

How to Dance Without Embarrassing Yourself?

By Tia Jiang & Nazli Uzgur

Outline

- Background
- Hustle Baseline Model
- Hustle Guess Model
- Box Steps Baseline Model
- Box Steps Guess Model
- Hustle Guess Model with Tuning Parameter

Notion

- X: Position of the lead dancer
- Y: Position of the follow dancer
- Buffer: The minimum distance between them we want to have
- D: Maximum distance between them that allows them to hold hands
- Veps: An error margin for the guesses on the other dancers speed

Safety

- Hustle:
 $x + \text{buffer} \leq y \ \& \ y - x \leq D$
- Box Step:
 $y1 + \text{buffer} \leq y2 \ \& \ (x2 - x1)^2 \leq D^2$

Hustle!



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Hustle: Baseline Model & Proof

Model Highlights

Step1:

- Controls

$v := *; ?(v > 0 \ \& \ 2 * v \leq (y - x - \text{buffer}) / T \ \& \ v < A);$

- ODEs $\{x' = v, y' = -v, t' = 1 \ \& \ t < T\}$

Step2:

- Controls

$v := *; ?(v > 0 \ \& \ 2 * v \leq (D - (y - x)) / T \ \& \ v < A);$

- ODEs $\{x' = -v, y' = v, t' = 1 \ \& \ t < T\}$

Proof Highlights

- Loop Invariant:

- Safety Condition

- Differential Cut After Step1 :

- Safety Condition

Cooperative hybrid system

Hustle

- Imperfect knowledge
 - $v_2 \neq v_1$
- Margin of error
 - $\text{abs}(v_{\text{guess}} - v_1) < \text{veps}$
 - $\text{Veps} > 0$

Hustle: Guess Model & Proof

Model Highlights (Omit Step2)

- Controls

$v1 := *; ?(v1 > 0 \ \& \ 2 * v1 \leq (y - x - \text{buffer}) / T \ \& \ v1 < A);$

$v\text{guess} := *;$

$?(((v\text{guess} - v1)^2 < \text{veps}^2) \ \& \ v\text{guess} > \text{veps});$

$v2 := v\text{guess} - \text{veps};$

- ODEs

$\{x' = v1, y' = -v2, t' = 1 \ \& \ t < T\}$

Proof Highlights

- Loop Invariant:

- Safety condition

- Differential Cut After Step1:

- Safety condition

Box Steps!



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Box Steps: Baseline Model & Proof

Model Highlights (Omit Step3 & 4)

Step1:

- Controls

$v := *; ?(v > 0 \ \& \ v < A);$

- ODEs $\{y1' = -v, y2' = -v, t' = 1 \ \& \ t < T\}$

Step2:

- Controls

$v := *; ?(v > 0 \ \& \ v < A);$

- ODEs $\{x1' = -v, x2' = -v, t' = 1 \ \& \ t < T\}$

Proof Highlights

- Loop Invariant:

- Safety condition

- Differential Cut:

- Safety condition

Cooperative hybrid system

Box Steps

- Imperfect knowledge
 - $v_2 \neq v_1$
- Margin of error
 - $\text{abs}(v_{\text{guess}} - v_1) < \text{veps}$
 - $\text{Veps} > 0$
- Supervised by lead
 - Lead make adjustment
 - No roll-over error

Box Steps: Guess Model & Proof

Model Highlights (Lead Adjustment)

oldx1 := x1;

oldx2 := x2;

{x1'=A, t'=1 & t<=(oldx2-oldx1)/A};

?(t=(oldx2-oldx1)/A);}

v1 := *; ?(v1>0 & v1<A);

vguess := *; ?(((vguess-v1)^2<veps^2) & veps*T<=D);

v2:=vguess;

Proof Highlights:

- Loop Invariant:
 - Safety condition
- Differential Cut:
 - Safety condition
- Monotonicity:
 - Initial condition
 - Safety condition
 - $x1=x2$

Guess Model with Tuning Parameter

Model Highlights (Omit Step3 & 4)

?vguess>veps;

v2 := vguess-veps;

t:=0;

{x'=v1, y'=-v2, t'=1 & t<T}

t:=0;

{x'=-v1, y'=v2, t'=1 & t<T}

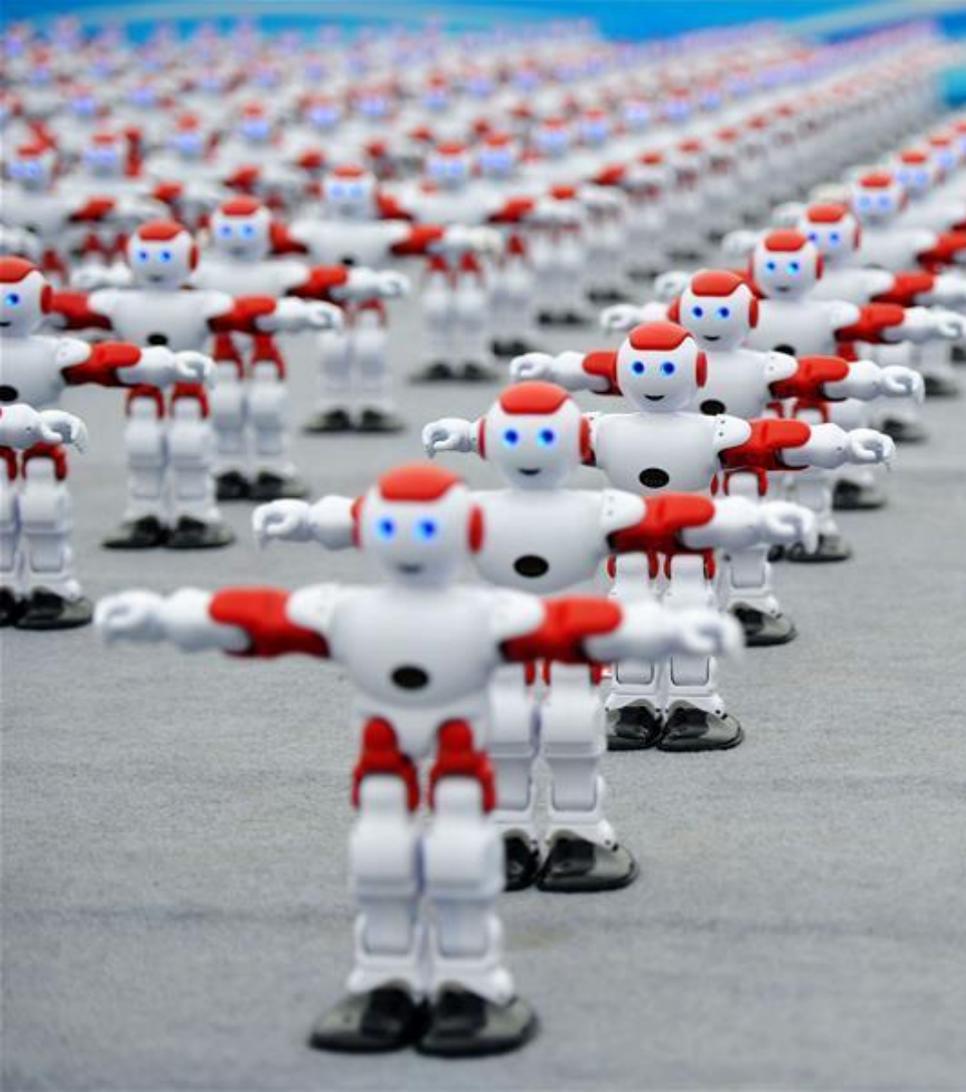
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?vguess<=veps;

veps:=veps/2;

Proof Highlights:

- Loop Invariant:
 - Safety condition
 - Veps > 0
- Differential Cut:
 - Safety condition
 - Veps > 0



Outside Ballroom



Conclusion

We proved that:

- It's safe to dance when both have perfect knowledge
- It's safe to dance when follow dancer has imperfect knowledge