Star Lab Project Proposal

KeYTuner: A Hybrid Guitar Tuning System

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ABSTRACT

KeYTuner is a hybrid system that models a clip-on tuner used to tune the strings of one the most common string instruments, an acoustic guitar. KeYTuner uses a tension sensor to measure the tension in a string and calculates the frequency of the sound produced by that string. The tuning mechanism is carried out by automating the rotation of the peg, ensuring that it rotates until the string is tuned. This paper describes the model and proof for KeYTuner.

INTRODUCTION

String instruments make sound with vibrating strings, and the pitch is modified by the thickness, tension, and length of the string. String instruments can be played in many ways, and come in many variations. All string instruments make sounds with tensioned strings. Longer strings produce a lower tone than shorter ones. Tighter strings produce a higher sound than looser ones. Thicker strings produce a lower sound than thinner strings.

KeYTuner is a hybrid system that models a clip-on tuner used to tune the strings of one the most common string instruments, an acoustic guitar. Most guitars need regular retuning due to changes in temperature, humidity and string condition. In the absence of an electronic tuner, tuning ‘by ear’ can only be achieved by an experienced musician, and often requires a fair amount of time and concentration. The proposed hybrid system aims to provide an automatic method of tuning.

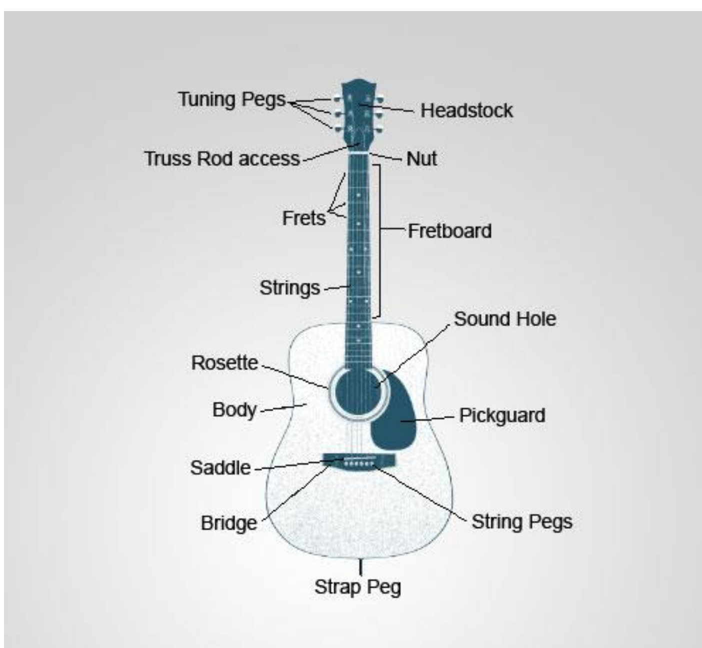

Figure1: Anatomy of an acoustic guitar[1]

RELATED WORK

The most prevalent guitar tuners today can be broadly classified into one of the three types:

1. Strobe: Strobe tuners are stroboscopes that flicker a light at the same frequency as the note. They compare the note played with the reference frequency using the stroboscopic effect.
2. Vibration (Piezo): These pick up the vibrations of the string to determine if the guitar is in tune.
3. Microphone: These rely solely on sound and not vibrations. They compare the sound produced by the string to the ideal sound that must be produced.

The KeYTuner works somewhat similar to a Strobe in the sense that it tunes a guitar based on the frequency of the standing wave generated by the string. While the Strobe uses light to achieve this, KeYTuner uses a clip-on tension sensor to calculate the frequency and then uses a controller to automatically tune the string using a tuning peg.

APPROACH

The problem of tuning a guitar could be approached in multiple ways, explaining the multitude of guitar tuners available in today's market. A string could be struck and the corresponding sound wave could be processed using FFT to compute its frequency components. Alternatively, the physical properties (frequency, tension) of the string could be modelled to match those of an ideal string. KeYTuner relies on the latter to tune guitar strings.

String	Note	Frequency (Hz)	Rounded up
6	E	82.407	82
5	A	110.000	110
4	D	146.832	147
3	G	195.998	196
2	B	246.942	247
1	E	329.628	330

Table1: Frequency of waves produced by guitar strings[2]

Tension in a string and the frequency of a wave produced by that string are related in the following way:

$$f = \frac{1}{2L}\sqrt{\frac{T}{u}}$$ [3]........equation (1)

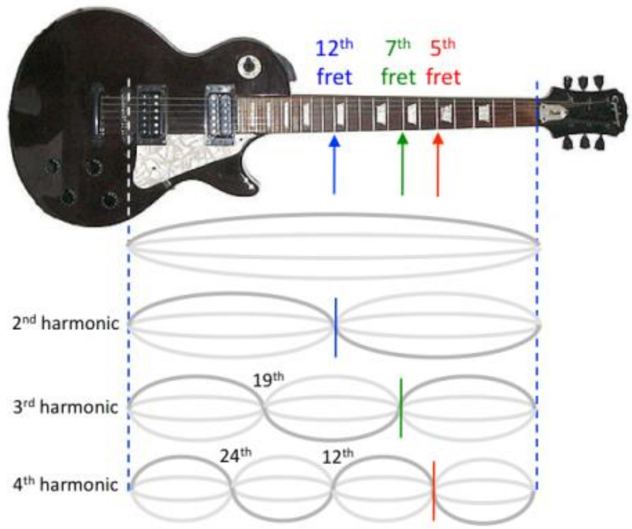

Figure2: Harmonics on a guitar[6]

KeYTuner uses the equation described in (1) to calculate the fundamental frequency (1st harmonic) of the wave generated when a guitar string is struck. The generated frequency is compared against the thresholds in Table 1. To assume that the string is always too slack is a reasonable assumption since all tuning techniques involve slackening the string first. The tuning peg is then rotated at an even rate to shorten the length of the string until the fundamental frequency of its standing wave equals the threshold. Figure 3 describes the relationship between length of an arc and the angle subtended by it.

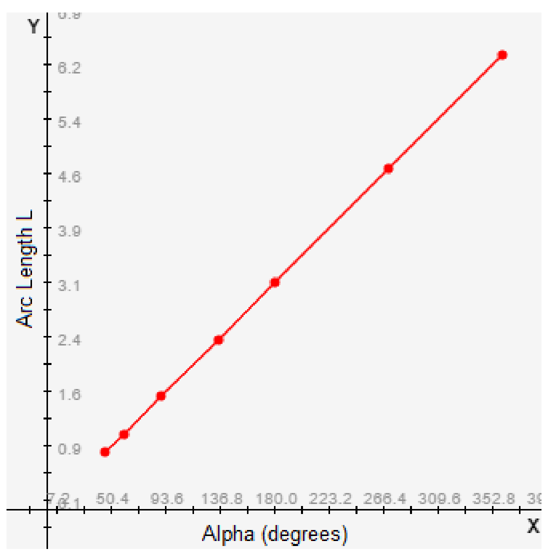

Figure3: Arc length (L) vs Angle subtended at the center (alpha)

Here, the length of the arc gives us the decrease in the length of the string. Ideally, the length by which the string is shortened is given by

L = pegR*alpha where alpha = $\sin^{-1}$(pegY/pegR)

pegR is the radius of the tuning peg and pegX and pegY are points on it circumference.

The rate of shortening of the string is proportional to the rate of change of alpha, and so,

L’ = pegR*alpha’

⇨ L’ = r*($\sin^{-1}$ pegY/pegR)’ * pegY’

But $$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$ [4]

Hence, L’ = pegR/((pegR^2 - pegY^2)^(1/2))*(pegV*pegX)

MODEL

A sensor clips on to the string to be tuned and measures the tension in the string. Using the formula described in (1), the fundamental frequency of the standing wave is calculated. A reasonable assumption in this case is that **the string is always too slack** and hence needs to be tightened. KeYTuner then implements an **event-driven model**, meaning the peg is then rotated at an even rate until the frequency of the string matches the ideal frequency.

The differential equations ***pegX' = -(pegV*pegY)/pegR*** and ***pegY' = (pegV*pegX)/pegR*** model a tuning peg being rotated at a constant velocity pegV.

L' = (pegR*pegV*pegX)/(pegR^2 – pegY^2)^$^{1/2}$models the rate of change of length of the string with the y coordinate of the tuning peg.

As the peg is rotated, the length of the string reduces under the constraint ***L >= (1/2*fideal)*(tension/u)$^{1/2}$*** where fideal is the threshold frequency of the string and u is the mass per unit length of the string (density). The constraint ensures that the peg is rotated only as long as the frequency of the wave is lower than the ideal frequency (since frequency is inversely proportional to length of the string). Once ***L = (1/2*fideal)*(tension/u)$^{1/2}$*** the differential equations stop evolving and the frequency of the string matches the ideal frequency for the string.

Initial conditions

pegR>0 : The radius of the peg is always greater than zero

tension < tmax: The tension on the string is always lesser than a max value

tension > 0: Tension is never negative

u > 0: The string always has a positive density

fideal > 0: The ideal frequency of the string is always positive

Invariants

pegR > 0: The radius of the tuning peg never changes

tension < tmax: Tension in the string must always be lesser than the max tension value

tension > 0: Tension is a positive parameter

u > 0: Density never changes

fideal > 0: The threshold frequency for a string does not change

Safety Conditions

The model ensures that the **string never snaps** as a result of excess tension and hence the following constraint is always applied:

tension < tmax

PROOF

The goal of the project was to develop a controller that would tune the strings of a guitar to match the ideal frequency of that particular string. KeYTuner achieves this by rotating the tuning peg continuously until the correct frequency is achieved. This is governed by equation (1). The proof employs the loop invariant rule with the invariants described in the previous section. The differential equations are proved using differential cuts with (tension<tmax) as the invariant.

CONCLUSION

The project submission contains a kyx file that models a controller that can successfully tune a guitar string based on the model described above. The associated kyt file contains the tactics for the proof.

Currently, KeYTuner models and proves the ability of the model to tune only one string. This model can be extended to incorporate ideal frequencies for all six strings of the guitar and then tune each string, one at a time. By changing the ideal frequency, one could use KeYTuner to tune just about any string instrument.

REFERENCES

[1] Acoustic Guitar Anatomy http://www.guitaristsource.com/guitars/guitar_Anatomy_Acoustic.shtml
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[3] Standing Waves on a String http://hyperphysics.phy-astr.gsu.edu/hbase/waves/string.html
[4] Derivative of Sine Inverse http://www-math.mit.edu/~djk/18_01/chapter20/proof01.html
[5] Harmonics on a Guitar http://clearscience.tumblr.com/