1 Modes

We often talk about modes in Prolog. Modes are a way of describing how a predicate will be used. We denote an argument of a predicate with + if that argument is provided to the predicate, and with - if that argument is outputted by the predicate. A defined Prolog predicate can often work with many different modes.

As a small example, recall the color predicate from the homework. There are two possible modes for this predicate: color(+col) and color(-col). The predicate works correctly with both modes. If we try color(red) or color(blue), we get confirmation that those satisfy the predicate (as we defined them to). If we try color(C) in Prolog, we will receive all possible colors we defined (there are four). In general, the mode of a predicate which modes all arguments positively is always just a verifier that some instance satisfies the predicate.

Note that not all possible modes work correctly for a given predicate in Prolog. Many predicates do not work with all arguments moded negatively. For example, given a predicate permutation(L, P), where L is a list and P is a permutation of L, the mode permutation(-L, -P) will not terminate because there are infinite pairs (L, P) such that P is a permutation of L.

Task 1. Let the predicate zip be defined as follows:

\[
\text{zip([], [], []).} \\
\text{zip([X|L], [Y|M], [(X, Y)|P]) :- zip(L, M, P).}
\]

Which modes does zip work correctly with?

Solution 1: There are 5 modes that work correctly; the other 3 do not terminate.

\[
\begin{align*}
& (+, +, +) \\
& (+, +, -) \\
& (\neg, +, +) \\
& (+, \neg, +) \\
& (\neg, \neg, +)
\end{align*}
\]

Task 2. Let the predicate mult be defined as follows:

\[
\begin{align*}
\text{nat(z).} \\
\text{nat(s(N)) :- nat(N).} \\
\text{plus(z, N, N).} \\
\text{plus(s(M), N, s(P)) :- plus(M, N, P).} \\
\text{mult(z, N, z).} \\
\text{mult(s(M), N, P) :- plus(Q, N, P), mult(M, N, Q).}
\end{align*}
\]
Which modes does \texttt{mult} work correctly with?

\textbf{Solution 2:} Note that (+, +, -) does not work, even though it might be expected to, due to the infinite possibilities in the \texttt{plus}(-, +, -) mode.

- (+, +, +)
- (+, -, +)
- (-, +, +)

\textbf{Task 3.} How might we rewrite \texttt{mult} if we wanted it to work correctly with the \texttt{mult}(+, +, -) modality?

\textbf{Solution 3:} Change the order of the \texttt{plus} and \texttt{mult} in the third clause of the predicate.