RECITATION 5: SEQUENT CALCULUS AND KEYMAERA I INTRODUCTION

Yesterday, we presented the sequent calculus as a formalism that will be useful in proof search. We saw that there is a tight correspondence between its rules, and the rules for giving verifications for natural deduction. Today, we will review the rules for the sequent calculus and work through a few example proofs.

1. The Rules

Recall that left rules correspond to "upside down elimination rules" and that right rules correspond to introduction rules.

$$\frac{\Gamma, A \wedge B, A \Longrightarrow C}{\Gamma, A \wedge B \Longrightarrow C} \wedge L_1 \qquad \frac{\Gamma, A \wedge B, B \Longrightarrow C}{\Gamma, A \wedge B \Longrightarrow C} \wedge L_2 \qquad \frac{\Gamma \Longrightarrow A \quad \Gamma \Longrightarrow B}{\Gamma \Longrightarrow A \wedge B} \wedge R$$

$$\frac{\Gamma, A \vee B, A \Longrightarrow C \quad \Gamma, A \vee B, B \Longrightarrow C}{\Gamma, A \vee B \Longrightarrow C} \vee L \qquad \frac{\Gamma \Longrightarrow A}{\Gamma \Longrightarrow A \vee B} \vee R_1 \qquad \frac{\Gamma \Longrightarrow B}{\Gamma \Longrightarrow A \vee B} \vee R_2$$

No $\top L$. $\quad \dfrac{}{\Gamma \Longrightarrow \top} \top R \quad \dfrac{}{\Gamma, \bot \Longrightarrow C} \bot L \quad$ No $\bot R$.

$$\frac{\Gamma, A \supset B \Longrightarrow A \quad \Gamma, A \supset B, B \Longrightarrow C}{\Gamma, A \supset B \Longrightarrow C} \supset L \qquad \frac{\Gamma, A \Longrightarrow B}{\Gamma \Longrightarrow A \supset B} \supset R$$

$$\frac{}{\Gamma, A \Longrightarrow A} \text{id}$$

2. KeYmaera I Introduction

[Demo of how to set up and use KeYmaera I]

3. Some Example Proofs

We will spend the remainder of the recitation working through some example proofs, both by hand and in KeYmaera I. [1]

Exercise 1. $\cdot \Longrightarrow A \supset A$

Proof.

$$\frac{\dfrac{}{A \Longrightarrow A} \text{id}}{\cdot \Longrightarrow A \supset A} \supset R$$

□

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[1] Most example problems are taken from 15-317 (Fall 2015) Recitation 5, whose notes were prepared by Evan Cavallo, Oliver Daids, and Giselle Reis. Responsibility for any errors herein lies with the present author.

Exercise 2. $\cdot \Longrightarrow A \wedge B \supset B \wedge A$

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \wedge B, B \Longrightarrow B}\,\text{id}}{A \wedge B \Longrightarrow B}\,\wedge L_2 \qquad \dfrac{\dfrac{}{A \wedge B, A \Longrightarrow A}\,\text{id}}{A \wedge B \Longrightarrow A}\,\wedge L_1}{A \wedge B \Longrightarrow B \wedge A}\,\wedge R}{\cdot \Longrightarrow A \wedge B \supset B \wedge A}\,\supset R$$

□

Exercise 3. $\cdot \Longrightarrow (A \supset (B \wedge C)) \supset (A \supset B)$

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(A \supset (B \wedge C)), A \Longrightarrow A}\,\text{id} \qquad \dfrac{\dfrac{}{(A \supset (B \wedge C)), A, B \wedge C, B \Longrightarrow B}\,\text{id}}{(A \supset (B \wedge C)), A, B \wedge C \Longrightarrow B}\,\wedge L_1}{(A \supset (B \wedge C)), A \Longrightarrow B}\,\supset L}{(A \supset (B \wedge C)) \Longrightarrow (A \supset B)}\,\supset R}{\cdot \Longrightarrow (A \supset (B \wedge C)) \supset (A \supset B)}\,\supset R$$

□

Exercise 4. $\cdot \Longrightarrow (A \supset B \supset C) \supset B \supset A \supset C$

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \supset B \supset C, B, A \Longrightarrow A}\,\text{id} \qquad \dfrac{\dfrac{}{A \supset B \supset C, B, A, B \supset C \Longrightarrow B}\,\text{id} \quad \dfrac{}{A \supset B \supset C, B, A, B \supset C, C \Longrightarrow C}\,\text{id}}{A \supset B \supset C, B, A, B \supset C \Longrightarrow C}\,\supset L}{A \supset B \supset C, B, A \Longrightarrow C}\,\supset L}{A \supset B \supset C, B \Longrightarrow A \supset C}\,\supset R}{A \supset B \supset C \Longrightarrow B \supset A \supset C}\,\supset R}{\cdot \Longrightarrow (A \supset B \supset C) \supset B \supset A \supset C}\,\supset R$$

□

Exercise 5. $\cdot \Longrightarrow (A \supset B) \supset ((A \wedge C) \supset (B \wedge C))$

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(A \supset B), (A \wedge C), A \Longrightarrow A}\,\text{id}}{(A \supset B), (A \wedge C) \Longrightarrow A}\,\wedge L_1 \quad \dfrac{}{(A \supset B), (A \wedge C), B \Longrightarrow B}\,\text{id}}{(A \supset B), (A \wedge C) \Longrightarrow B}\,\supset L \qquad \dfrac{\dfrac{}{(A \supset B), (A \wedge C), C \Longrightarrow C}\,\text{id}}{(A \supset B), (A \wedge C) \Longrightarrow C}\,\wedge L_2}{(A \supset B), (A \wedge C) \Longrightarrow B \wedge C}\,\wedge R}{(A \supset B) \Longrightarrow ((A \wedge C) \supset (B \wedge C))}\,\supset R}{\cdot \Longrightarrow (A \supset B) \supset ((A \wedge C) \supset (B \wedge C))}\,\supset R$$

□