RollerCoaster Tycoon X

Like the original, but safer

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Who has taken a roller coaster? Who wants to be confident that they are safe?

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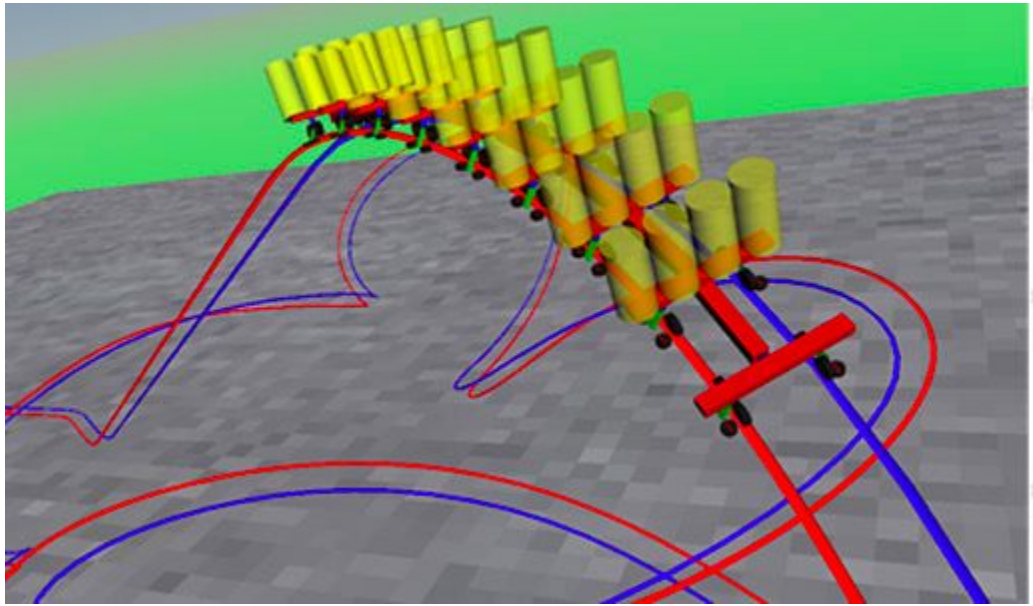
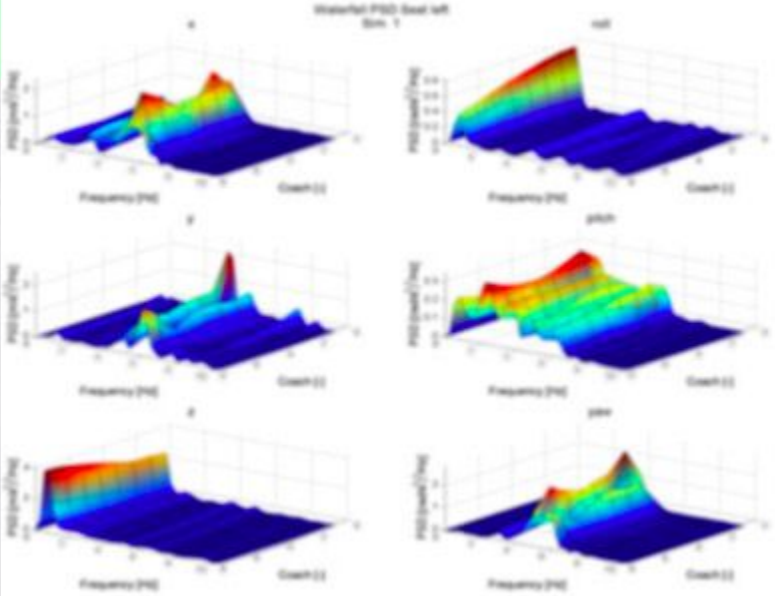
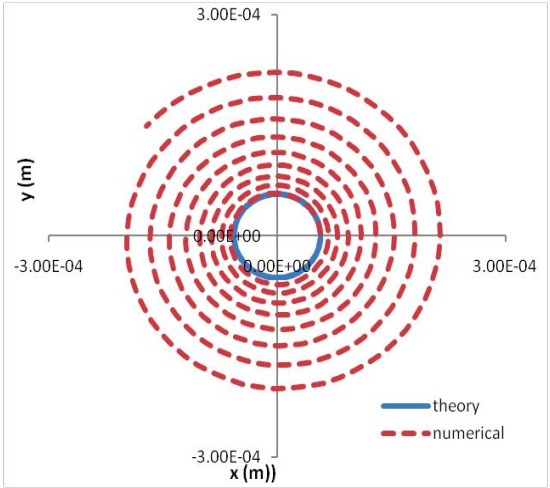
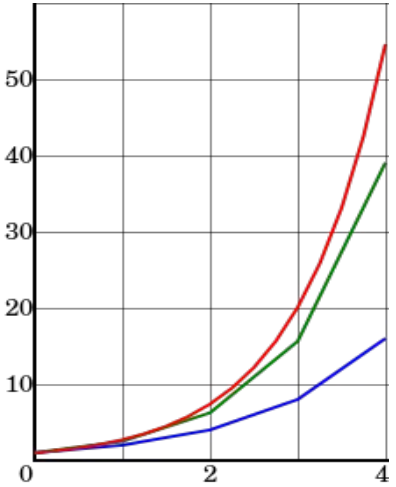
The safety of coasters is verified by numerical simulation





But numerical simulation is subject to error





$\{x' = -y,\ y' = x\}$

$\{x' = x\}$

Approach

We break a coaster down into track sections and prove safety individually

- Reduces complexity
- More generalizable
- Prove properties for individual sections
- Ending conditions for one section are the starting conditions for next
- Piece the different proofs back together to form a complete proof

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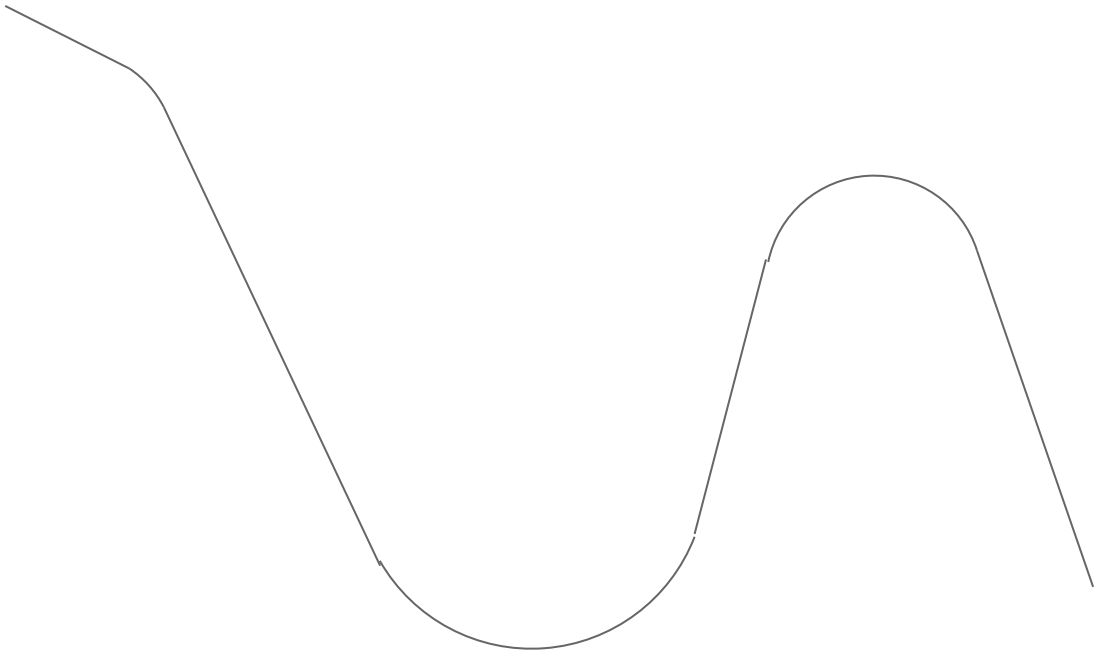
Coasters can be modeled with straight lines and arcs



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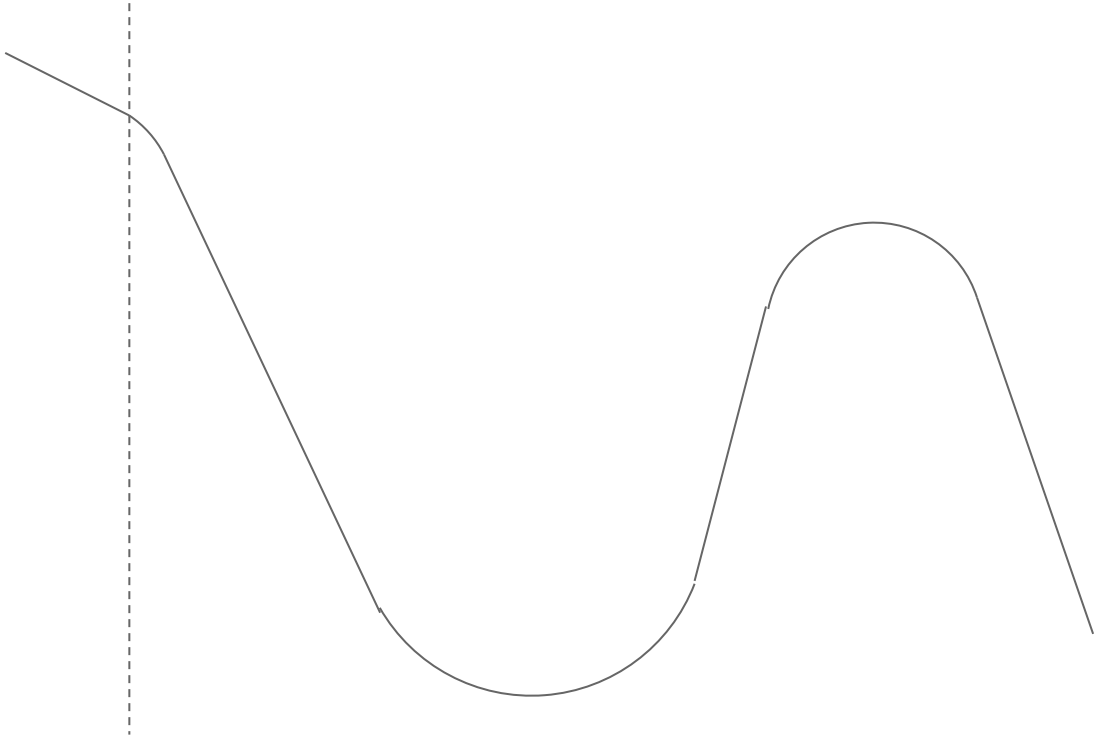

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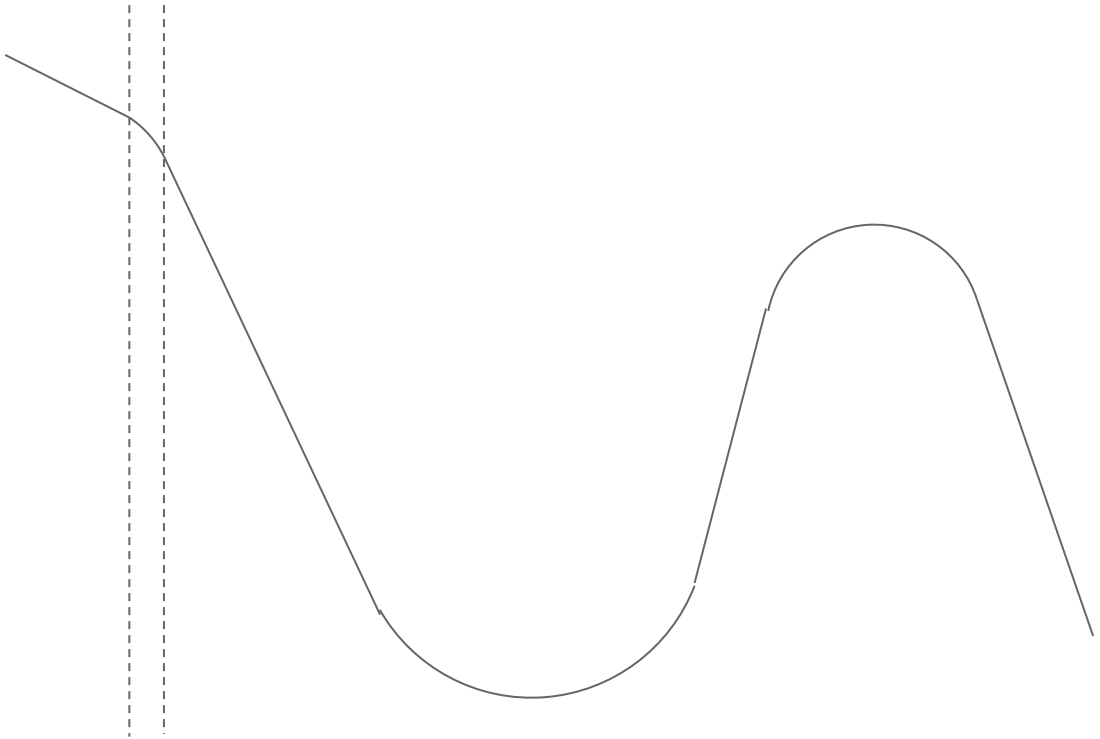

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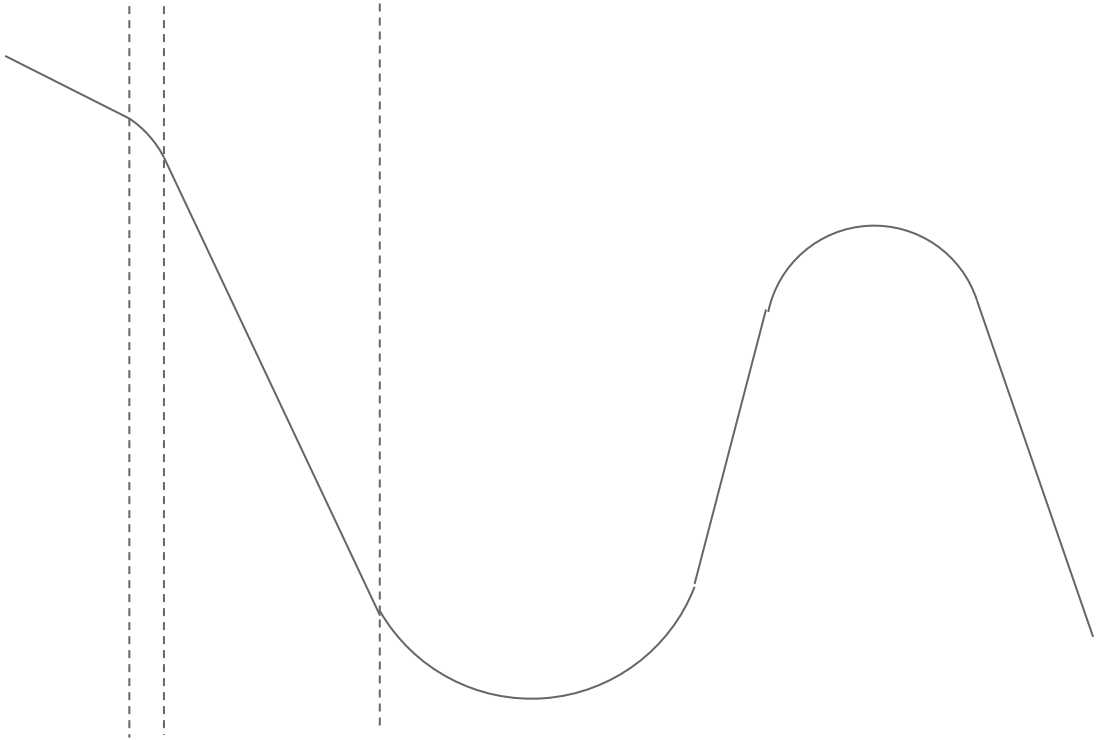

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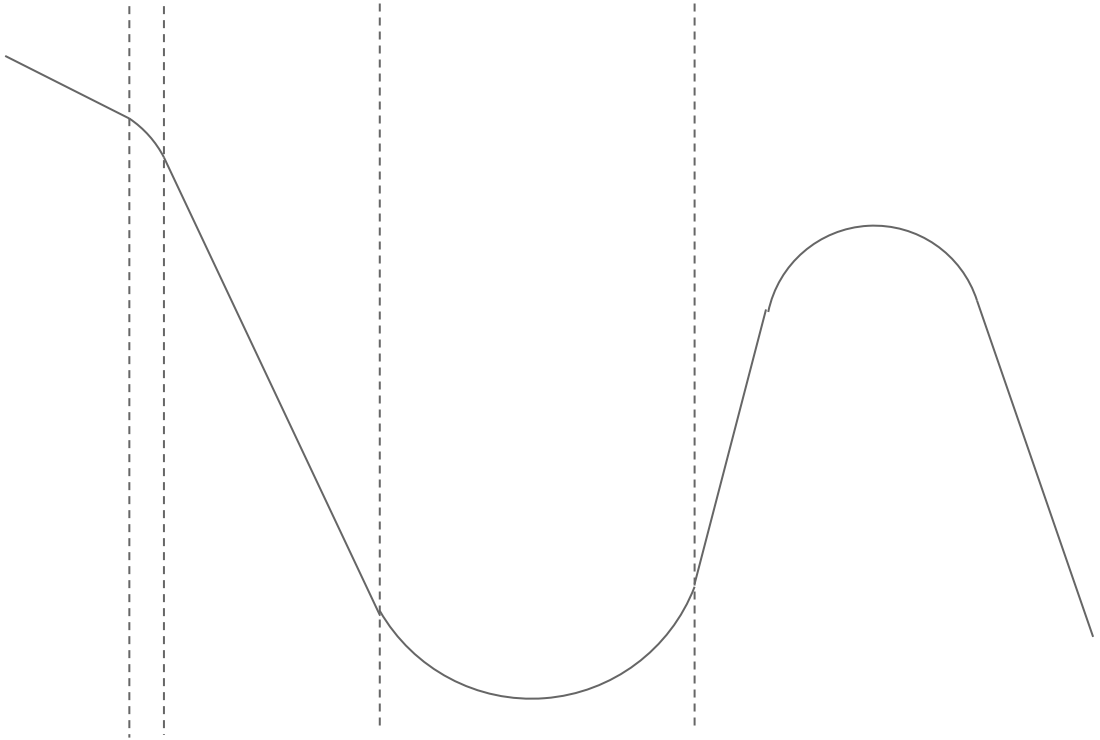

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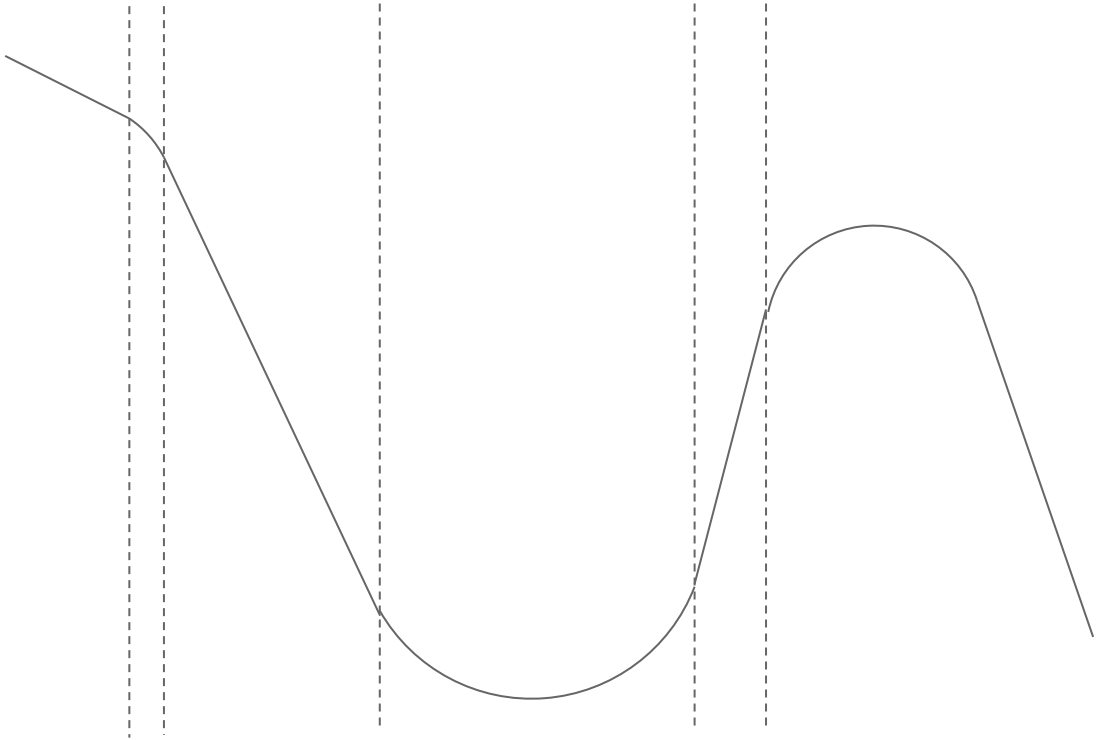

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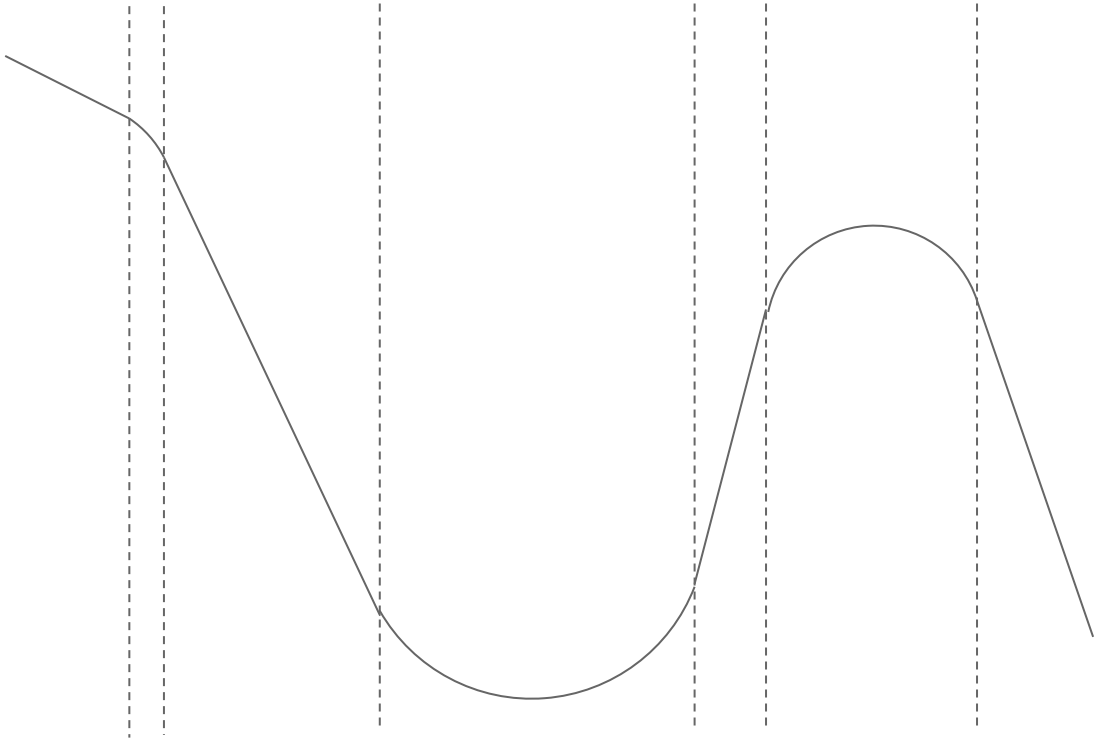

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Coasters can be modeled with straight lines and arcs

- Straight lines: $y = mx + c$
- Arcs: $(cx-x)^2 + (cy-y)^2 = r^2$

We want to prove several properties about roller coasters

- Train goes forward
- Energy is conserved
- Train stays on the track

Models

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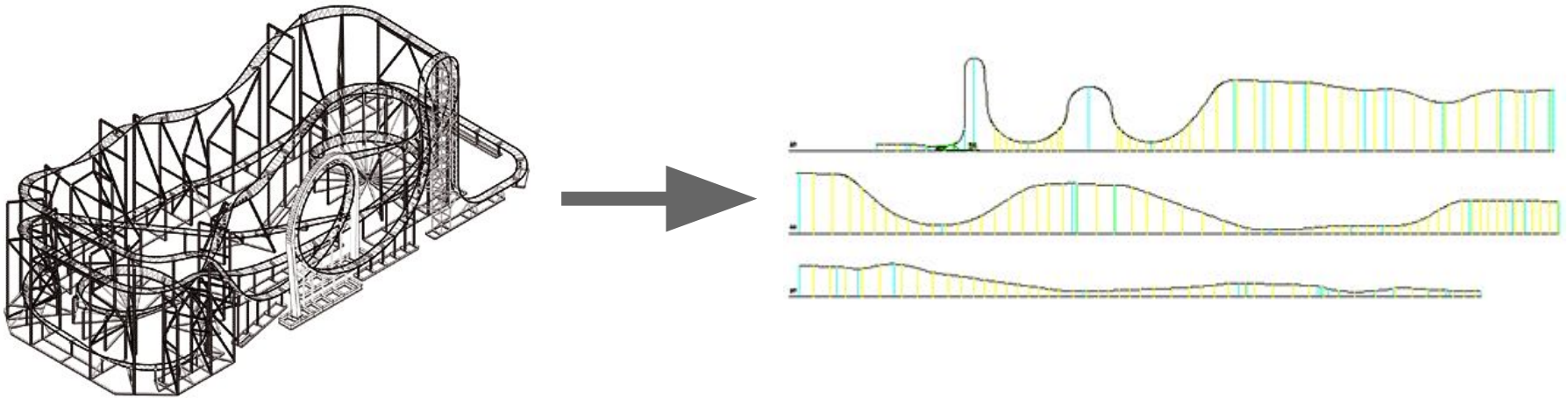
Modeling: Basic Assumptions

- Assumptions: zero friction, unit gravity, point mass, two dimensions



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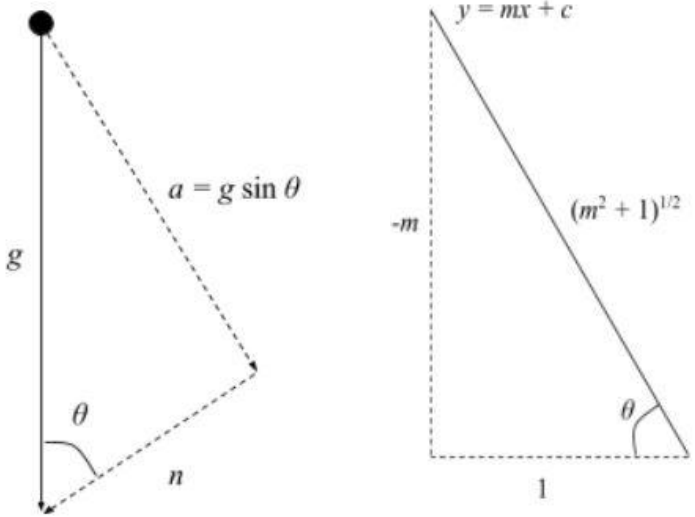
Modeling: Straight Line Dynamics

- For straight lines:
  - x’ = v*dx
  - y’ = v*dy
  - v’ = g sin θ



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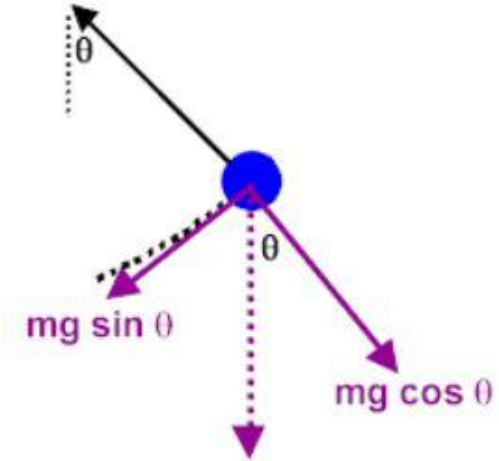
Modeling: Arc Dynamics

- For arcs (clockwise):
  - x’ = v*(y-cy)/r
  - y’ = -v*(x-cx)/r
  - v’ = (x-cx)/r



Proof

Proof approach

- If ODEs are solvable, solve!
- Identify which properties would be proved through similar means
- Proving properties that were essential in proving others
  - Positive velocity is a powerful property!
- Useful proof rules
  - Differential invariants
  - Differential ghosts

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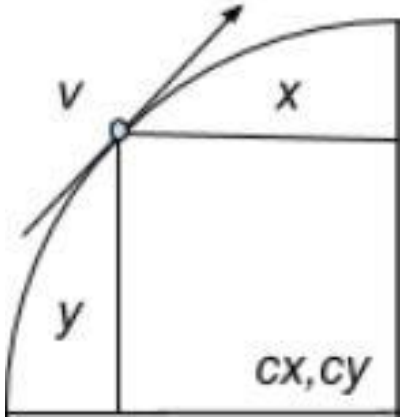
Proof Example: Arc motion

1. Stays on track and energy is conserved
    - Proved with invariants
2. Strictly positive velocity
    - Proved with ghosts as velocity is decreasing
3. Stays within the quadrant
    - Proved with invariants using information about velocity



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RollerCoaster Tycoon X is a safe roller coaster design tool

We modeled the Top Thrill Dragster and proved that it is safe