04: Safety & Contracts

15-424: Foundations of Cyber-Physical Systems

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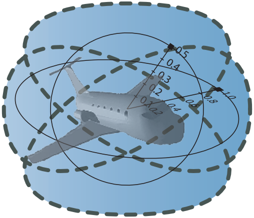

Outline

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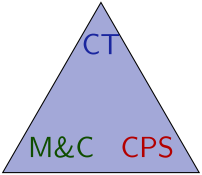
rigorous specification
contracts
preconditions
postconditions
differential dynamic logic

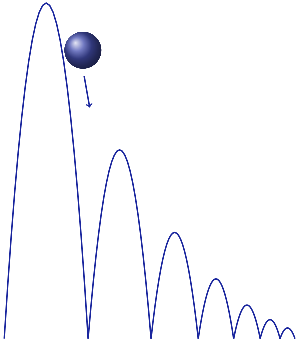


discrete+continuous
analytic specification

model semantics
reasoning principles

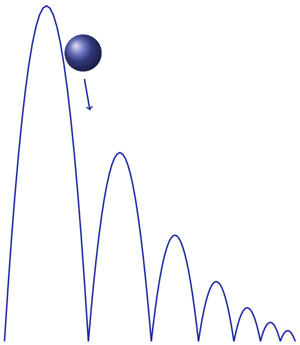
Outline

Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

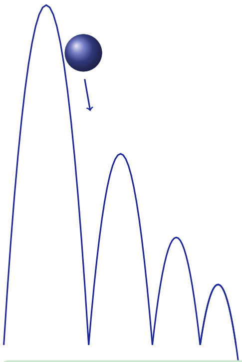
Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

$$x' = v, v' = -g$$

Quantum the Acrophobic Bouncing Ball



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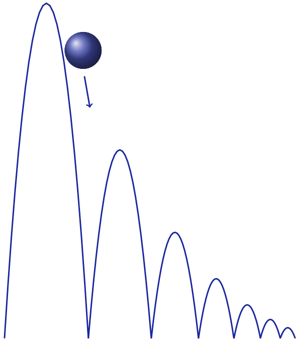

Example (Quantum the Bouncing Ball)

$$x' = v, v' = -g \,\&\, x \geq 0$$

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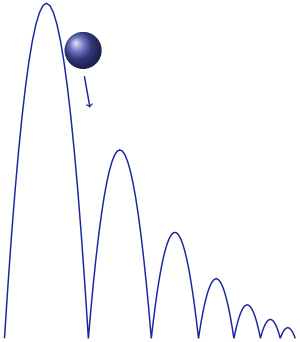
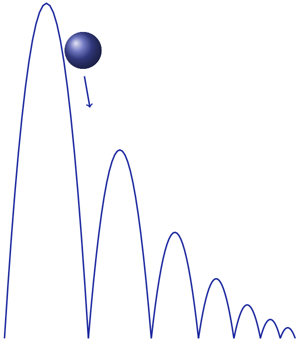


Example (Quantum the Bouncing Ball)

$$x' = v, v' = -g \,\&\, x \geq 0;$$

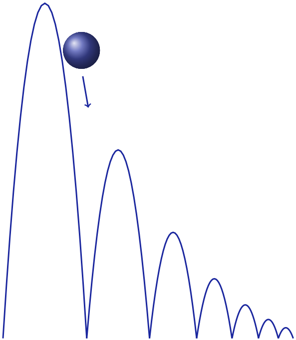
$$\text{if}(x = 0)\; v := -cv$$

Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

$$\left(x' = v, v' = -g \,\&\, x \geq 0;\right.$$
$$\left.\text{if}(x = 0)\; v := -cv\right)^*$$



Example (Quantum the Bouncing Ball)

$$\big(x' = v, v' = -g \,\&\, x \geq 0;$$
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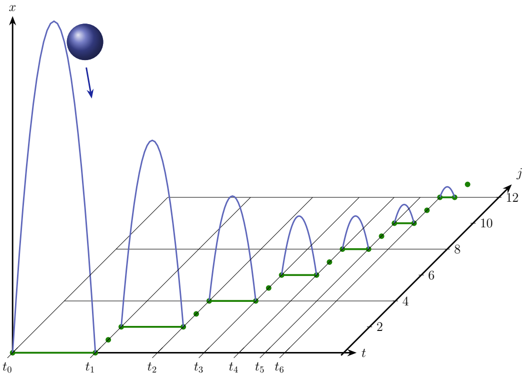
Quantum Discovered a Crack in the Fabric of Time



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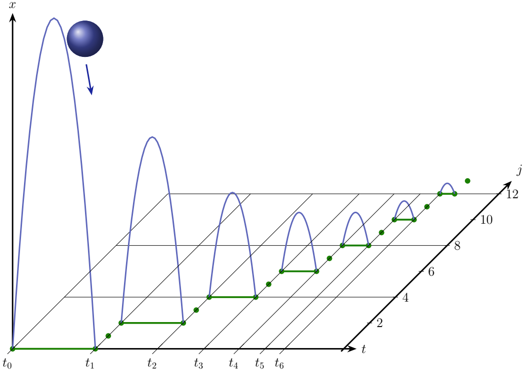

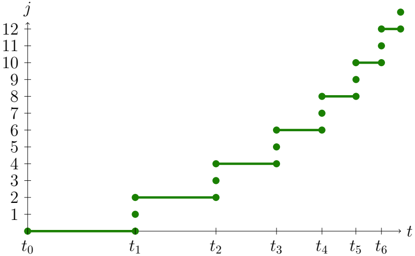

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Outline

Safety of Robots

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Three Laws of Robotics — Isaac Asimov

1. A robot may not injure a human being or, through inaction, allow a human being to come to harm.
2. A robot must obey the orders given to it by human beings, except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

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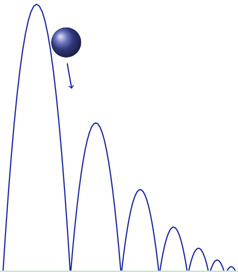
Three Laws of Robotics are not the answer.
They are the inspiration!

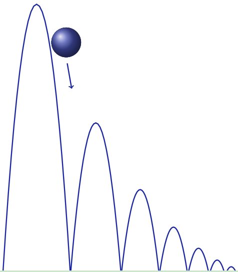
Contracts for Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

$$\begin{gathered}\big(x' = v, v' = -g \,\&\, x \geq 0;\\ \mathrm{if}(x = 0)\, v := -cv\big)^*\end{gathered}$$

Contracts for Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

$$@\text{ensures}(0 \le x)$$

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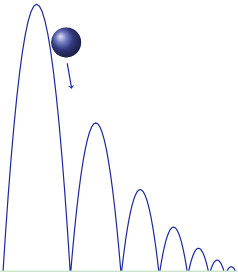
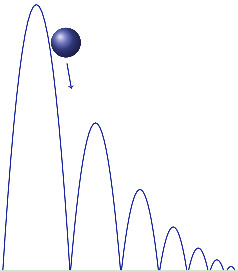
Contracts for Quantum the Acrophobic Bouncing Ball



Example (Quantum the Bouncing Ball)

$$
\begin{aligned}
&@\text{ensures}(0 \le x)\\
&\color{red}{@\text{ensures}(x \le H)}\\
&\big(x' = v, v' = -g \,\&\, x \ge 0;\\
&\quad \text{if}(x = 0)\, v := -cv\big)^*
\end{aligned}
$$



Example (Quantum the Bouncing Ball)

$$@\mathrm{requires}(x = H)$$

$$@\mathrm{ensures}(0 \le x)$$

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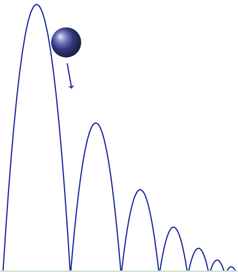
Contracts for Quantum the Acrophobic Bouncing Ball



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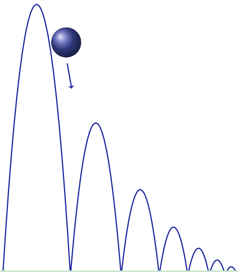

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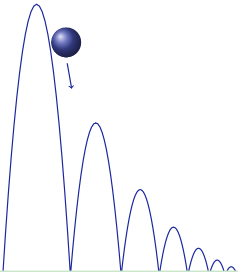
Contracts for Quantum the Acrophobic Bouncing Ball



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&@\mathrm{ensures}(0 \le x)\\
&@\mathrm{ensures}(x \le H)\\
&\big(x' = v, v' = -g \,\&\, x \ge 0;\\
&\quad \mathrm{if}(x = 0)\, v := -cv\big)^{*} @\mathrm{invariant}(x \ge 0)
\end{aligned}
$$

Developed on the board:

1. Differential dynamic logic dL as a precise specification language for CPS
2. Translation of contracts for bouncing ball to logical formula in dL
3. Syntax and semantics of dL

See lecture notes for details [1].

André Platzer.
Foundations of cyber-physical systems.
Lecture Notes 15-424/624, Carnegie Mellon University, 2016.
URL: `http://www.cs.cmu.edu/~aplatzer/course/fcps16.html`.

André Platzer.
Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics.
Springer, Heidelberg, 2010.
doi:10.1007/978-3-642-14509-4.