In the Spotlight

Verifying Automated Theatrical Follow-Spots

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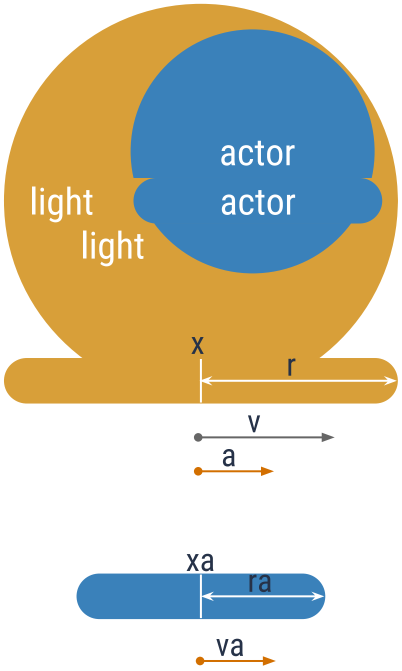
15-424 Foundations of Cyber Physical Systems

Background

- Follow-spot lights are one of the only non-automated areas of theatrical lighting
  - Humans have been superior to computers at following actors smoothly and accurately
- BlackTrax and Wybron AutoPilot II are two modern systems which have tackled this problem

Images: http://designedbybrandon.com/portfolio/nadeem/series3/in-the-spotlight_large.jpg http://www.avequipment.com/upload/images/large/chauvet_followspot_400g.png

Assumptions



- Radii are constant, $r > ra > 0$
- The actor has a maximum velocity, $V > 0$
- Time-triggered model
  - Maximum time step, $T > 0$
- The light beam moves via cartesian coordinates
  - $x' = v, v' = a$ $\qquad x(t) = x + v \cdot t + \frac{1}{2} \cdot a \cdot t^2$
  - $xa' = va$ $\qquad xa(t) = xa + va \cdot t$

- light_on_actor ≡ $ra + abs(x - xa) \leq r$

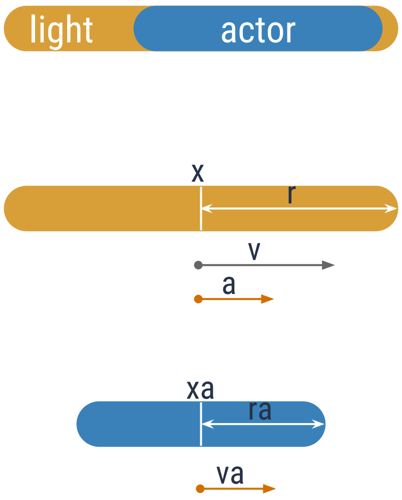
Model

Pre → [(Ctrl;Physics)*@Invariant]Post

- Pre ≡ $V > 0$ & $T > 0$ & $r > ra > 0$ & light_on_actor
- Invariant ≡ Post ≡ light_on_actor
- Physics ≡ $\{x' = v,\ v' = a,\ xa' = va,\ t' = 1\ \&\ t \le T\}$

Ctrl ≡ Environment; Actor; Light

- Environment ≡ $t := 0$
- Actor ≡ $va := *$; ?(abs(va) ≤ V)
- Light ≡ $a := *$; ?(safe_acc)



Determining if acceleration is safe

Intuition: extreme values

- light_on_actor ≡ abs(x - xa) ≤ r - ra
  - Check at endpoints and local extrema

Implementation:

- light_on_actor(t, va) ≡ abs(x(t) - xa(t, va)) ≤ r - ra
  - $x(t) \equiv x + v \cdot t + ½ \cdot a \cdot t^2$
  - $xa(t, va) \equiv xa + va \cdot t$
  - $t_{ext}(va) \equiv (va - v)/a$
- safe_acc ≡ light_on_actor(T, V) & light_on_actor(T, -V) & light_on_actor(t_{ext}(V), V) & light_on_actor(t_{ext}(-V), -V)

Model Recap & Proof

$V > 0$ & $T > 0$ & $r > ra > 0$ & light_on_actor $\rightarrow$
[
{
t ≔ 0; va ≔ *; ?(abs(va) ≤ V); a ≔ *;
?(light_on_actor(T, V) & light_on_actor(T, -V)
& light_on_actor(t_{ext}(V), V) & light_on_actor(t_{ext}(-V), -V));
{x' = v, v' = a, xa' = va, t' = 1 & t ≤ T}
}* @invariant(light_on_actor)
] light_on_actor

light_on_actor(t, va) ≡ abs(x(t) - xa(t, va)) ≤ r - ra
t_{ext}(va) ≡ (va - v)/a
x(t) ≡ $x + v \cdot t + \frac{1}{2} \cdot a \cdot t^2$
xa(t, va) ≡ $xa + va \cdot t$

Future Work

- Expand the model to higher dimensions
- Weaken and eliminate assumptions
  - Allow actor acceleration
  - Model the light with angular movement
  - Augment the model to account for potentially faulty signals from the actor to the light
- Add more realistic restrictions
  - Place an upper bound on the light's velocity and acceleration

Thank You!

Questions?