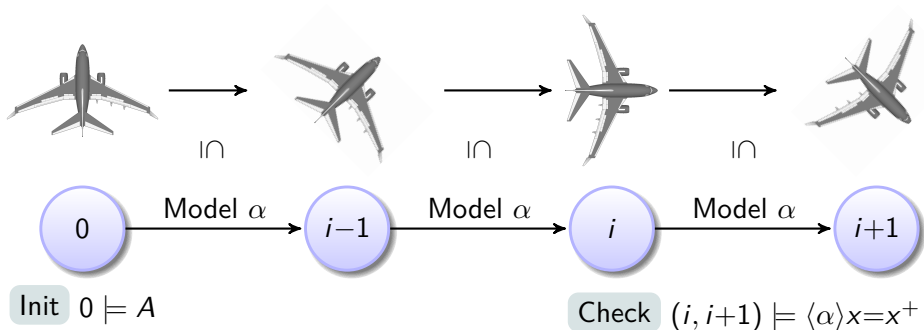


$d\mathcal{L}$ proof $A \rightarrow [\alpha^*]S$



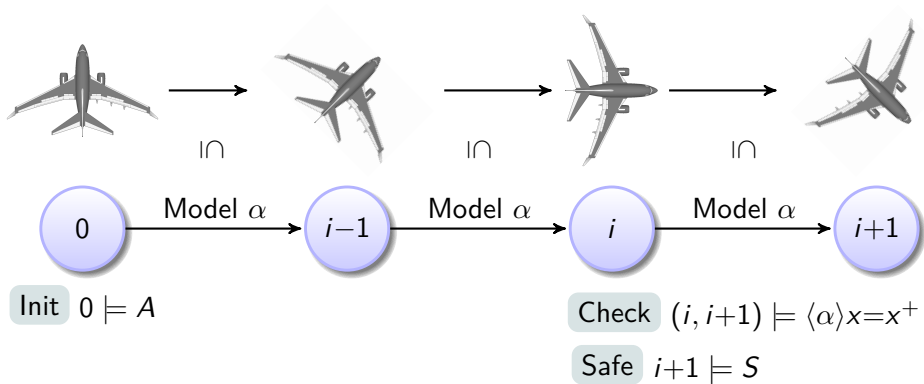


$d\mathcal{L}$ proof $A \rightarrow [\alpha^*]S$



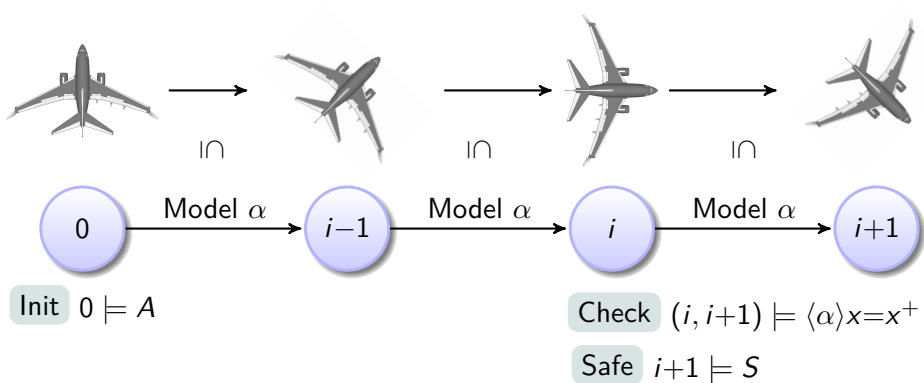


$d\mathcal{L}$ proof $A \rightarrow [\alpha^*]S$





$d\mathcal{L}$ proof $A \rightarrow [\alpha^*]S$



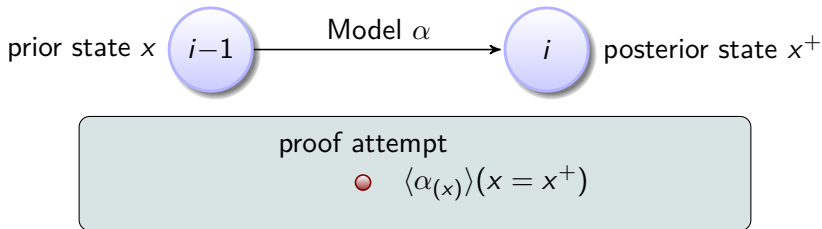
Theorem (Model Monitor Correctness)

(FMSD'16)

"System safe as long as monitor satisfied."

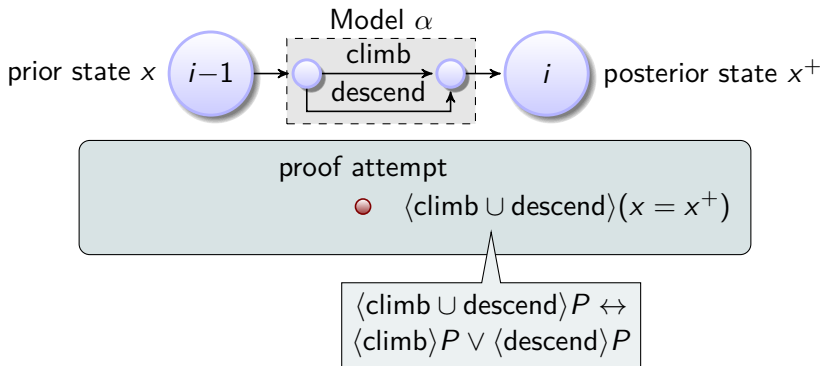


- Proof calculus of $d\mathcal{L}$ executes models symbolically



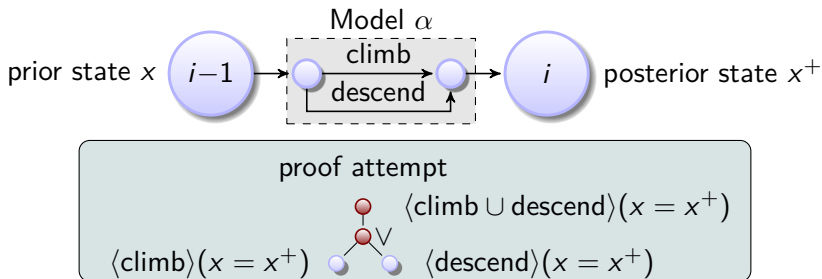


- Proof calculus of $d\mathcal{L}$ executes models symbolically



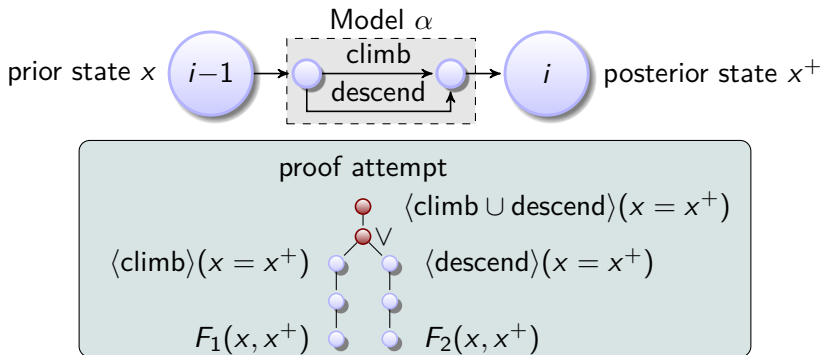


- Proof calculus of $d\mathcal{L}$ executes models symbolically



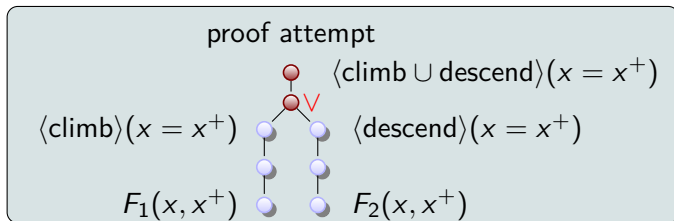
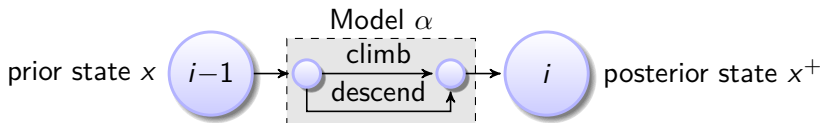


- Proof calculus of $d\mathcal{L}$ executes models symbolically

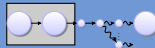




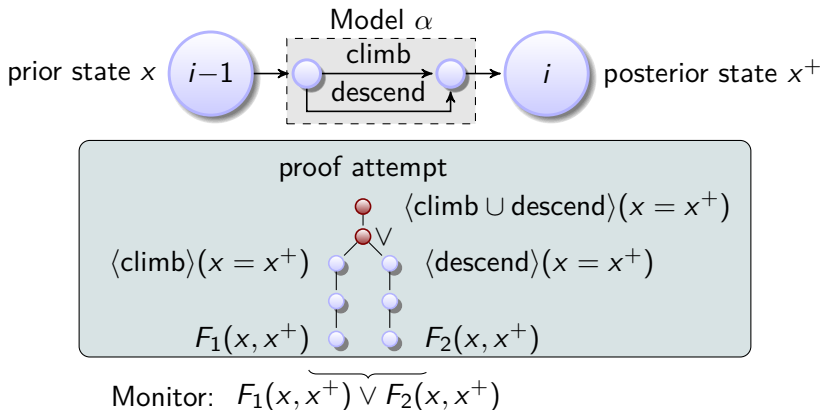
- Proof calculus of $d\mathcal{L}$ executes models symbolically



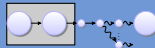
Monitor: $F_1(x, x^+) \vee F_2(x, x^+)$



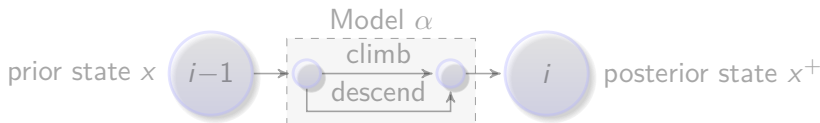
- Proof calculus of $d\mathcal{L}$ executes models symbolically



- The subgoals that cannot be proved express all the conditions on the relations of variables imposed by the model \rightsquigarrow close at runtime



- Proof calculus of $d\mathcal{L}$ executes models symbolically



Model Monitor

Immediate detection of model violation

\rightsquigarrow Mitigates safety issues with safe fallback action

$$F_1(x, x^+) \quad \text{---} \quad \text{---} \quad F_2(x, x^+)$$

$$\text{Monitor: } F_1(x, x^+) \vee F_2(x, x^+)$$

- The subgoals that cannot be proved express all the conditions on the relations of variables imposed by the model \rightsquigarrow close at runtime

Water Tank Example: Monitor Conjecture

Variables

x current level

ε control cycle

m maximum level

f flow

Model and Safety Property

$$\underbrace{0 \leq x \leq m \wedge \varepsilon > 0}_A \rightarrow \left[\left(f := *; ? \left(-1 \leq f \leq \frac{m-x}{\varepsilon} \right); \right. \right. \\ \left. \left. t := 0; (x' = f, t' = 1 \ \& \ x \geq 0 \wedge t \leq \varepsilon) \right)^* \right] \\ \underbrace{(0 \leq x \leq m)}_S$$

Model Monitor Specification Conjecture

$$\underbrace{\varepsilon > 0}_{A|_{\text{const}}} \rightarrow \left\langle f := *; ? \left(-1 \leq f \leq \frac{m-x}{\varepsilon} \right); \right. \\ \left. t := 0; (x' = f, t' = 1 \ \& \ x \geq 0 \wedge t \leq \varepsilon) \right\rangle \overbrace{(x = x^+ \wedge f = f^+ \wedge t = t^+)}^{\Upsilon_{V_m}^+}$$

Water Tank Example: Nondeterministic Assignment

Proof Rules

$$\langle * \rangle \frac{\Gamma \vdash \exists X \langle x := X \rangle P, \Delta}{\Gamma \vdash \langle x := * \rangle P, \Delta} \quad (X \text{ is a new logical variable})$$

$$\exists R \frac{\Gamma \vdash p(e), \exists x p(x), \Delta}{\Gamma \vdash \exists x p(x), \Delta} \quad (e \text{ is any arbitrary term})$$

$$WR \frac{\Gamma \vdash \Delta}{\Gamma \vdash P, \Delta}$$

Sequent Deduction

$$\begin{array}{c} \text{Opt. 1} \\ \text{WR} \frac{A \vdash \langle f := F \rangle \langle ?-1 \leq f \leq \frac{m-x}{\epsilon} \rangle \langle \text{plant} \rangle \Upsilon^+}{A \vdash \exists F \langle f := F \rangle \langle ?-1 \leq f \leq \frac{m-x}{\epsilon} \rangle \langle \text{plant} \rangle \Upsilon^+} \quad \text{Opt. 1} \quad A \vdash \langle f := f^+ \rangle \langle ?-1 \leq f \leq \frac{m-x}{\epsilon} \rangle \langle \text{plant} \rangle \Upsilon^+ \\ \langle * \rangle \frac{A \vdash \langle f := *; ?-1 \leq f \leq \frac{m-x}{\epsilon} \rangle \langle \text{plant} \rangle \Upsilon^+}{A \vdash \langle f := *; ?-1 \leq f \leq \frac{m-x}{\epsilon} \rangle \langle \text{plant} \rangle \Upsilon^+} \quad \text{WR} \quad \dots \end{array}$$

with Opt. 1 (anticipate $f = f^+$ from Υ^+)

Water Tank Example: Differential Equations

Proof Rules

$$\langle'\rangle \frac{\exists T \geq 0 ((\forall 0 \leq t \leq T \langle x := y(t) \rangle Q) \wedge \langle x := y(T) \rangle P)}{\langle x' = f(x) \& Q \rangle P} (y(t) \text{ solution } T, t \text{ new})$$

$$\text{QE} \frac{\text{QE}(P)}{P} \quad (\text{iff } P \leftrightarrow \text{QE}(P) \text{ in first-order real arithmetic})$$

Sequent Deduction

$$\begin{array}{l} A \vdash F = f^+ \wedge x^+ = x + Ft^+ \wedge t^+ \geq 0 \wedge x \geq 0 \wedge \varepsilon \geq t^+ \geq 0 \wedge Ft^+ + x \geq 0 \\ \text{QE} \frac{}{A \vdash \forall 0 \leq \tilde{t} \leq T (x + f^+ \tilde{t} \geq 0 \wedge \tilde{t} \leq \varepsilon) \wedge F = f^+ \wedge x^+ = x + Ft^+ \wedge t^+ = t^+} \\ \exists R, WR \frac{}{A \vdash \exists T \geq 0 ((\forall 0 \leq \tilde{t} \leq T (x + f^+ \tilde{t} \geq 0 \wedge \tilde{t} \leq \varepsilon)) \wedge F = f^+ \wedge (x^+ = x + FT \wedge t^+ = T))} \\ \langle'\rangle \frac{}{A \vdash \langle f := F; t := 0 \rangle \langle \{x' = f, t' = 1 \& x \geq 0 \wedge t \leq \varepsilon\} \rangle \Upsilon^+} \end{array}$$

Water Tank Example: Synthesized Model Monitor

Input: Model and Safety Property

$$\underbrace{0 \leq x \leq m \wedge \varepsilon > 0}_A \rightarrow \left[\left(f := *; ?(-1 \leq f \leq \frac{m-x}{\varepsilon}); \right. \right. \\ \left. \left. t := 0; (x' = f, t' = 1 \ \& \ x \geq 0 \wedge t \leq \varepsilon) \right)^* \right] \\ \underbrace{(0 \leq x \leq m)}_S$$

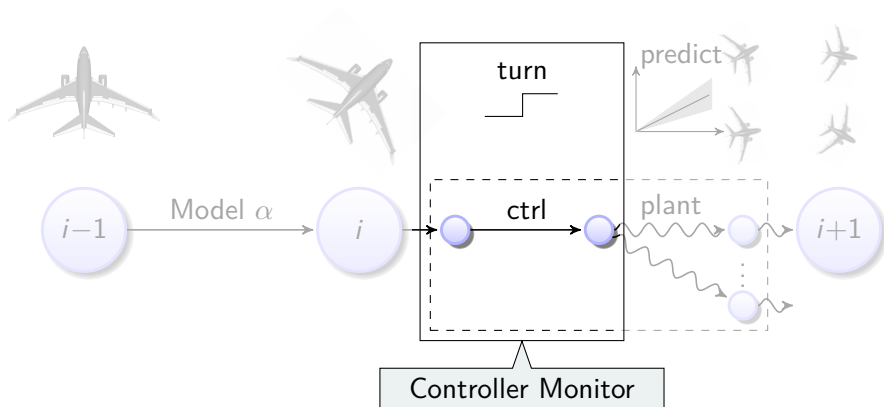
Output: Synthesized Model Monitor

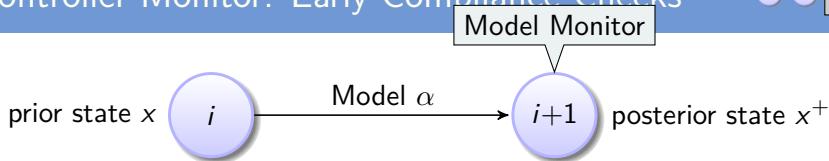
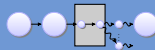
$$-1 \leq f^+ \leq \frac{m-x}{\varepsilon} \wedge x^+ = x + f^+ t^+ \wedge x \geq 0 \wedge x + f^+ t^+ \geq 0 \wedge \varepsilon \geq t^+ \geq 0$$

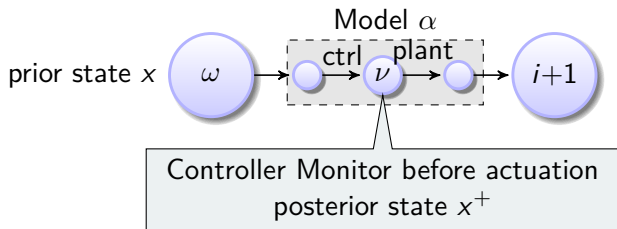
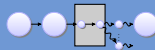
Proof (Generated by ModelPlex tactic).

A proof of correctness of the synthesized model monitor. □

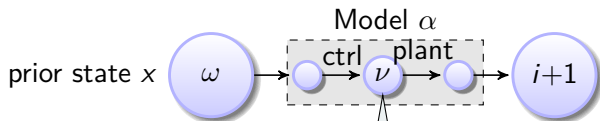
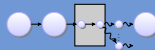
For typical models `ctrl`; `plant` we can check earlier







Semantical: $(\omega, \nu) \in \llbracket \text{ctrl} \rrbracket$ reachability relation of ctrl



Controller Monitor before actuation
posterior state x^+

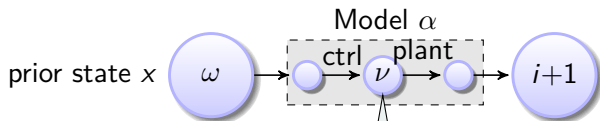
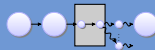
Offline

Semantical: $(\omega, \nu) \in \llbracket \text{ctrl} \rrbracket$

\Updownarrow Theorem

Logical dL: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+)$

exists a run of ctrl to
a state where $x = x^+$



Controller Monitor before actuation
posterior state x^+

Offline

Semantical: $(\omega, \nu) \in \llbracket \text{ctrl} \rrbracket$

\Updownarrow Theorem

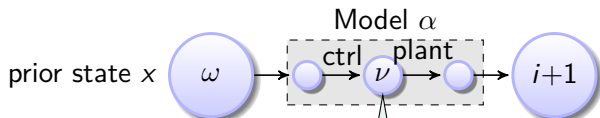
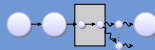
Logical $d\mathcal{L}$: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+)$

\Uparrow $d\mathcal{L}$ proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

exists a run of ctrl to
a state where $x = x^+$

check at runtime (efficient)



Controller Monitor before actuation
posterior state x^+

Offline

Semantical: $(\omega, \nu) \in \llbracket \text{ctrl} \rrbracket$

\Updownarrow Theorem

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+)$

\Uparrow d \mathcal{L} proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

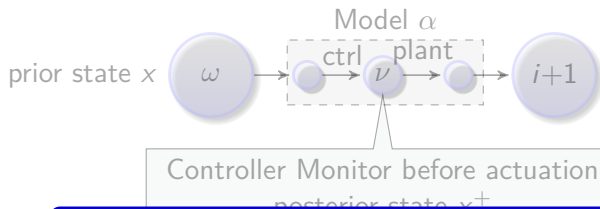
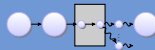
exists a run of ctrl to
a state where $x = x^+$

check at runtime (efficient)

Theorem (Controller Monitor Correctness)

(FMDS'16)

"Controller safe & in plant bounds as long as monitor satisfied."



Controller Monitor

Immediate detection of unsafe control before actuation
 \rightsquigarrow Safe execution of unverified implementations
 in perfect environments

Logical d \mathcal{L} : $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+)$

\Uparrow d \mathcal{L} proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

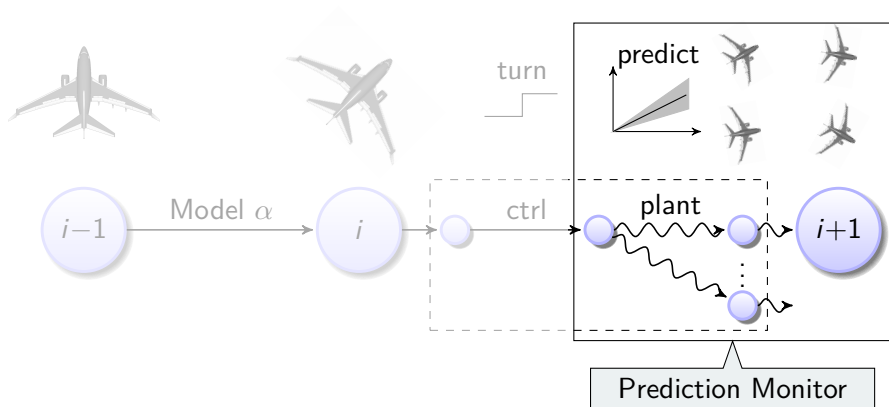
check at runtime (efficient)

Theorem (Controller Monitor Correctness)

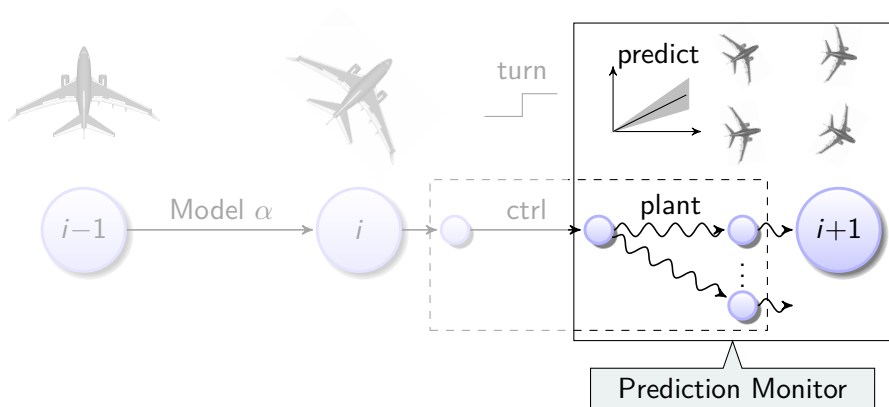
(FMSD'16)

"Controller safe & in plant bounds as long as monitor satisfied."

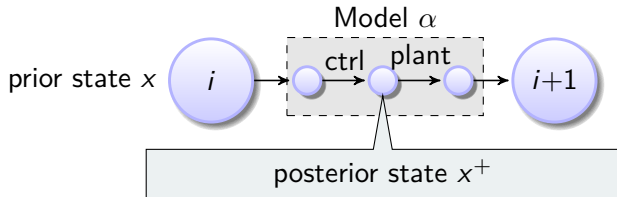
Safe despite evolution with disturbance?

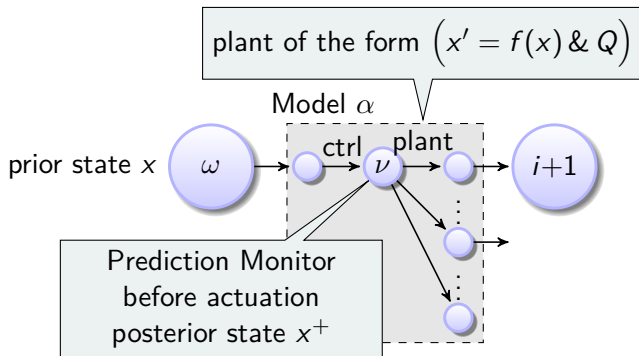
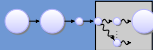


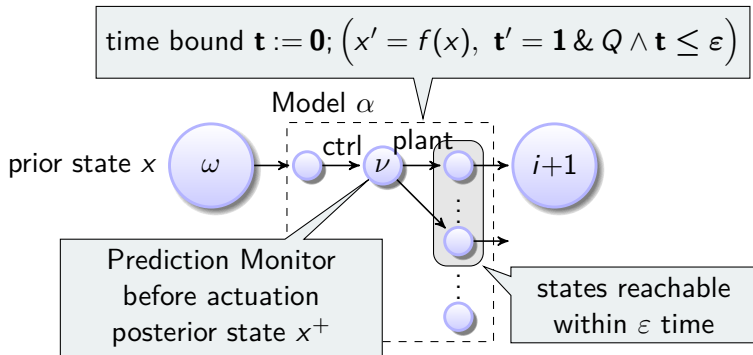
Safe despite evolution with disturbance?



“Prediction is very difficult, especially if it's about the future.” [Nils Bohr]

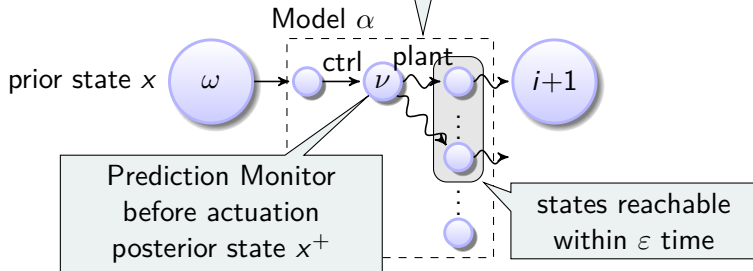




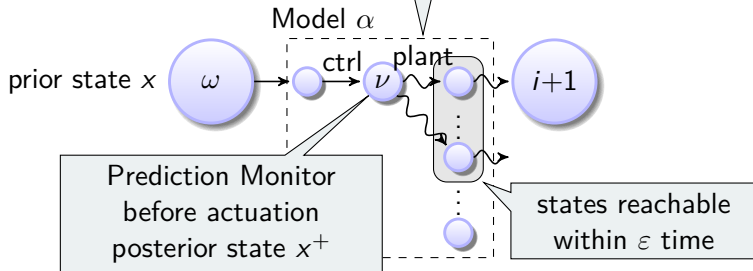


Prediction Monitor: Compliance with Disturbance

disturbance $t := 0; \left(f(x) - \delta \leq x' \leq f(x) + \delta, t' = 1 \& Q \wedge t \leq \varepsilon \right)$



disturbance $t := 0; \left(f(x) - \delta \leq x' \leq f(x) + \delta, t' = 1 \ \& \ Q \wedge t \leq \varepsilon \right)$



Offline

Logical $d\mathcal{L}$: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+ \wedge [\text{plant}] \varphi)$

\uparrow $d\mathcal{L}$ proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

Invariant φ implies safety S
(known from safety proof)

disturbance $t := 0; \left(f(x) - \delta \leq x' \leq f(x) + \delta, t' = 1 \& Q \wedge t \leq \varepsilon \right)$



Prediction Monitor with Disturbance

Proactive detection of unsafe control before actuation
despite disturbance

\rightsquigarrow **Safety in realistic environments**

Offline

Logical $d\mathcal{L}$: $(\omega, \nu) \models \langle \text{ctrl} \rangle (x = x^+ \wedge [\text{plant}] \varphi)$

\uparrow $d\mathcal{L}$ proof

Arithmetical: $(\omega, \nu) \models F(x, x^+)$

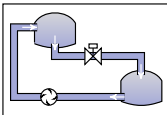
Invariant φ implies safety S
(known from safety proof)

Outline

- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

- Evaluated on hybrid system case studies

Water tank



Cruise control



© Volvo

Traffic control



© ASFINAG

Ground robots



© Black-I Robotics

Train control



© Harald Eisenberger

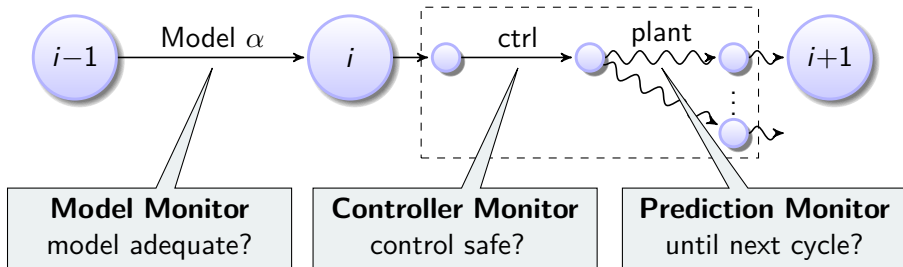
- Model sizes: 5–16 variables
- Monitor sizes: 20–150 operations
- Synthesis duration: 0.3–23 seconds (axiomatic) 6.2–211 (sequent)
- ModelPlex tactic produces correct-by-construction monitor in KeYmaera X
- Theorem:** ModelPlex is decidable and monitor synthesis fully automated for controller monitor synthesis and for important classes

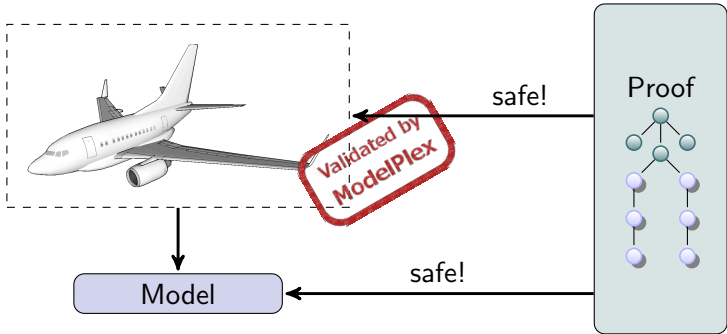
Outline

- 1 Motivation
- 2 Learning Objectives
- 3 ModelPlex Runtime
 - ModelPlex Runtime Monitors
 - ModelPlex Compliance
- 4 ModelPlex
 - Logical State Relations
 - Model Monitors
 - Correct-by-Construction Synthesis
 - Example: Water Tank
 - Controller Monitors
 - Prediction Monitors
- 5 Evaluation
- 6 Summary

ModelPlex ensures that proofs apply to real CPS

- Validate model compliance
- Characterize compliance with model in logic
- Prover transforms compliance formula to executable monitor
- Provably correct runtime model validation







Stefan Mitsch and André Platzer.

ModelPlex: Verified runtime validation of verified cyber-physical system models.

Form. Methods Syst. Des., 49(1):33–74, 2016.

Special issue of selected papers from RV'14.

doi:10.1007/s10703-016-0241-z.



Stefan Mitsch and André Platzer.

ModelPlex: Verified runtime validation of verified cyber-physical system models.

In Borzoo Bonakdarpour and Scott A. Smolka, editors, *RV*, volume 8734 of *LNCS*, pages 199–214. Springer, 2014.

doi:10.1007/978-3-319-11164-3_17.



André Platzer.

Logics of dynamical systems.

In *LICS*, pages 13–24. IEEE, 2012.

doi:10.1109/LICS.2012.13.



André Platzer.

A complete uniform substitution calculus for differential dynamic logic.

J. Autom. Reas., 2016.

doi:10.1007/s10817-016-9385-1.



Nathan Fulton and André Platzer.

A logic of proofs for differential dynamic logic: Toward independently checkable proof certificates for dynamic logics.

In Jeremy Avigad and Adam Chlipala, editors, *Proceedings of the 2016 Conference on Certified Programs and Proofs, CPP 2016, St.*

Petersburg, FL, USA, January 18-19, 2016, pages 110–121. ACM, 2016.

doi:10.1145/2854065.2854078.



André Platzer.

Differential dynamic logic for hybrid systems.

J. Autom. Reas., 41(2):143–189, 2008.

doi:10.1007/s10817-008-9103-8.



André Platzer.

A uniform substitution calculus for differential dynamic logic.

In Amy Felty and Aart Middeldorp, editors, *CADe*, volume 9195 of *LNCS*, pages 467–481. Springer, 2015.
[doi:10.1007/978-3-319-21401-6_32](https://doi.org/10.1007/978-3-319-21401-6_32).