

# 08: Events & Responses

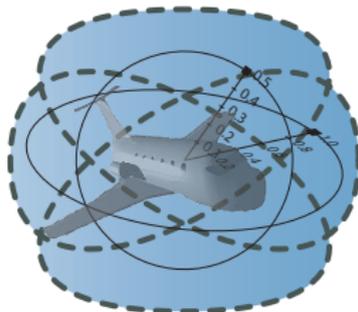
## 15-424: Foundations of Cyber-Physical Systems

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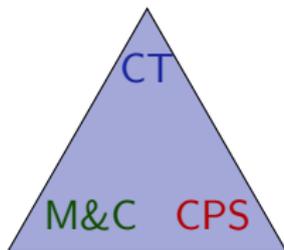
- 1 Learning Objectives
- 2 The Need for Control
  - Quantum the Ping Pong Ball
  - Cartesian Demon
  - Determinizing Ping Pong Balls
- 3 Event-triggered Control
  - Evolution Domains Detect Events
  - Non-negotiability of Physics
  - Splitting and Connecting Evolution Domains
  - Firing of Events
  - Physics vs. Control
- 4 Proof
  - Loop Invariants
- 5 Summary

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# Learning Objectives

## Events & Responses

using loop invariants  
design event-triggered control

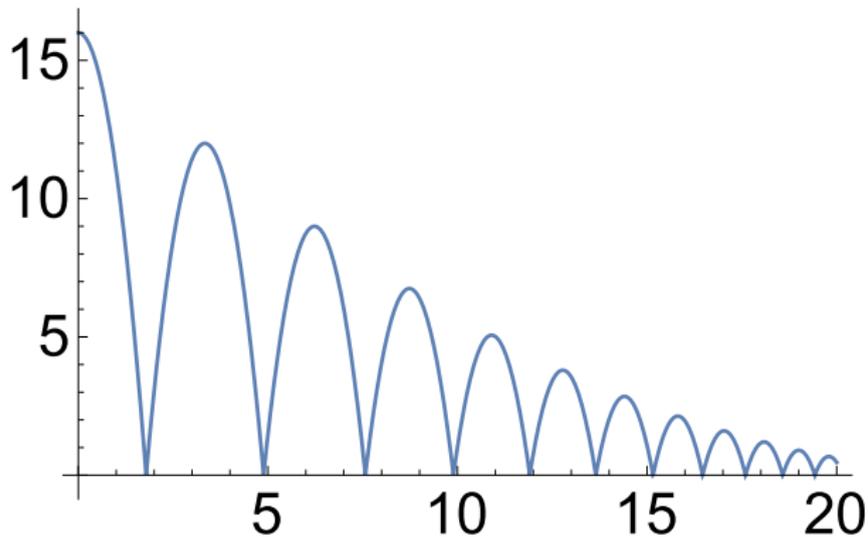


modeling CPS  
event-triggered control  
continuous sensing  
feedback mechanisms  
control vs. physics  
Cartesian Demons

semantics of event-triggered control  
operational effects  
model-predictive control

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# Quantum the Safely Bored Bouncing Ball

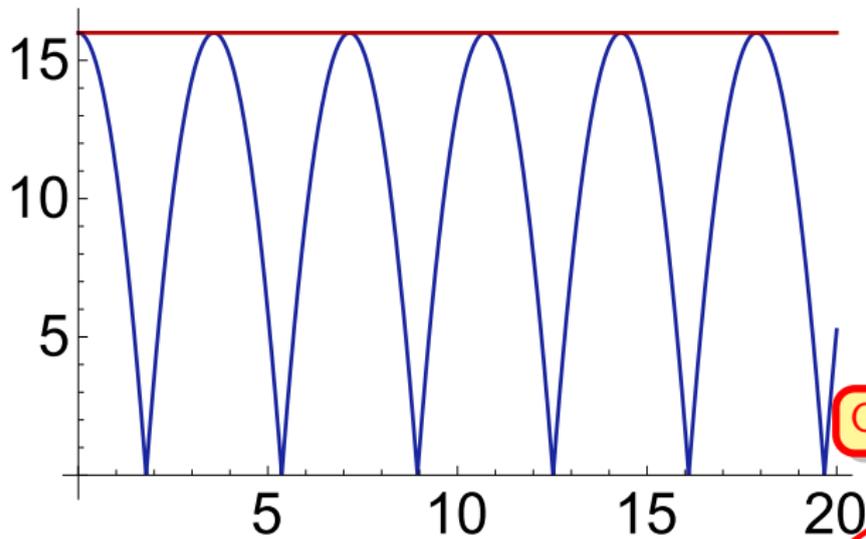


Proposition (Quantum can bounce around safely)

$$0 \leq x \wedge x = H \wedge v = 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \rightarrow$$
$$[(x' = v, v' = -g \ \& \ x \geq 0; (?x=0; v := -cv \cup ?x \neq 0))^*](0 \leq x \wedge x \leq H)$$

Proof  $\text{@invariant}(2gx = 2gH - v^2 \wedge x \geq 0)$

# Quantum the Safely Bored Bouncing Ball



Can be improved...

Proposition (Quantum can bounce around safely)

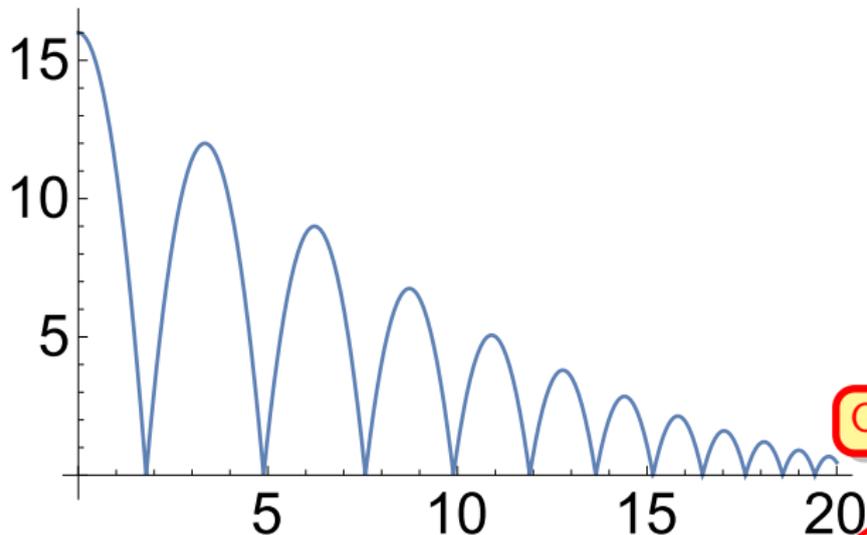
$$0 \leq x \wedge x = H \wedge v = 0 \wedge g > 0 \wedge \mathbf{1} = \mathbf{c} \rightarrow$$

$$[(x' = v, v' = -g \ \& \ x \geq 0; (?x=0; v := -cv \cup ?x \neq 0))^*](0 \leq x \wedge x \leq H)$$

Proof

$$\textcircled{\text{invariant}}(2gx = 2gH - v^2 \wedge x \geq 0)$$

# Quantum the Safely Bored Bouncing Ball



Can be improved...

Proposition (Quantum can bounce around safely)

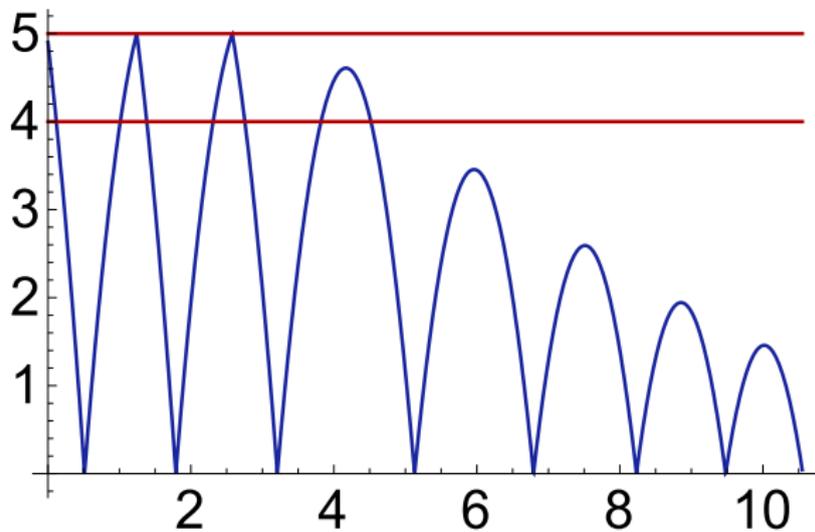
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$$[(x' = v, v' = -g \ \& \ x \geq 0; (?x=0; v := -cv \cup ?x \neq 0))^*](0 \leq x \wedge x \leq H)$$

Proof

$$\text{invariant}(2gx = 2gH - v^2 \wedge x \geq 0)$$

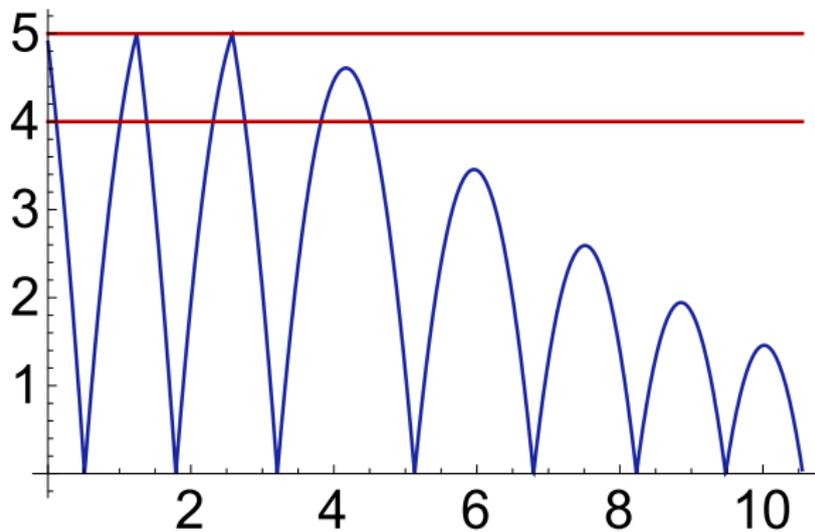
# Quantum the Daring Ping Pong Ball



Conjecture (Quantum can play ping pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
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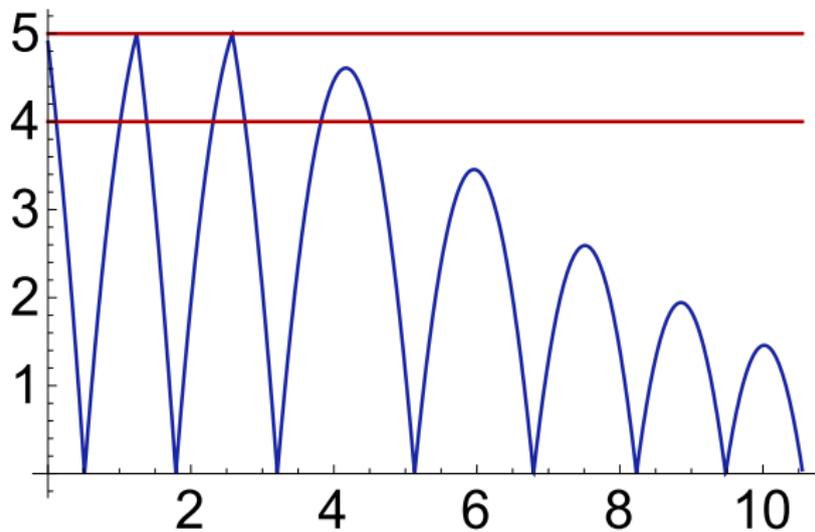
Proof?

Ask René Descartes

## Outwit the Cartesian Demon

Skeptical about the truth of all beliefs until justification has been found.

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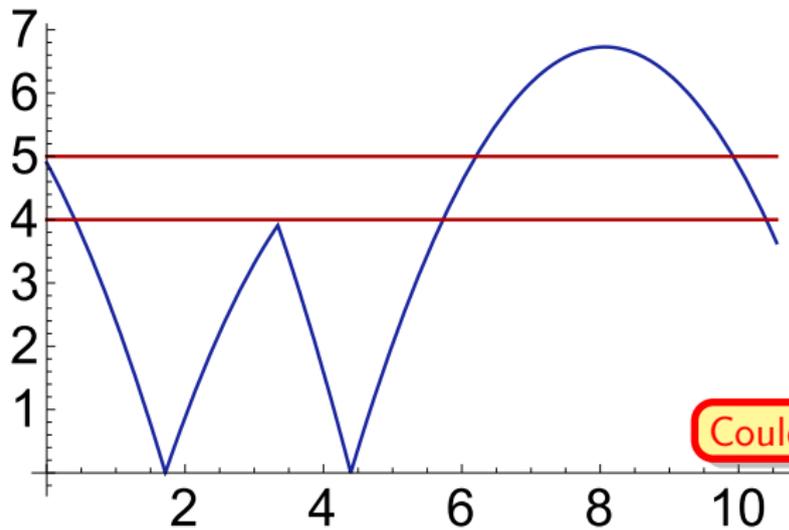
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Proof?

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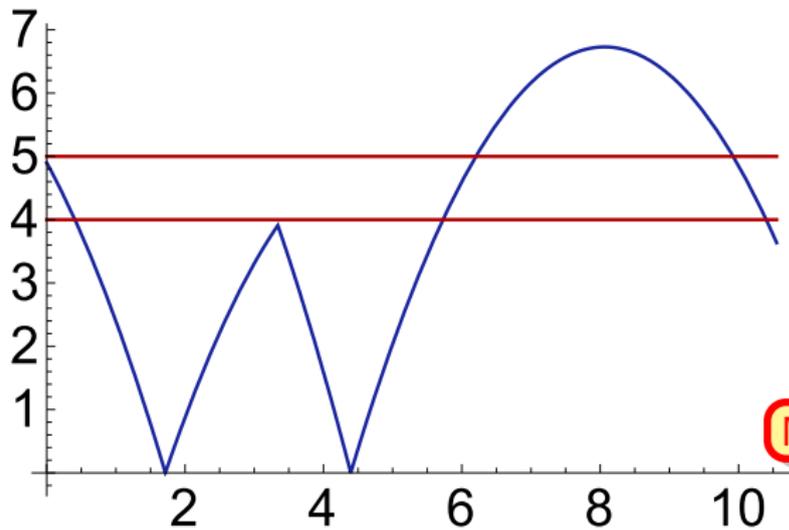
Could run instead of control

Conjecture (Quantum can play ping pong safely)

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Proof? Ask René Descartes who says no!

# Quantum the Daring Ping Pong Ball



No bounce at event

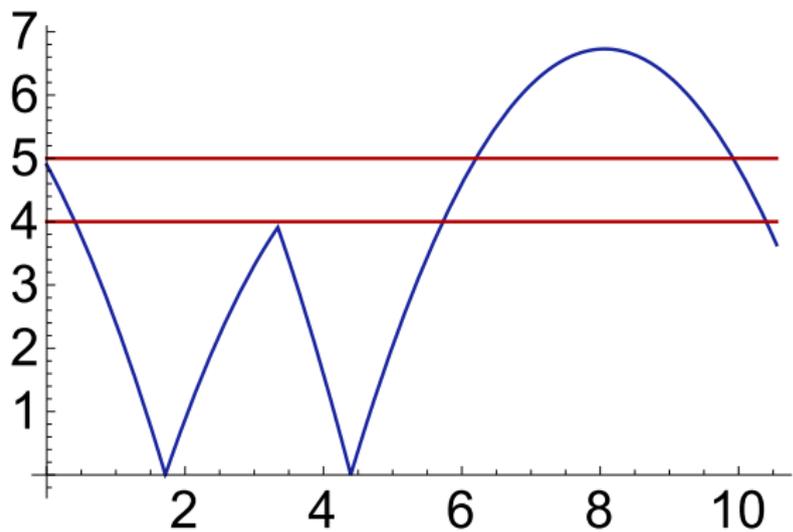
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Proof?

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# Quantum the Daring Ping Pong Ball



Could miss

Conjecture (Quantum can play ping pong safely)

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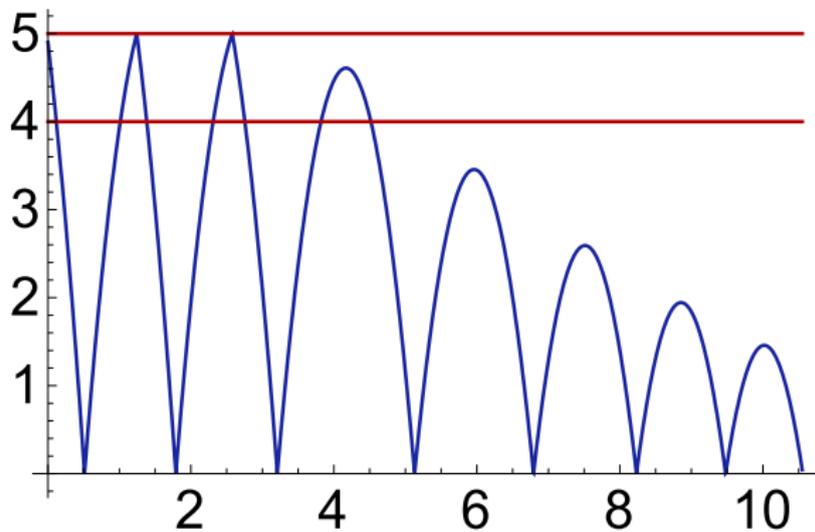
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# Quantum the Deterministically Daring Ping Pong Ball



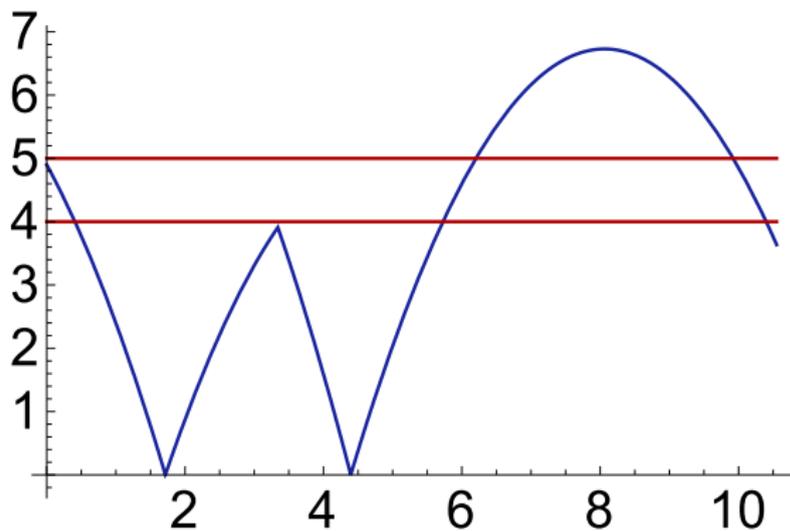
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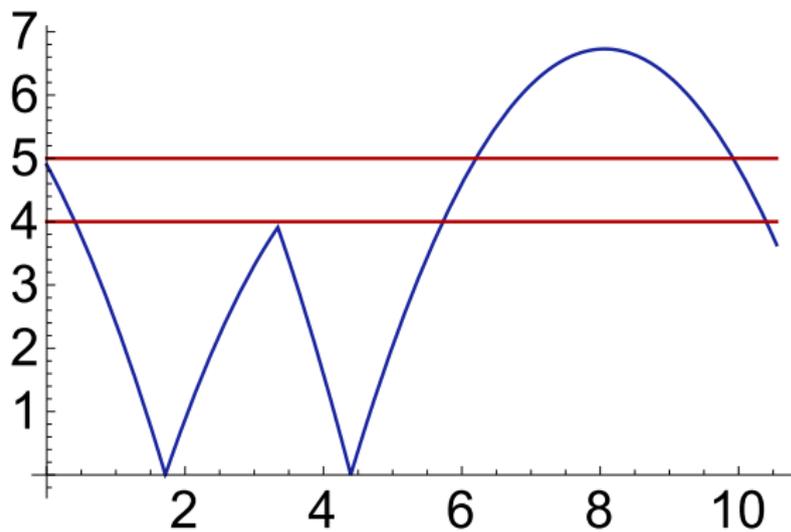


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Proof? Ask René Descartes who says no!

# Quantum the Deterministically Daring Ping Pong Ball



Could also miss if-then even

Conjecture (Quantum can play ping pong safely)

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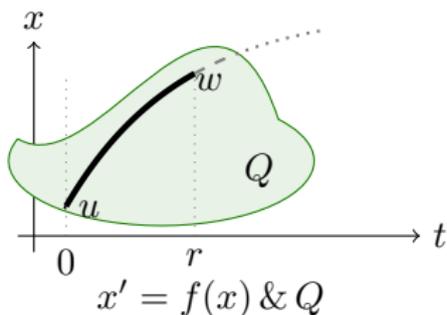
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# Evolution Domains Detect Events

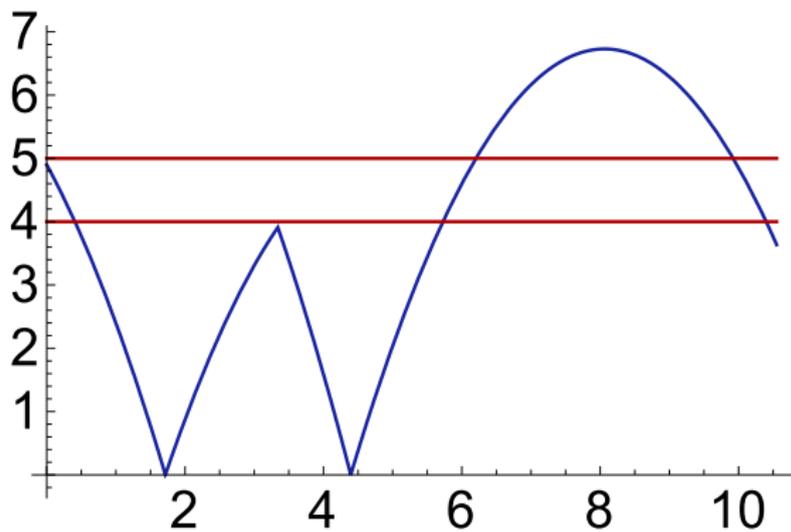
## Evolution domains detect events

$$x' = f(x) \& Q$$

Evolution domain  $Q$  of a differential equation is responsible for detecting events.  $Q$  can stop physics whenever an event happens on which the control wants to take action.



# Quantum the Deterministically Daring Ping Pong Ball



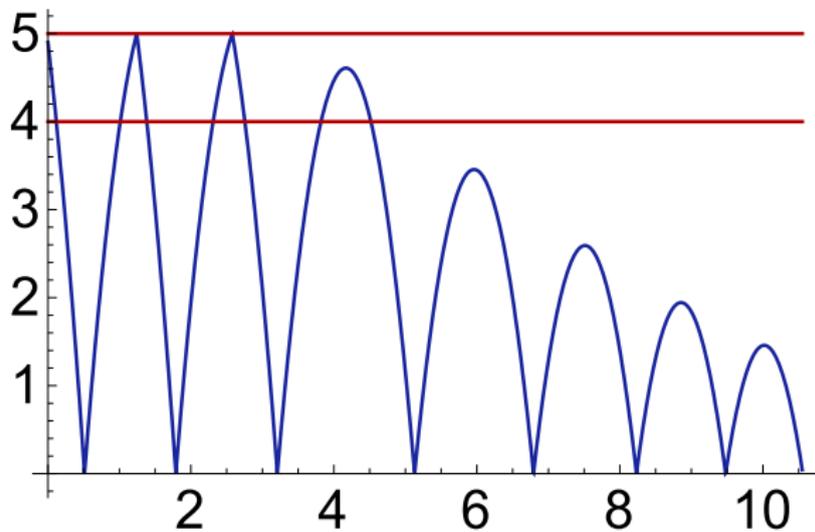
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# Quantum the Deterministically Daring Ping Pong Ball



Domain as event trap?

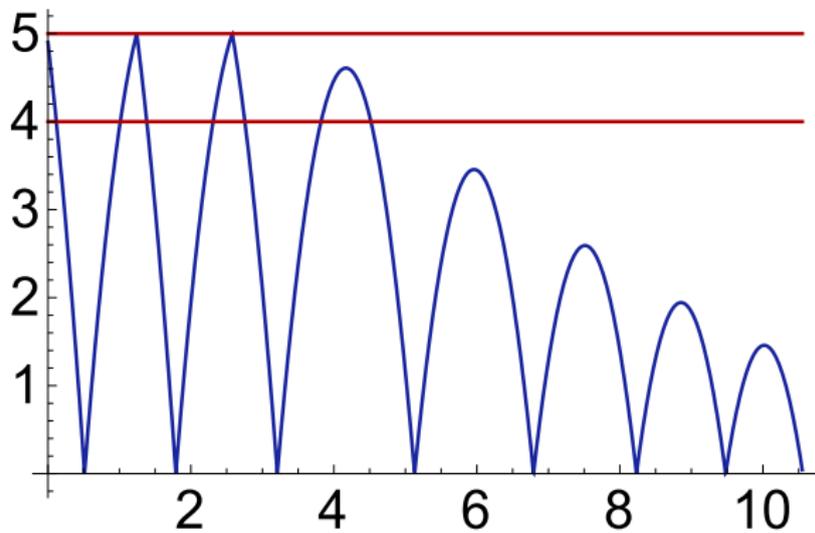
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# Quantum the Deterministically Daring Ping Pong Ball



Broken physics: Always ev

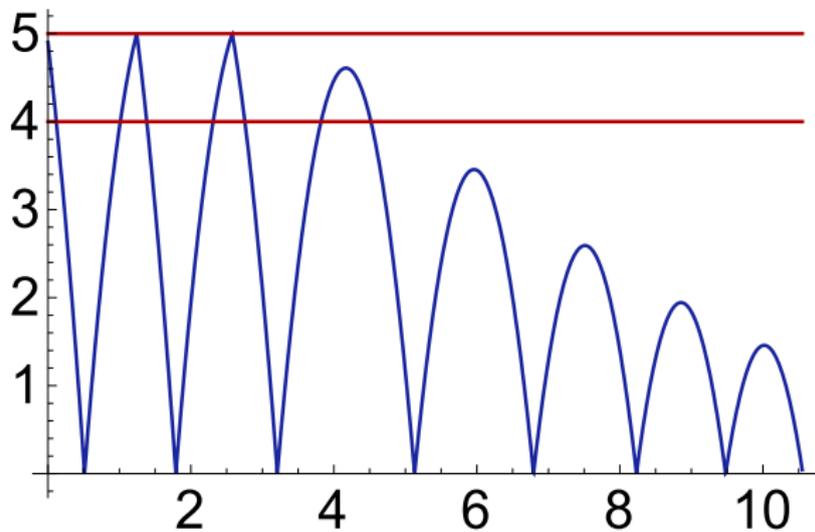
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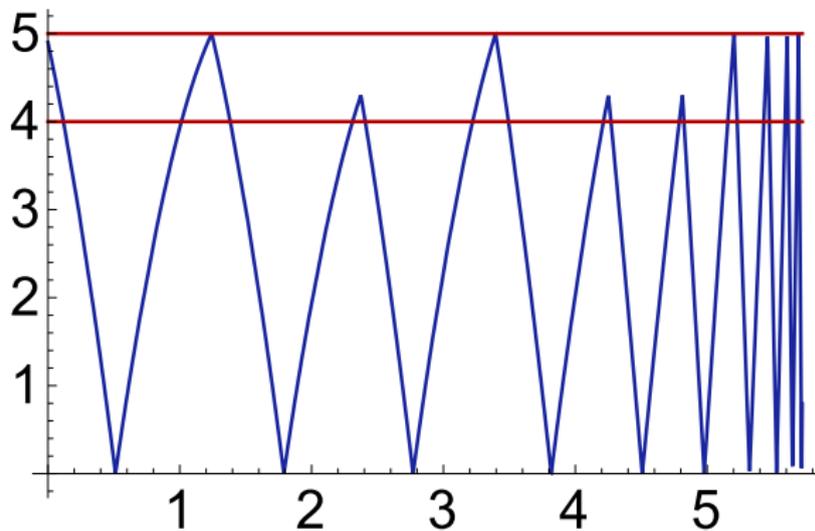
Limiting constraint

Conjecture (Quantum can play ping pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
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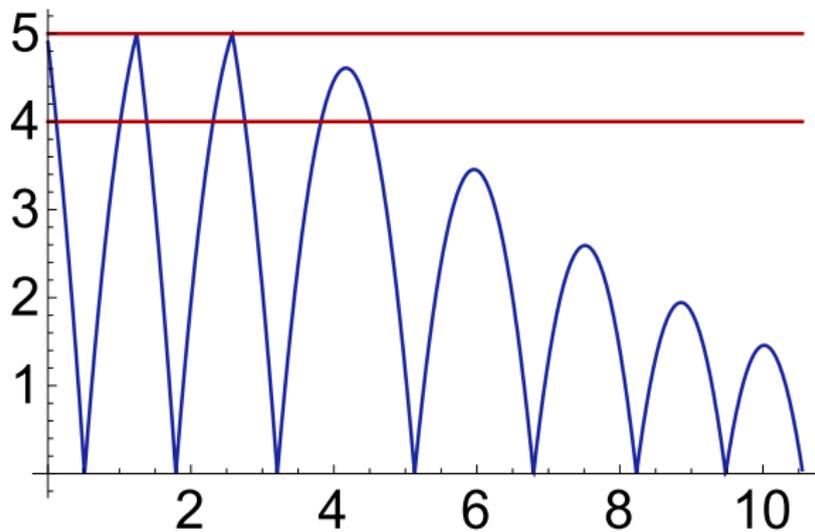
May miss 4 but not 5

Conjecture (Quantum can play ping pong safely)

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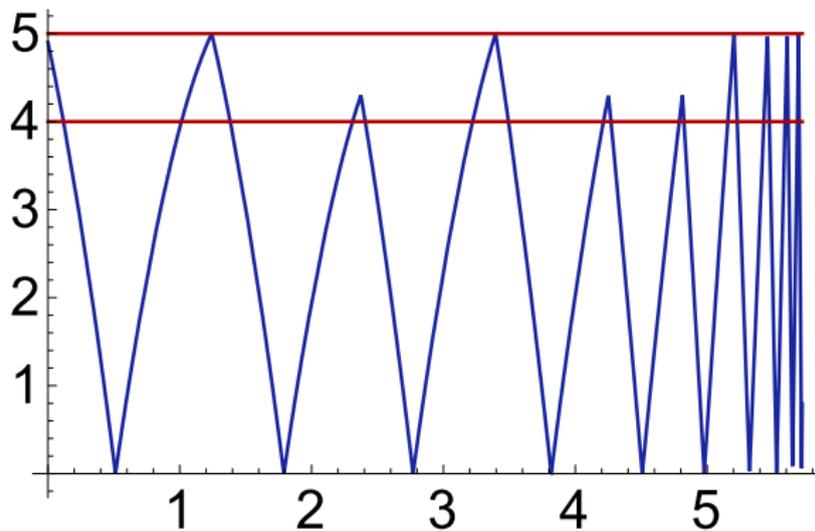
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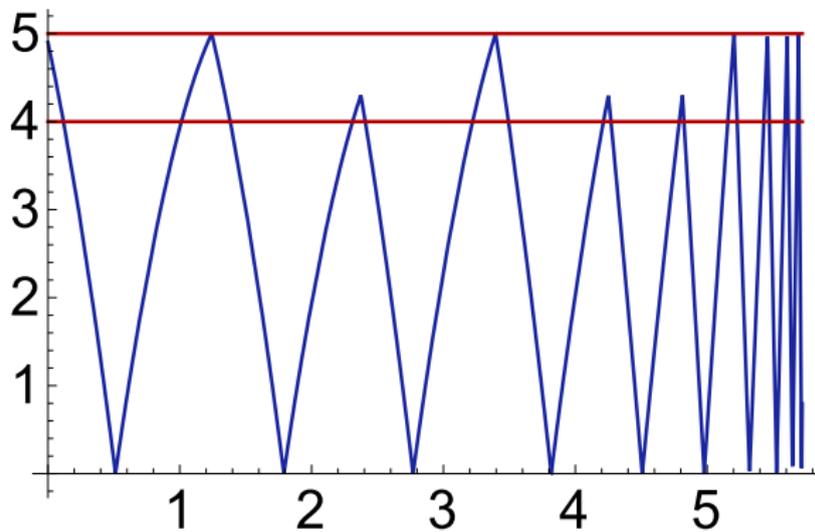
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Proof?

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## Non-negotiability of Physics

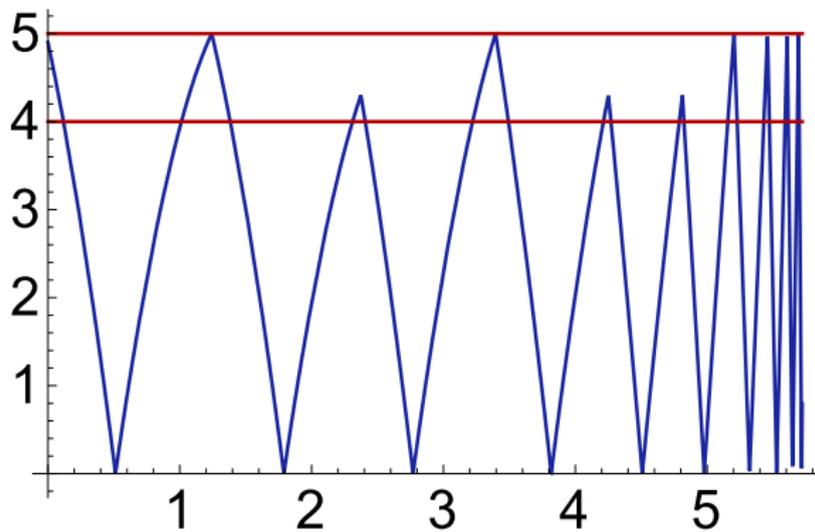
- 1 Making systems safe by construction is a great idea. For control!
- 2 But not by changing the laws of physics.
- 3 Physics is unpleasantly non-negotiable.
- 4 If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control

We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.

# Quantum the Deterministically Daring Ping Pong Ball



Can't

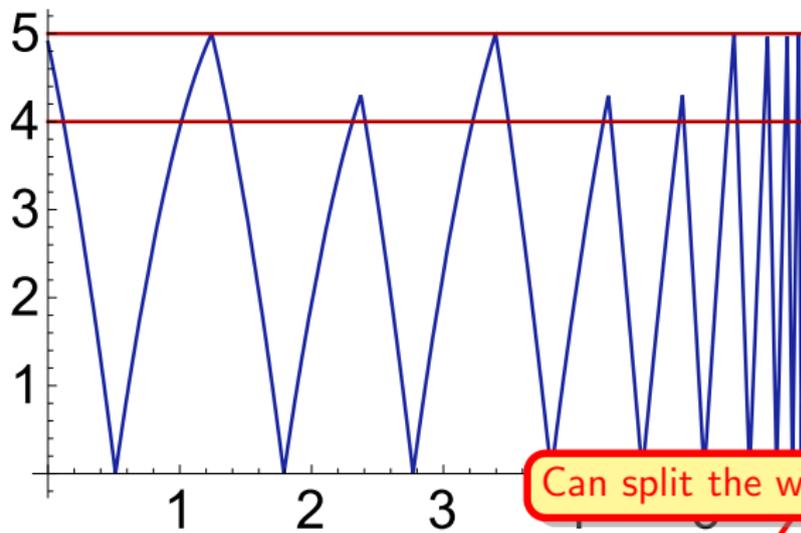
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Proof?

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# Quantum the Deterministically Daring Ping Pong Ball



Can split the world for an event

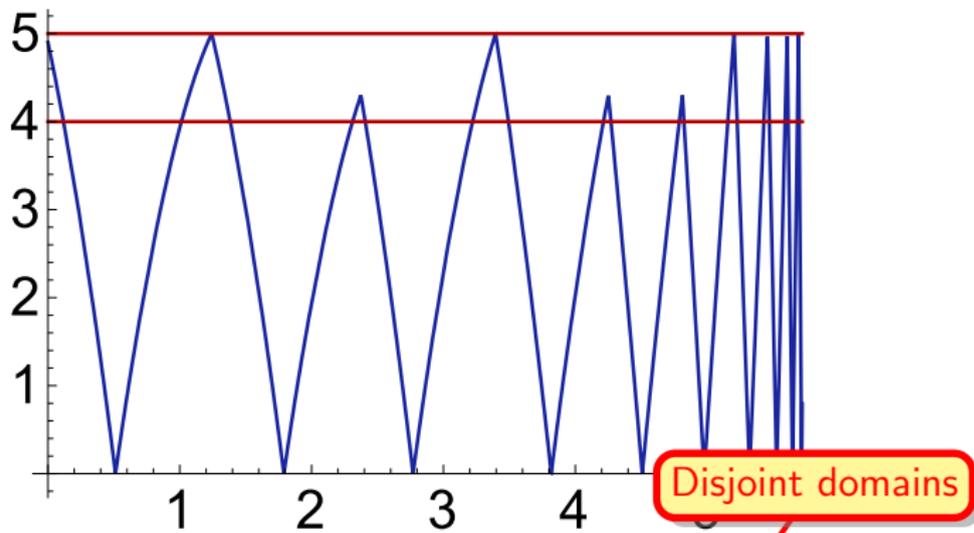
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$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5) v := -fv)^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes

# Quantum the Deterministically Daring Ping Pong Ball



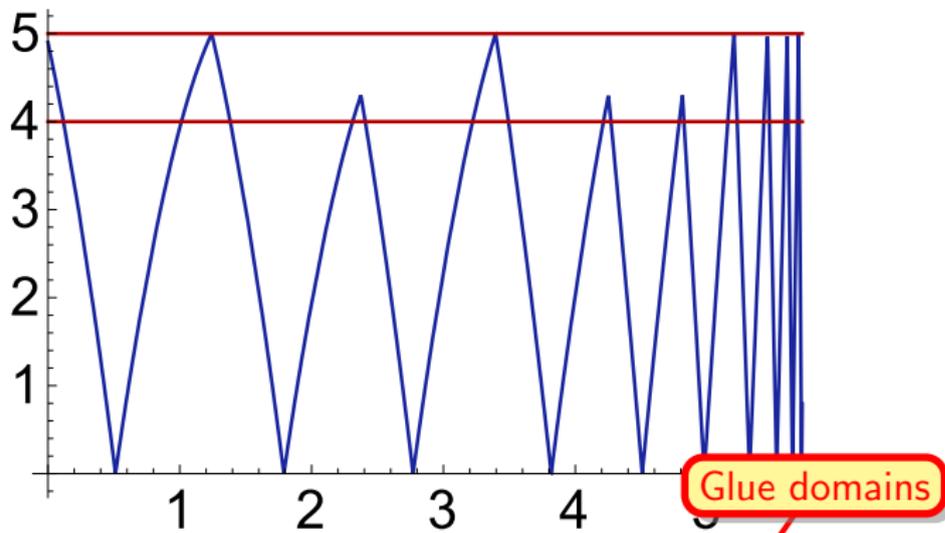
Conjecture (Quantum **Shattered the world** ping safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
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Proof?

Ask René Descartes

# Quantum the Deterministically Daring Ping Pong Ball



Conjecture (Quantum Reunite the world ping safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\{x' = v, v' = -g \& x \geq 0 \wedge x \leq 5 \cup x' = v, v' = -g \& x \geq 5\};$$
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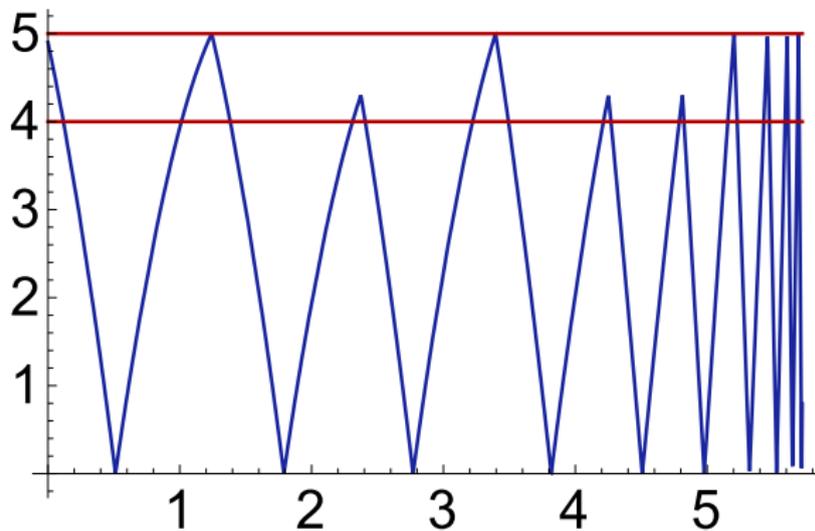
Proof?

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## Connected evolution domains

- 1 Evolution domain constraints need care.
  - 2 Determine regions within which the system can evolve.
  - 3 Disconnected/disjoint disallows continuous transitions.
- 
- 1 Splitting the state space into different regions to detect events is fine.
  - 2 Destroying the world is not.
  - 3 Not even by poking infinitesimal holes into the time-space continuum.

# Quantum the Deterministically Daring Ping Pong Ball



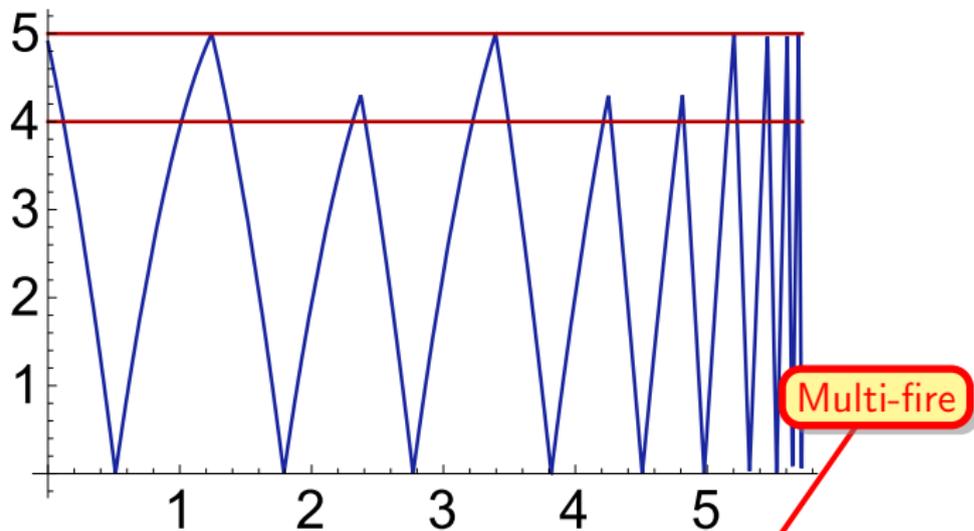
Conjecture (Quantum can play ping pong safely)

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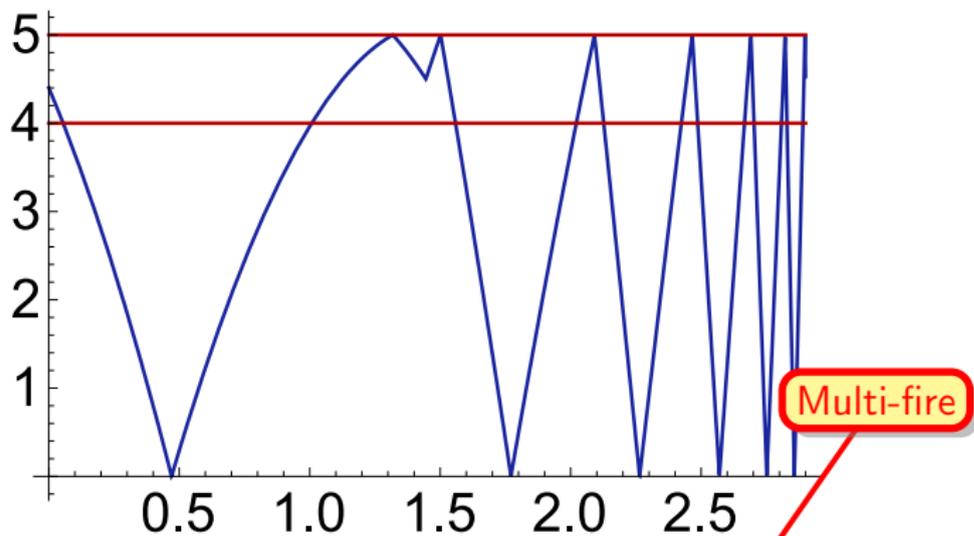
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Ask René Descartes

# Quantum the Deterministically Daring Ping Pong Ball

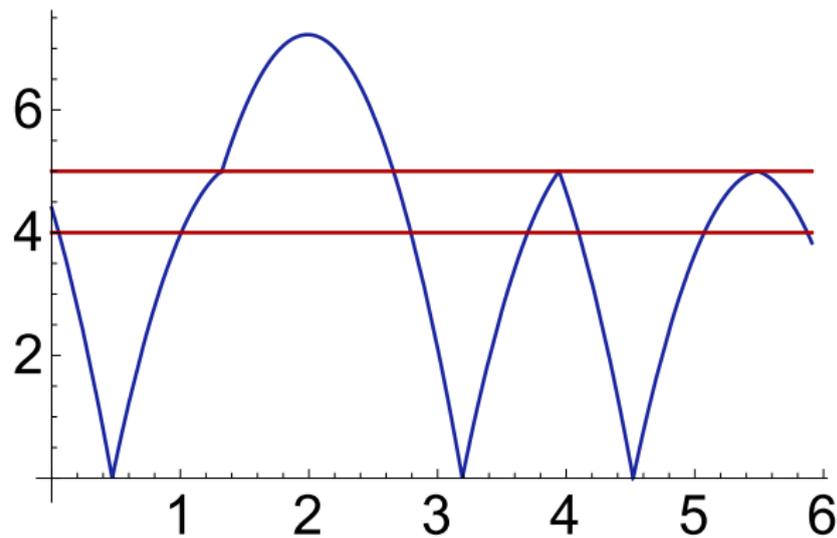


Conjecture (Quantum can play ping pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
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Proof? Ask René Descartes

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Multi-fir

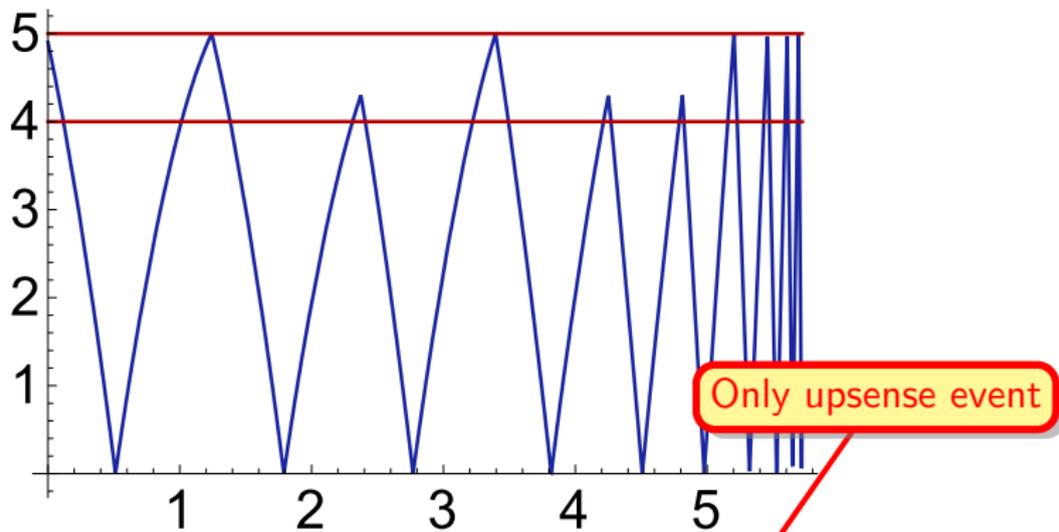
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Proof?

Ask René Descartes who definitely says no!

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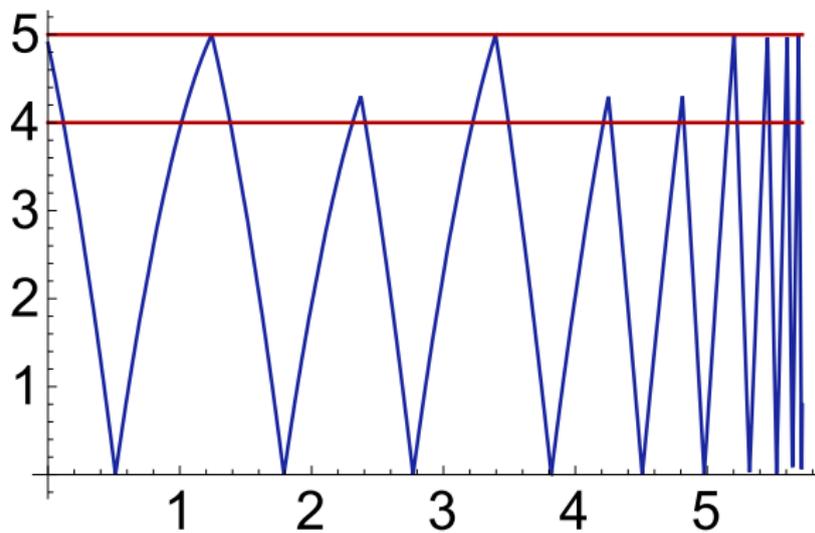
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Proof? Ask René Descartes

## Multi-firing of events

- ① If the same event is detected multiple times:
- ② Are multiple responses acceptable?
- ③ Or is a single response crucial?

# Physics vs. Control: Classification



**control:** robust, all cases

**physics:** precise

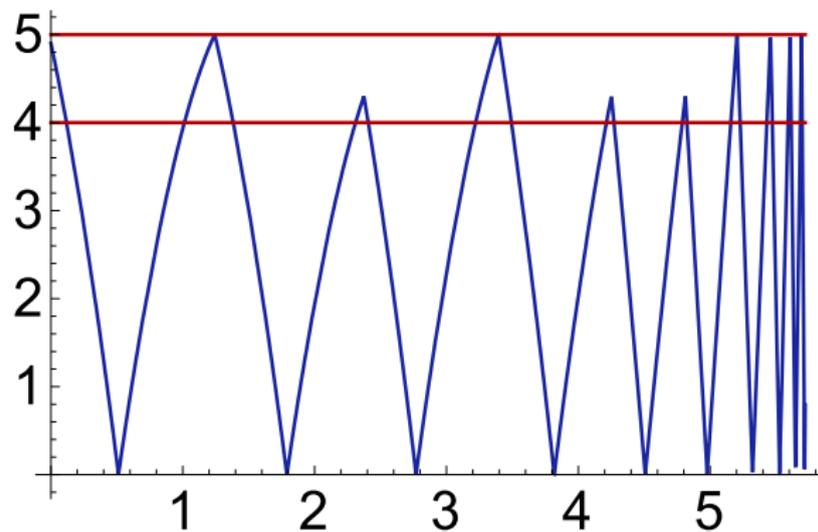
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- 2 The Need for Control
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  - Evolution Domains Detect Events
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- 4 **Proof**
  - **Loop Invariants**
- 5 Summary

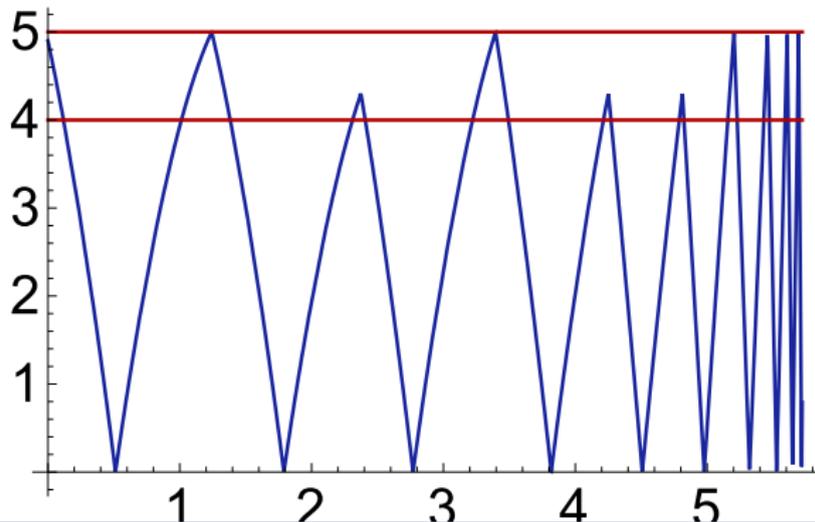
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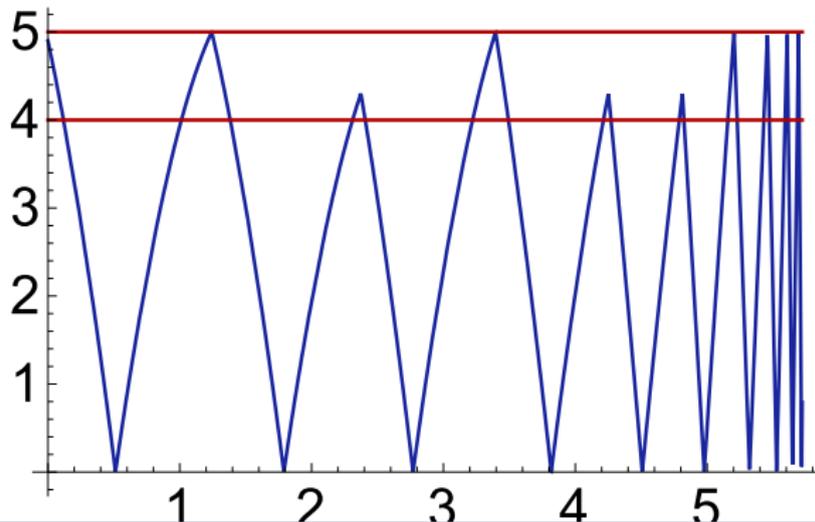
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Loop invariant  $j(x, v)$ :

①  $0 \leq x \leq 5$  not inductive

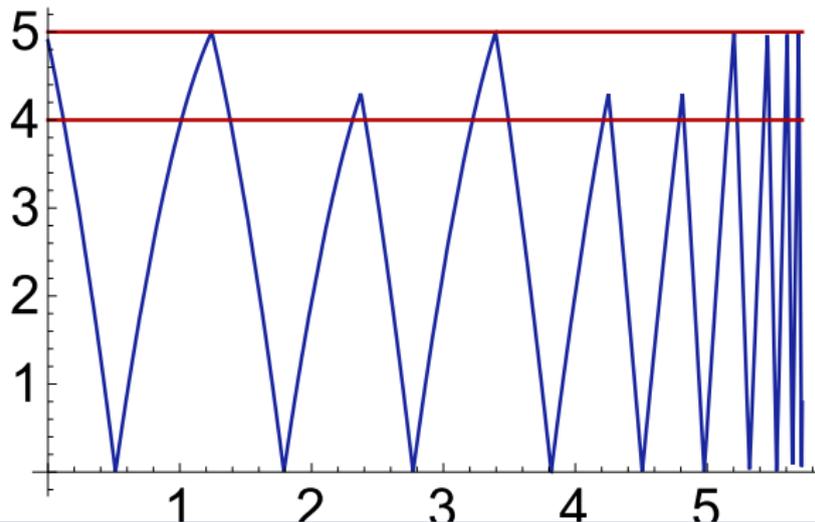
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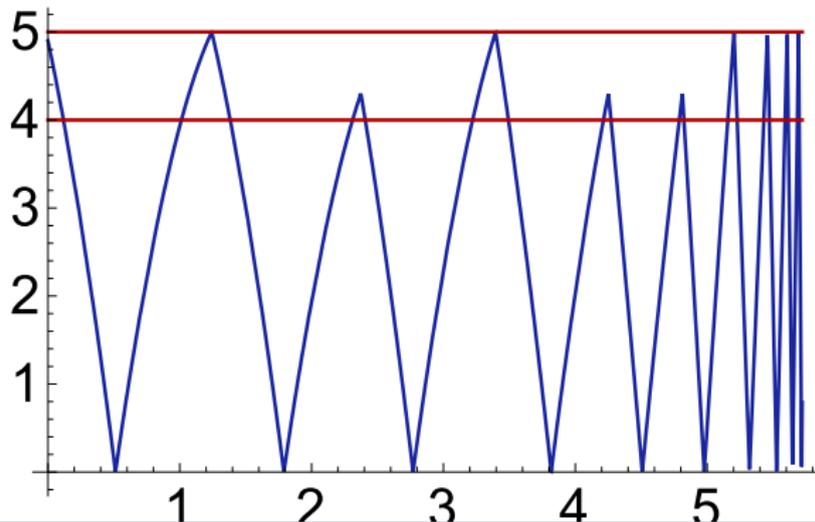
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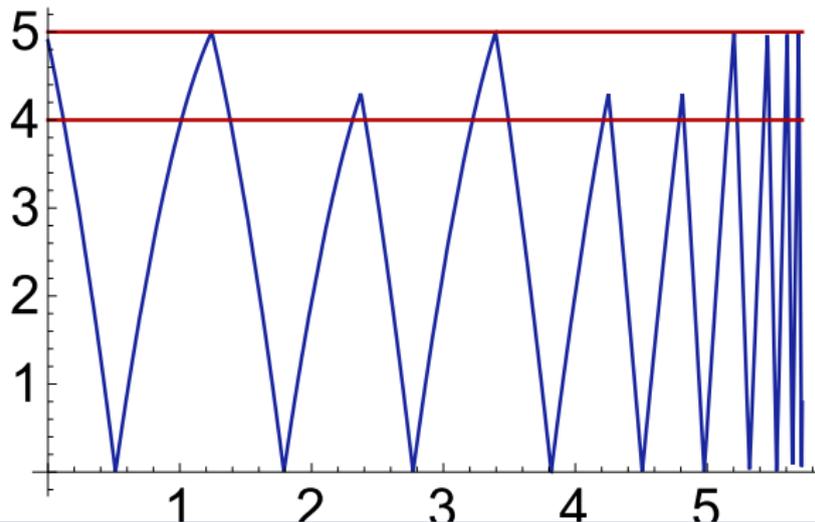
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Loop invariant  $j(x, v)$ :

- 1  $0 \leq x \leq 5$  not inductive
- 2  $0 \leq x \leq 5 \wedge v \leq 0$  not inductive
- 3  $0 \leq x \leq 5 \wedge (x=5 \rightarrow v \leq 0)$

# Quantum's Ping Pong Proof Invariants

Proposition (Quantum can play ping pong safely)

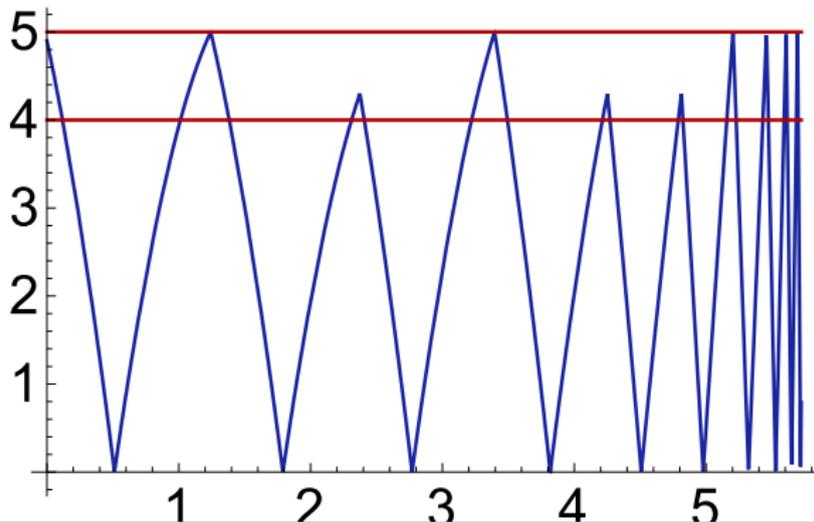
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Proof

@invariant( $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$ )



Loop invariant  $j(x, v)$ :

- ①  $0 \leq x \leq 5$  not inductive
- ②  $0 \leq x \leq 5 \wedge v \leq 0$  not inductive
- ③  $0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$  yes

# Quantum's Ping Pong Proof Invariants

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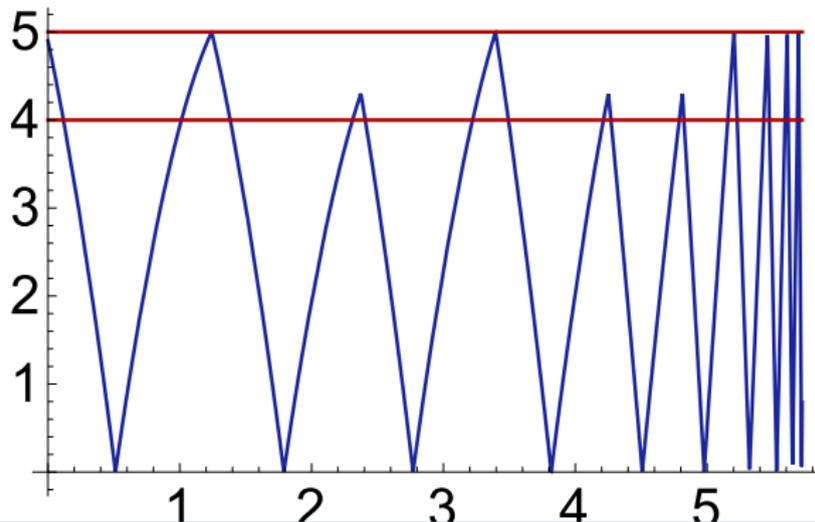
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# Summary: Event-triggered Control

- 1 Common conceptually simple paradigm for designing controllers
- 2 Assumes all events are surely detected
- 3 Implementation: Requires continuous sensing  
Tell me if you ever find a faithful implementation platform ...
- 4 Robust events, not just  $\text{if}(x = 9.8696)$  ...
- 5 Events have subtle models, but make design and verification easier!

Non-negotiability of Physics

Connected domains

Multi-firing ●

- 6 Verify event-triggered model as first step
- 7 Then refine toward realistic implementation based on safe event-triggered design
- 8 Physics  $\neq$  Control

## Non-negotiability of Physics

- 1 Making systems safe by construction is a great idea. For control!
- 2 But not by changing the laws of physics.
- 3 Physics is unpleasantly non-negotiable.
- 4 If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control

We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.



André Platzer.

Foundations of cyber-physical systems.

Lecture Notes 15-424/624/824, Carnegie Mellon University, 2017.

URL: <http://1fcps.org/course/fcps17.html>.



André Platzer.

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Springer, Heidelberg, 2010.

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