# Modeling Safety Zones for Scuba Divers in $dL$

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**Abstract**

SCUBA (Self Contained Underwater Breathing Apparatus) Diving is a sport that necessitates many safety measures. Although the breathing apparatus (regulator) takes care of regulating the pressure of air supply as a diver changes their depth, it does not have a check on whether the diver has enough oxygen to get back to the surface safely. We propose an alternative model for a new design for SCUBA computers. This model uses heart rate as the primary input for determining remaining safe dive time. Today, pulse sensors are readily available and can be purchased for as little as $10. This is much cheaper than the airflow sensors and underwater wireless transmitters used in today's computers. Such systems cost around $2000 up to ten times more expensive than a simple computer, which is can only be used for avoiding Decompression Sickness.

## 1 Introduction

A few SCUBA diving equipment manufacturers such as Suunto have created diving computers that can pair with wireless underwater transmitters attached to your air tank. These transmitters monitor the rate of air leaving the divers oxygen tank and provide an estimation of your remaining dive time. These systems can cost upwards of $2000, over ten times he price of simpler dive computers that only help one to avoid the bends (Decompression Sickness). Most of this cost is due to the wireless transmitter/receiver system.

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Decompression Sickness (DCS) as well as Arterial Gas Embolism (AGE) are diseases that can kill a diver when they ascend too quickly. Both conditions occur when the nitrogen absorbed by tissues through the inhaled air is diffused out of the solution faster than the blood can carry it away to the lungs, and forms bubbles in the tissues (DCS) and blood vessels (AGE). Our model will account for this using a maximal safe rate of ascent.

To reduce the cost of diving safety systems, we looked at ways to estimate the airflow, i.e., a divers oxygen consumption. One way, is through ones heart rate. Heart rate and oxygen consumption are closely correlated, despite, there the delay between the two. Your heart can change much faster than your rate of breathing, however, once one changes the other follows.

To model safety zones (times when a diver still has enough air to safety ascend and avoid DCS/AGE) we used a three-piece model.

Before we delve any further, it is important to note the limitations of our model. We will be using mathematical models from various sources. These models contain parameters that should be optimized for the sport you are interested in by obtaining data while performing the sport and using a regression analysis method to have the model fit the data. Due to constraints of budget and time, we were not able to optimize all model parameters. Where applicable, we used information we could find online and verified this with Karim's SCUBA diving experience.

## 2 Related Work

Cyber Physical Systems (CPS) is still a relatively new scientific field. It is however, growing at an astonishing rate. The National Science Foundation (NSF) says, "... demand for innovation in these domains continues to grow, and is driving the need to accelerate fundamental research to keep pace" [3].

Despite the need for CPS analysis in differential Logic, much of today's engineering of CPS is completed without rigorous proofs. Part of the reason is because of the scale of growth. Take cellphones for example. There are billions of them, across the world. Despite, the number of cellphones in the world, there has not been a lot of work around proving the safety, security and sustainability [1] of cellphones. Part of this is because of the sheer number of phones and various models in existence but another part is construction of these models. The models are complex and proofs of these models tend to be even more complex. Finally, systems engineering has not worked on integrating CPS into hardware [3]. It is a process that takes time and presents large architectural challenges. Couple this with the speed technology is moving and we find that there is a large portion of CPS that are completed without rigorous proofs and as such many systems are subject to recall.

Thanks to the youth of the field of CPS, we are actually one of the first few to analyze an application of CPS in SCUBA diving. However, there has been research performed on the most integral part of our model, the human heart.

One such group is from the Department of Electrical and System Engineering at the University of Pennsylvania. They modeled and analyzed implantable cardiac medical devices. They were motivated by the fact that there currently is no standard for the software that goes into such devices. In fact, in 2006 21% of all medical devices were recalled due to software bugs [6]. This team created a virtual heart model, which was used to test the integrity of cardiac medical devices. They use a simpler model than ours, which focuses more on the blood flow and electrical signals in the heart. They also use a simplified form for examining heart rate. They use an abstraction, which does not use any differential equations of large number of finite elements. Finally, the verification of their heart model is performed via electrophysiology and clinical studies. This contrasts our work in that we have analyzed a much more accurate model that responds to physical demands and prove our SCUBA diver can safely exit the water via analysis of their heart rate.

According to Haque et al. there is a lack of verification tools in general for CPS [5]. Most of the current methods rely on simulation models analyzing sensor data. They do not verify mathematical models in differential Logic.

Banerjee et al. proposed a framework on how to create solutions in CPS [1]. In particular, they define three properties, which ultimately a finalized CPS should have (safety, security and sustainability). Our work presented will focus on safety. The other two properties are implementation dependent and will be left for future work.

# 3 Our Model

We use a three-piece model to define safety zones. They are the heart or heart rate, volume of oxygen consumption and nitrogen absorption.

Our model will also be time-triggered control. This is because, when we built the wrist computer, our ultimate limiting factor will be the frequency with which we will be scanning for the divers heart rate. This rate will be at least twice the maximum heart rate for a SCUBA diver. This is because of the Nyquist Sampling Theorem, which states that in order to be able to construct a digital signal of a continuous signal, you should sample at twice the maximum frequency to be able to fully reconstruct the signal. For us the maximum frequency is the maximum rate at which a heart can beat. Furthermore, this frequency should also be the limit for our time-triggered control. That is, we should allow the model to evolve for at most $1/HR_{max} + ct$, where $ct$ is the time required for the model to compute the necessary decompression time for a given depth. This will allow us to make the proper control decisions within a reasonable amount of time.

### 3.1 Heart Rate

There are several requirements to modeling heart rate. Firstly, the model should be continuous and differentiable in order to exploit the benefits differential equations (ODE) and ODE representation in KeyMarea X (the theorem prover we will be using). Amongst other things, the model includes velocity as a variable for heart rate. This is because when scuba diving, your velocity is the largest factor that determines your heart rate. As you chase a fish or rush to see something exciting, your heart beats faster. You then consume your air supply at a greater rate and at some point; you may consume your air so fast that you will exit the safety zone.

- Both the minimum and maximum heart rate of an individual are modeled as repellers. Meaning, as you approach either extreme, it becomes increasing difficult to reach that value.
- The model accounts for the physical conditioning of an individual. The less fit you are, the faster you reach your bodys limit.
- The model has memory. Your current heart rate is a function of your previous heart rate and your current activity level. This models the fact that your heart does not change as fast as your movement.

There are some drawbacks to using Zakynthinakis model. Namely, accuracy of her model presents a challenge in proving properties of the system. Since heart rate is modeled as the product of three functions (minimum heart rate, maximum heart rate and heart rate demand), the derivative turns out to be quite messy. Thus the analysis and proof of the heart rate model will be difficult.

We have however found ways to simplify the model. For instance, Zakynthinaki accounts for the blood lactate level an individual has before beginning their exercise routine. Eventually, this term goes to zero after some time. In scuba diving, one has to prepare their equipment, put it on, and then surface swim to their dive location before beginning the dive. A diver knows that in order to maximize their dive time, they must try to perform all of these initial steps as calmly as possible. By minimizing their heart rate before the dive, they will reduce the lactate in their blood. Lactate is a chemical compound that is responsible for the burning feeling in your muscles during exercise but also responsible for the increased energy production during exercise. Since, the diver will work to minimize their heart rate, we will assume that the initial effects of heart rate demand are minimal thus simplifying the function for heart rate demand. In terms of Zakynthinakis model, this means we will only be using her function of demand during steady state.

Additionally, Zakynthinaki's data was fitted for a runner. We will present the locations, where possible, that we changed her model parameters to better-fit data that would be seen during a dive. Once again, we remind you that we cannot do this for all parameters because we do not have a data set for conditions seen during scuba diving.

We begin by examining the first equation of three, which comprise the equation for change of heart rate. $f_{max}(HR) = -(1 - e^{(\frac{HR-HR_{max}}{stdev})^2})$. Through experimentation, Zakynthinaki found that an appropriate value of stdev is 10 beats per minute. The $f_{max}$ function acts as a repeller function. That is, as one approaches the maximum heart rate (we use a value of 200 beats per minute, from the popular Fox and Haskell formula using an age of 20, Virens age), it becomes increasing difficult to increase your heart rate.

The next equation uses a similar form to $f_{max}$, but is a repeller function for the minimum heart rate.

$$f_{min}(HR) = 1 - e^{(\frac{HR-HR_{min}}{stdev})^2}$$

We use the same value for stdev. An individual's minimum heart rate is a function of their athletic conditioning. To represent ones conditioning, Zakynthinaki introduces a parameter that she calls lambda but we will call cond. Cond varies from zero to one, with one being a perfect athlete. Our model will allow for varying athletic conditions and as such, the minimum heart rate of each individual will vary. Ones minimum or resting heart rate is inversely proportional to their athleticism. Thus, for males, $HR_{min}(cond) = \frac{35}{cond}$ beats per minute.

The last equation in the heart rate model, is heart rate demand. This demand function includes both lactate and velocity, i.e., exercise intensity. Demand on the heart is normally examined under two cases: during exercise and during recovery. For the purposes of our model, we will only be analyzing the demand case. Thus, our simplified lactate function is $L(v,t) = \alpha_3 * L_{cond} * L_t(t)$. As mentioned previously, Zakynthinakis model accounts for lactate levels before the exercising begins, however, we do not need to account for this. $\alpha_3$ is used to "correct the units of the equation and simulates slow kinetmatics" [13]. $\alpha_3$ has units of beats/min/mM (milliMolar). We use the recommended value of 4. $L_{cond}(v)$ models the "cardiovascular condition and exercise intensity" and $L_t(t)$ is only "time dependent" [13].

$$L_{cond}(v) = L_{base} + (L_{max} - L_{base}) * e^{\alpha_6 * (v - v_{max})}$$
$$L_t(t) = 1 - e^{\frac{-t}{\alpha_7}}$$

$L_{base}$ is the concentration of lactate in arterial blood when the body is at rest which is about 1 mM [13]. We use a value of 9 mM for $L_{max}$ which is noted as the violation exhaustion point (a point no SCUBA diver wants to reach [4]. Thus, $L_{max}$ is actually lower than what Zakynthinaki uses in her model. We also present a different value for $v_{max}$ because humans cannot swim underwater as fast as they can run. The average velocity a diver swims at is around 1 km/hr and a reasonable maximum is 3 km/hr as per Karim's diving experience. To further increase the model's accuracy, we also multiply the maximum velocity by the square root of *cond* to represent that only perfect athletes can only reach this velocity. $\alpha_6$ controls the curvature of the blood lactate curve [13]. In order to match the curvature of Zakynthinaki's original model (since our maximum velocities are different), we found 1.8 hr/Km to be appropriate. $\alpha_7$ represents the time constant of the exponential decay. This value varies with the type of dive an individual is going on. We chose $\alpha_7 = 2700s^{-1}$ to match Zakynthinakis curve and give a value more appropriate to a typical diving scenario.

This now brings us to the final form of the demand equation.

$$f_d(HR, v, t) = -\alpha * cond * (HR - L(v,t))$$

Where $\alpha$ "corrects the units and correctly simulates the heart rate kinetics" and is determined through Zakynthinaki's experiments to be 0.08 $sec^{-1}$ [13].

Finally, we unite all three equations into differential form

$$HR' = f_{min} * f_{max} * f_d$$

As you can see, this differential equation has a very complicated dependence on heart rate, velocity and time. The first and most important hurdle to overcome before we could use this differential equation as the dynamics of system was to find a way to model the exponential terms. This was problem because e doesn't exist in the Dl syntax. Our first idea to define e was by writing an ODE of the form x' = x to indirectly produce an exponential term in the solution; However, this was not possible because:

1. The expansion of the $HR' = f_{min} * f_{max} * f_d$ does not have one exponential term, it has multiple exponential terms added and multiplied together, each dependent on velocity, time, Heart Rate or a combination of them.

2. The differential equation itself has exponential terms and not just the solution.

Thus we had to examine this model and come up with a simpler differential equation that replicates most of the features to a reasonable extent. We came up with the equation:

$$HR' = -(HR - HR_{ss})\frac{c}{-\frac{1}{3} * (\frac{v}{v_{max}}) + .5} \tag{1}$$

$$\tag{2}$$

where

$$2.71828^c = HR_{ss} - HR$$

and

$$HR_{ss} = HR_{min} + \frac{v}{v_{max}}(HR_{max} - HR_{min})$$

Notice, firstly the solution of the (1):

$$HR = HR_{ss} - (HR_{ss} - HR_{init})e^{(-\frac{1}{3}*(\frac{v}{v_{max}})+.5)^{-1}ct}$$

Which looks like (see Figure 1):

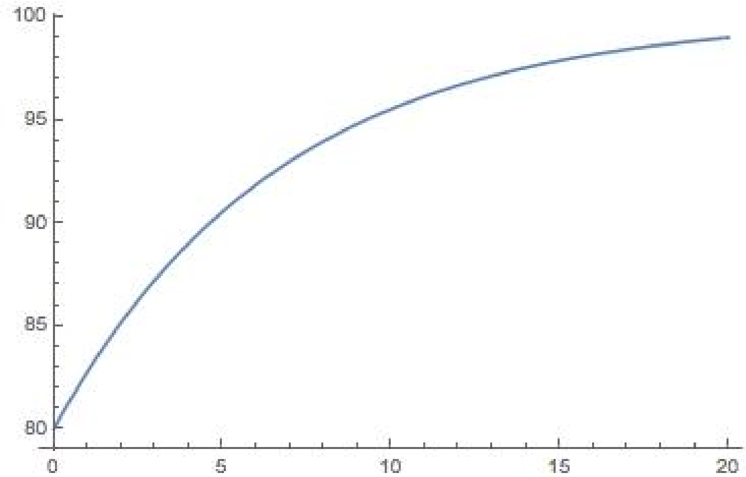


Figure 1: Example solution of (1). This is how heart rate would evolve to a steady state of 100 beats per minute from 80 beats per minute in 20 seconds.

- We can see that the Heart Rate converges to the steady state $HR_{ss}$ as t $\to \infty$.

- The way we calculate the steady state is dependent on the velocity of the diver. $HR_{ss} = HR_{min} + \frac{v}{v_{max}}(HR_{max} - HR_{min})$ so that there is a 1:1 relationship between the velocity as a percentage of $v_{max}$ and the steady state as a percentage of $HR_{max}$. It is then scaled to be between $HR_{min}$ and $HR_{max}$.

- The constant multiplied with t in the exponential term is $(-\frac{1}{3} * (\frac{v}{v_{max}}) + .5)^{-1} * ln(HR_{ss} - HR_{init})$. The first part of the product determines the time scale to reach the steady state.This was determined by setting a reasonable value of 10s to reach $HR_{max}$ given an exercise intensity of $v_{max}$ and making the relationship between time taken to reach $HR_{ss}$ and percentage of $V_{max}$ linear. For Example, 20s for .5 $v_{max}$.

- The second half of the product determines what it means to be "close" to $HR_{ss}$ within the time determined by the linear equation. We set this as $ln(HR_{ss} - HR_{init})$ so that $exp(-ln(HR_{ss} - HR_{init})) = \frac{1}{HR_{ss} - HR_{init}}$ which makes the solution $HR_{ss} - 1$ in the time prescribed by the linear equation.

- Being within 1 of the steady state is reasonably close given the values of steady states end up being in the hundreds, making it within 1% of the steady state value after the prescribed time.

- Also note that the way we calculate the value of $ln(HR_{ss} - HR_{init})$ is using differential ghosts:

$$c = *; ?(2.71828^c = HR_{ss} - HR_{init}).$$

We needed to do this since the natural log does not exist in the dL syntax. Since $e$ also doesn't exist in the syntax of dL, we approximated it to 5 decimal places of it actual value. It was reasonable for us to do so because c was a constant in the differential equation, and even if the $e$ was part of the syntax, the computer would approximate its value before using it.

### 3.2 Volume of $O_2$

According to the work done by Sterling et Al. [11] the relation between heart rate and oxygen consumption can be analyzed by splitting the function for oxygen consumption into two domains. That is, where the demand for oxygen is less than time derivative of maximum oxygen consumption and where it is greater. With scuba diving, there is rarely a case where a diver would have to maintain a level of exertion where his or her demand of oxygen would be greater than the maximum oxygen consumption. Thus, for simplification of our model, we will only be examining the case where the demand is less than the maximum heart rate.

We found another team that worked on finding a relationship between volume of oxygen taken in and heart rate. Swain et Al. worked in the Human Performance Laboratory in Marshall University in 1994 [12]. They looked at 81 men and women aged between 18 and 34. They performed an incremental exercise test up until exhaustion. They recorded each patient's maximum heart rate and volume of oxygen taken in. They then performed linear regression to have the data fit a linear equation with a coefficient of determination of 0.988, i.e., it is a very good estimation. The equation we use is [12]:

$$\%HR_{max} = 0.6463 * \%VO_{2max} + 36.8$$

Where the relationship between heart rate and maximum oxygen consumption can be expressed as percentages of their maximum. As stated earlier, the maximal heart rate our model will use is 200 *beats/min*. We will use a maximal $VO_2$ of 60 *milliliters/kg/min* as this is the typical maximum for an athlete between ages 20 and 30. This turns into 288 *litres/hr* for a person with the weight of 80 *Kgs*. Shuffling the equation results in:

$$VO2 = \frac{VO2_{max}}{0.6463 * HR_{max}} * HR - \frac{36.8}{0.6463}$$

Upon differentiating this becomes:

$$VO2' = \frac{VO2_{max}}{0.6463 * HR_{max}} * HR'$$

Clealry, we can see that the rate at which the oxygen tank of the diver depletes is -VO2'. Thus the final ODE for the oxygen in the tank becomes:

$$\begin{aligned} Tank' &= -VO2' \\ &= -(\frac{VO2_{max}}{0.6463 * HR_{max}} * HR') \\ &= -(\frac{VO2_{max}}{0.6463 * HR_{max}} * -(HR - HR_{ss})\frac{c}{-\frac{1}{3} * (\frac{v}{v_{max}}) + .5}) \end{aligned}$$

### 3.3 Nitrogen Absorption

When ascending to the surface, the ambient pressure around the diver decreases. If this change in pressure is, too rapid it can cause two major problems:

- Decompression Sickness (DCS)
- Arterial Gas Embolism (AGE)

The tissues in our body absorb nitrogen from the air proportional to the ambient pressure around the body. The nitrogen is stored as a solution in the tissues. As long as the partial pressure of inhaled air remains constant there is no threat from the absorbed nitrogen; however, when there is a sudden and rapid decrease in pressure, the nitrogen diffuses out of the solution and forms bubbles in the tissues and the bloodstream. This happens because the body is does not get enough time to equalize the pressures by carrying the excess gas from the tissues to the lungs to be exhaled. DCS, also referred to as the bends, is a result of the bubbles growing in the tissue. DCS can result in fatigue, itchiness of the skin, joint pain, and shortness of breath, dizziness, and even paralysis. AGE, on the other hand, is caused when the bubbles enter lung circulation and cause tissue damage by blocking blood flow. This is often fatal. Hence, it is imperative that we address the diver's ascent and decompression in our model.

When reviewing literature on mathematical models of decompression, we realized that it is best to assume a safe rate of ascent for the diver according to diving tables and pre-made calculations. Assuming a safe rate of ascent from any depth bypasses complicated science of modeling nitrogen absorption by tissues, and therefore required maximum rate of ascent, as a function of depth and time. Making this assumption, however, does not affect our final goal of ensuring enough air for the diver to reach the surface safely because if we can determine how much oxygen is needed to ascend at the maximum rate, we can alert the diver in time. Hence, we are trying to model the relationship between the velocity of the diver, heart rate, and oxygen consumption so that at any given depth, we know how much minimum oxygen is needed for the diver to ascend to the surface at the maximum permissible velocity.

According to the US Navy Diving Manual, a safe rate of ascent is 30 feet/min (fpm) [7].

### 3.4 Controller and the Diver's Motion

We designed the controller in order to model the diver as:

1. going towards the surface and therefore reducing his depth. In this case the rate of change of depth of the diver,$vDepth = -1 * v$, where v nondeterministically assigned but then constrained to be equal to $v_{ascent} = 30$ feet per min or .54 kmph) for this case. Safety in this case is assured by the initial conditions.

   OR

2. staying at a certain depth and moving around. In this case $vDepth = 0 * v = 0$. Safety in this case is ensured by a guard which ensures that after time T, given the worst case scenario of a maximum intensity workout that results in VO2max, the diver will still have enough oxygen in the tank to come up to the surface with $speed = v_asc$, while spending VO2max (more conservative than assumption VO2asc):

$$?(Tank - VO2max * T >= depth/v_asc * VO2max);$$

It is important to note that even though $vDepth = 0$, it is not neccessary that $v = 0$. v is constrained to be $0 \leq v \leq vMax (= 3kmph)$ using the guard:

$$?(0 \leq v \& v \leq v_{max});$$

This means the oxygen is still being used according to the dynamics for VO2'(v).

OR

3. diving deeper and thereby increasing his depth. In this case $vDepth = +1 * v$ where v is again constrained to be equal to $v_ascent$ ( = 30 feet per min or .54 kmph). Here there is guard that ensures that after time T, given maximum exhaustion resulting in VO2max, the tank has enough oxygen to bring the diver up to the surface from the new depth, in a similar way as the previous case:

$$?(Tank - VO2max * T >= (depth - vMax * T)/vMax * VO2max);$$

The dynamics of the movement of the diver are pretty simple since we only control his depth: Depth = depthV.

# 4 Proof of the Model

The main property we would like to prove about our system is that the SCUBA diver can safely reach the surface. This means that starting from the depth the diver is at, they have enough air left in their tanks to ascend at a safe rate (avoid DCS and AGE). In order to do this, we must recognize that it is impossible to predict a divers heart rate in the future. Thus, the only true measure of safety is that, we assume that the diver will be panicked when leaving the water and thus will be consuming oxygen at their maximal rate ($VO2_{max}$). We cannot compute the necessary air supply from their current heart rate because it is subject to changes at any point in time. Thus our safety condition is:

$$Tank \geq depth/vAscent * VO2_{max}$$

Where tank, represents the volume of air left in the SCUBA divers tank. Thus, safety in our model is defined such that, no matter the current depth of the diver, we can provide them with indication on their wrist computer on whether or not they can proceed to go deeper (they will have enough air for safe exit).

Proving the model turned out to be quite the challenge. In order to simplify our proof, we further estimate our model to bring it down to a simpler mathematical expression. As you can see from the previous sections, this meant defining a new equation for the change of heart rate as well an equation to describe how the volume of air remaining in the tank, decreases with respect to heart rate.

The main technique we used to prove our model was by coming up with an invariant, i.e., something that holds true during all runs of the model. Our invariant was centered on the fact that our controller helped ensure the safety condition by not allowing the diver to go deeper or even stay at the same depth if they did not have enough air for it. Thus our invariant is:

$$Tank \geq depth/vAscent * VO2_{max} \; \& \tag{1}$$

$$VO2 \leq VO2_{max} \; \& \tag{2}$$

$$v < 3 \; \& \; v \geq 0 \; \& \tag{3}$$

$$depth < depthMax \tag{4}$$

where:

1. This is the safety condition. It should hold for each iteration of the program.

2. The diver should not be able to consumer more oxygen than their maximal amount.

3. The speed at which the diver is moving. The maximum speed a diver can move at is 3 Km/hr.

4. The diver should never pass depthMax. This is computed before the diver starts his/her dive and is based on how much air they have in their tank.

# 5 Improvements

The main improvement that we need for our model is data from SCUBA divers. As you have seen, there were multiple points in the model where regression analysis was needed to fit our differential equations to diving data. Unfortunately, since we did not have any data, we had to use data from runners or use parameters that felt right.

Another point of the model that we would like to increase the accuracy of is the nitrogen absorption. It turns out that in todays diving computers, used advanced algorithms that are more conservative that the US Navys standard for decompression. These algorithms are in fact so conservative that decompression is actually part of your sightseeing trip while you dive. They use safety stops (points during your dive where you maintain your depth for several minutes to allow for maximal decompression). See figure 2 for an example of how these algorithms determine if safety stops (decompression) is required.

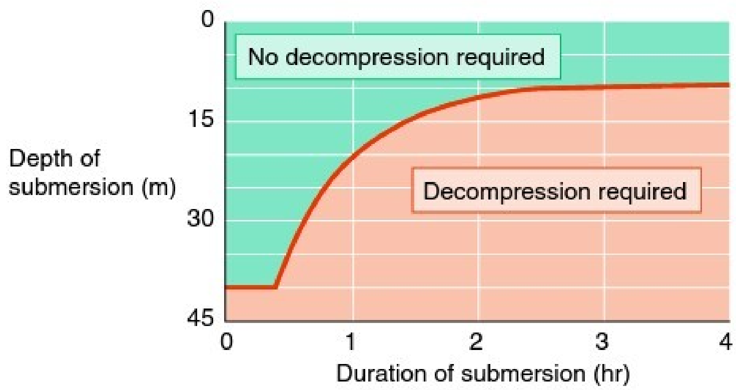


**Figure 61-5** The need for decompression as a function of depth and duration of dive. If the dive is sufficiently brief or sufficiently shallow, no decompression is required *(teal area)*. For deeper depths or longer durations, a decompression protocol is required *(salmon area)*. *(Data from Duffner GJ: Ciba Clin Symp 1958; 10:99-117.)*

Figure 2: When decompression is required according to depth and length of dive. From Duffner GJ: Ciba Clin Symp 1958; 10:99-117

## 6 Conclusion

Thanks to the model we developed[1], we have been able to prove that when using heart rate as an indicator of oxygen consumption we can identify safety zones for SCUBA Divers. These safety zones give warning to divers when their rate of oxygen consumption and time required for decompression is greater than the remaining air supply in their tanks. We chose to use heart rate because today, pulse sensors have become very cheap (under $10 per sensor). They are in smartwatches and fitness bands with some that are even waterproof. By taking this new low cost technology, we can help reduce the chance of Decompress Sickness. In the U.S. alone, over a thousand cases of DCS occur annually [8]. This low cost technology will help put safety in the reach of casual SCUBA divers.

Casual SCUBA Divers are the most as risk divers because it is harder for them to remember the signs of DCS because of their infrequent underwater trips. They also, want to maximize their dive time on their occasional trips so they try to rush out of proper decompression techniques since it does take several minutes during a dive. Finally, most casual divers do not own their own diving computer. Either they use cheap rental computers or they own cheap wrist computers (under $300). In any case, these cheap computers offer proper decompression instructions but no indication whatsoever of the remaining dive time for a SCUBA diver based off their air consumption. There are many cases where a diver will check their air tank pressure gauge (this offers them an indication of how much air is remaining in their tanks) and realize they are past the safe point they originally planned for because they either lost track of time or overexerted themselves chasing a beautiful aquatic animal. At this point, they must immediately terminate their dive and try to ration their air supply for a shortened decompression cycle. If they do not plan it correctly or do not have enough air, they will suffer from DCS or AGE unless rushed to a hospital and placed in a decompression chamber for several hours.

By proving that this low cost technology works, we will be able to offer divers ample warning that their air supply is running low for a proper decompression. Safety will be cheaper for people to purchase and ultimately will help save many lives.

Some of the goals that we accomplished are:

- We created a model that had an ideal tradeoff between accuracy and analyzability. This came from the initial complex model but we were

[1]Equal work was performed by both project members.

able to replicate most of that models properties via simpler mathematical functions.

- The most fundamental driver for us was that we wanted to create something useful, that we could use in our lives. SCUBA diving is an activity that only a lucky few get to participate in. By having worked on this, we might be able to make it safer and more accessible for all.

- Our original goal was to prove our complex (three-piece) heart model. This model included high order differential equations with multiple exponential terms, which were dependent on more than one parameter (velocity, heart rate, and depth). The complexity behind proving such a model was even more complex. Although, we were not able to prove this model, it gave us valuable insight on the properties we needed for a model on heart rate.

Some of things we learned while doing this project is that

- It is extremely difficult to prove accurate models. This was our original goal and we were not able to meet it.

- We also, found that it was hard to combine sources of information. Various authors used different units and different variables for their models. We had to find ways to combine all three parts of our model despite these challenges.

- We gained an appreciation for the importance of CPS. The models are complex enough to give you a better intuition for what goes on in your body while SCUBA diving. They also give us a way to prove something (our safety condition) without risking a persons life. They provide this information, without having to build any hardware or sending a human into the water (which may result in death).

# References

[1] Ayan Banerjee, Krishna K. Venkatasubramanian, Tridib Mukherjee, and Sandeep Kumar S. Gupta. Ensuring safety, security, and sustainability of mission-critical cyber-physical systems. *Proceedings of the IEEE*, 100(6061910), January 2012.

[2] Michel de Lara. Diving into mathematics or some mathematics of scuba diving.

[3] National Science Foundation. Cyber-physical systems (cps).

[4] Matthew L. Goodwin, James E. Harris, Andrs Hernndez, and L. Bruce Gladden. Blood lactate measurements and analysis during exercise: A guide for clinicians. *Journal of Diabetes Science and Technology*, 1:558–569, 2007.

[5] Shah Ahsanul Haque, Syed Mahfuzul Aziz, and Mustafizur Rahman. Review of cyber-physical system in healthcare. *International Journal of Distributed Sensor Systems*, 2014(217415):20.

[6] Zhihao Jiang, Miroslav Pajic, and Rahul Mangharam. Cyber-physical modeling of implantable cardiac medical devices. *Proceedings of the IEEE*, 100(5985460), January 2012.

[7] United States Navy. Us navy decompression us navy decompression tables and procedures, tables and procedures, part three.

[8] Divers Alert Network and DAN Assistant Medical Director Dr. E.D. Thalmann. Decompression illness: What is it and what is the treatment?, March/April 2004.

[9] Colin Please, Katie Newhall, Michael Levinson, Shelby Stanhope, Yi Wang, Michael Cromer, Jayanth Jagalur-Mohan, and Jacek Wrobel. Mathematical modeling of decompression during scuba diving, 2009.

[10] Ph.D. Scott A. Stevens, Ph.D. William D. Lakin, and M.D. Wolfgang Goetz, Ph.D. A differentiable, periodic function for pulsatile cardiac output based on heart rate and stroke volume. *Mathematical Biosciences*, 2002.

[11] J.R. Stirling, Maria S. Zakynthinaki, and Bengt Saltin. A model of oxygen uptake kinetics in response to exercise: Including a means of calculating oxygen demand/deficit/debt. *Bulletin of Mathematical Biology*, 67:9891015, 2005.

[12] David P. Swain, Kimberly S. Abernathy, Carla S. Smith, Shirley J. Lee, and Shelly A. Bunn. Target heart rates for the development of cardiorespiratory fitness. *Medicine & Science in Sports & Exercise*, January 1994.

[13] Maria S. Zakynthinaki. Modeling heart rate kinetics. *PLoS One*, 2015 April 13.