Ship Outta Luck

Saving the Crew of a Sinking Ship

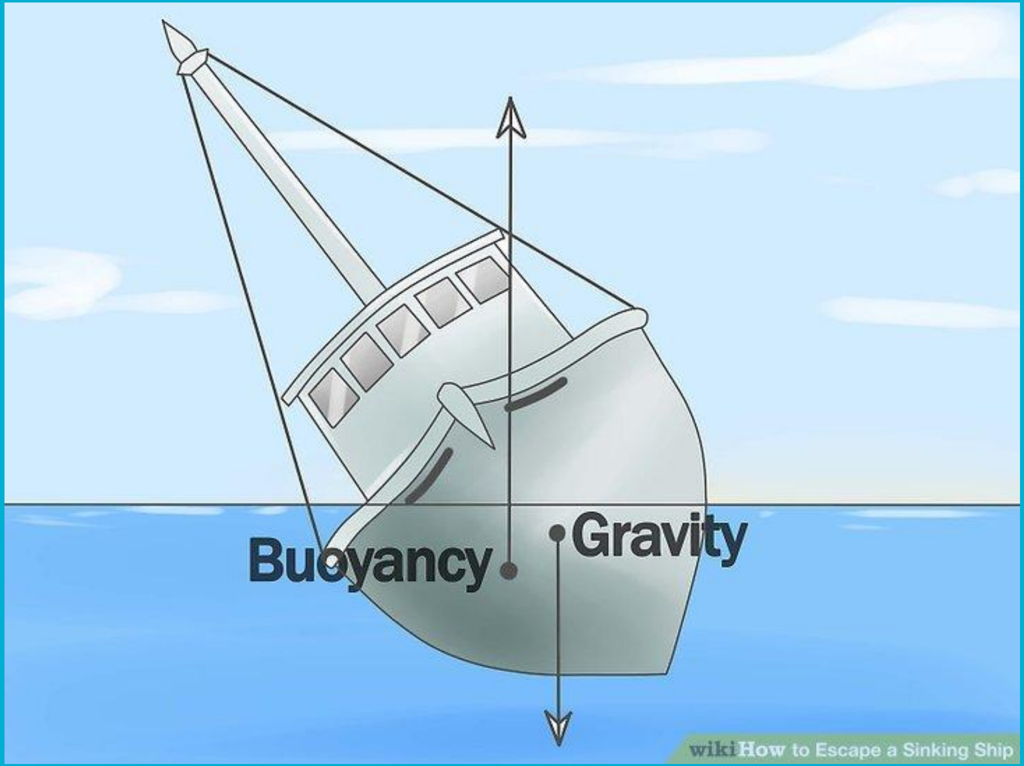
Project by Irene Li and Jordan Tick

The Setup

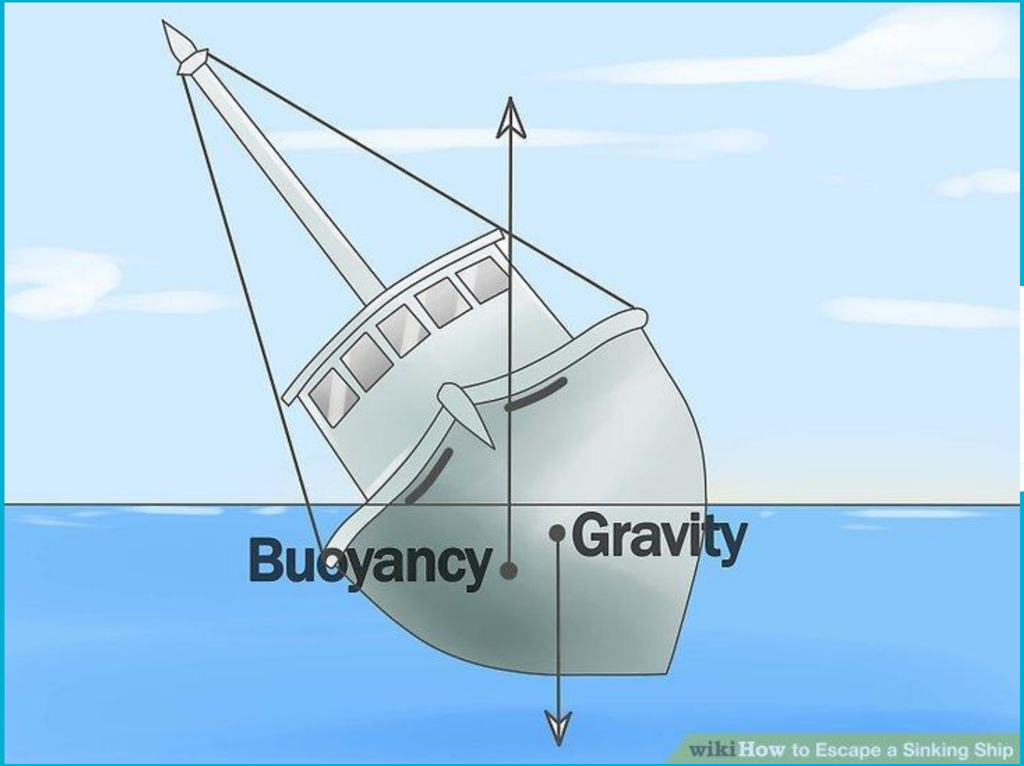
A ship is sinking due to a hole at the center of its bottom. There's nothing we can do to stop this from happening.

...But, can we save everyone in time?

The Setup — Forces at Play



The Setup — Forces at Play



$$F_{gravity} = (m + waterOnBoa$$

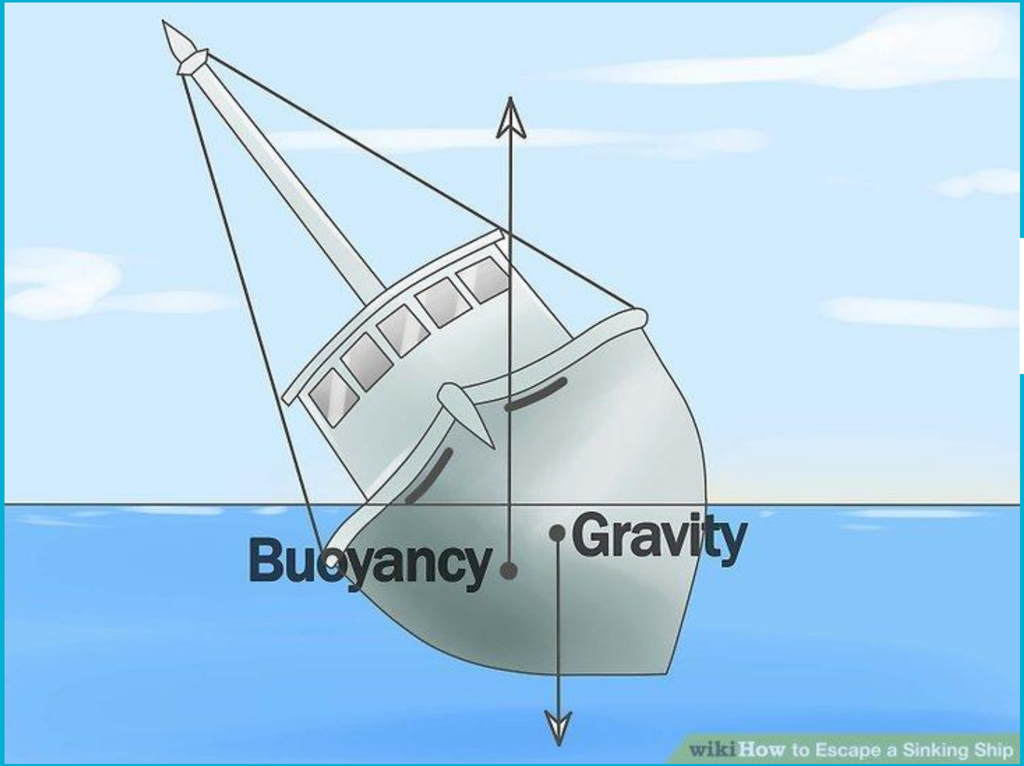
$$waterOnBoard = hole * t$$

$$F_{buoyancy} = (d * A) * p * g$$

$$d''(t) = dV'(t) = (m + hole * t * p - d * A * p) * g / m$$

Newton's
Second law:
F = ma

The Setup — Forces at Play



$$d''(t) = dV'(t) = (m + hole * pt - pA * d(t))g/m$$
$$d''(t) + (Apg/m)d(t) = (hole * pg/m)t + g$$

The Math

Start from: $d''(t) + (Apg/m) * d(t) = (hole * pg/m) * t + g$

Differentiate both sides: $dV''(t) + (Apg/m) * dV(t) = hole * p * g/m$

Solve it:

$$dV(t) = Bsin(\sqrt{\frac{pAg}{m}}t + \varphi) + \frac{hole}{A}$$

$$d(t) = -\sqrt{\frac{m}{p*A*g}} * B * cos(\sqrt{\frac{p*A*g}{m}} * t + \varphi) + \frac{hole}{A} * t + \frac{m}{p*A}$$

Using trigonometric identity sin^2 +cos^2=1 to obtain the invariant:

$$(\sqrt{\frac{pAg}{m}}(d(t) - \frac{waterOnBoard}{A} - \frac{m}{pA}))^2 + (dV(t) - \frac{hole}{A})^2 = B^2$$

The Math (continued)

Lower bound on time (using either invariant or exact solution of d(t)):

$$d \geq h \rightarrow t \geq (h - B\sqrt{\frac{m}{pAg}} - \frac{m}{pA})A/hole$$

The Model — Ship Sinking

The Constraints:

- m,p,A,hole,g,h all positive
- waterOnBoard = 0, t = 0 to start
- d < h and pAhg >= mg

The Controls:

None. The system determines how fast everything sinks!

At the start:

$d(t_0) = -(B*cos/w) + m/(p*A)$

$dV(t_0) = B*sin + hole/A$

If we set cos = 0, sin = -1, B = hole/A then $d(t_0) = m/(p*A)$ and $dV(t_0) = 0$

The Model: [{waterOnBoard'=hole, d' = dV, t' = 1,
dV' = (m+waterOnBoard*p-p*d*A)*g/m}] (t < (h-B/w-m/(p*A))*A/hole -> d < h)

The Invariants:

waterOnBoard=hole*t
w = sqrt(p*A*g/m)
(w*(d-waterOnBoard/A-m/(p*A)))^2+(dV-hole/A)^2=B^2

The Model — Ship Saving

The Constraints:

- rS, rQ, passengers, bP all positive
- rP=0, rV = rS, survivors=0 to start
- T >= bP/rS + (passengers/rQ)*2*bP/rS

The Controls:

rP ≥ bP (at boat)	**rP ≤ 0 (at shore)**	**else (in between)**

If passengers-survivors> rQ:
    survivors += rQ
Else:
    Survivors = passengers
rV = -rS
	rV = rS	(keep going)

The Model: [{rP' = rV, t' = 1 & rP>=0 & rP<=bP}] (t >= T —> passengers=survivors)

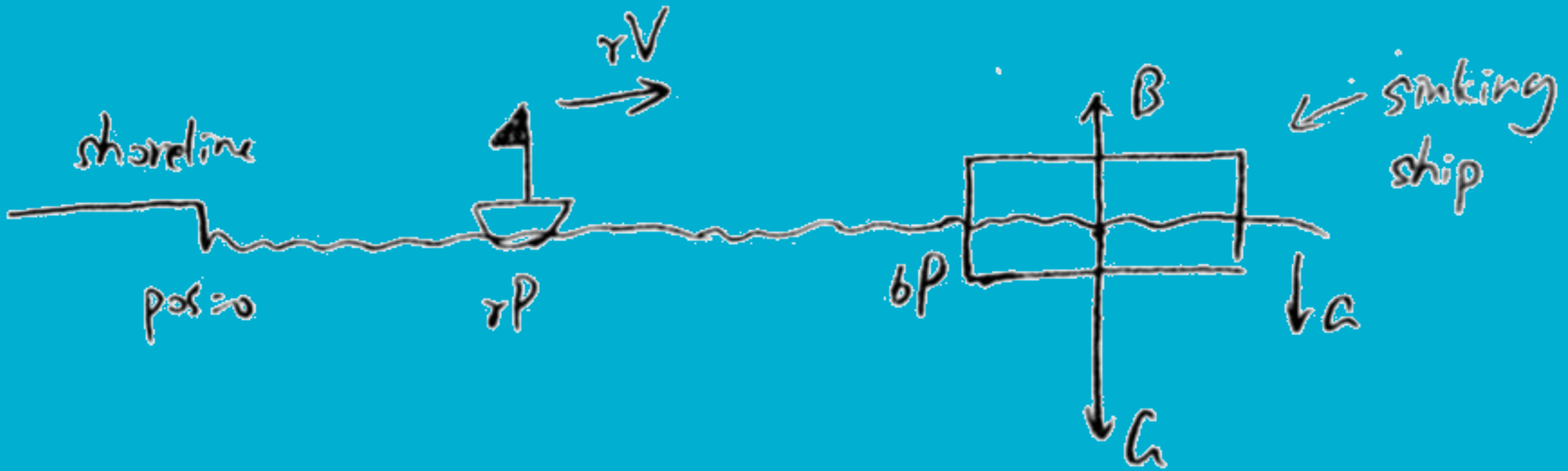
The Invariants:

survivors = passengers

OR

survivors<passengers
t<T
(rV=rS | rV=-rS)
T-t >= (-rP+bP*rV/rS)/rV + ((passengers-survivors)/rQ)*2*bP/rS))

The Model – Altogether



$$T = (h - \frac{B}{w} - \frac{m}{pA}) * \frac{A}{hole} \geq \frac{bP}{rS} + \frac{passengers}{rQ}(2\frac{bP}{rS})$$

Addressing the shortcomings

- Ships aren't prisms…. We tried more complex shapes but they couldn't be bounded.
- F=ma doesn't quite hold true if m varies with time

True equation:

$$F_{external} = m\frac{dV}{dt} + v\frac{dm}{dt}$$

→

$$a = (F_{external} - v * dm/dt)/m(t)$$

_Thank you for your time!

Any questions?