

# Constructive Logic (15-317), Spring 2020

## Assignment 3:

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Due: Tuesday, February 4, 2020, 11:59 pm

### 1 Harmony

**Task 1** (11 points). Consider a connective  $\odot$  with the following elimination rules:

$$\frac{\frac{\overline{A \text{ true}}^u \quad \overline{B \text{ true}}^v}{A \odot B \text{ true}} \quad \frac{\vdots}{C \text{ true}}}{C \text{ true}} \odot E^{u,v}$$

(Normally we take the verificationist perspective that introduction rules come first to define a connective, but this time we'll go in the opposite direction.)

- a. Come up with a set of zero or more introduction rules for this connective.
- b. Show that the connective is locally sound and complete for your choice of introduction rules.
- c. Is it possible to come up with a notational definition  $A \odot B \triangleq \underline{\hspace{2cm}}$  so that both your defined introduction rule(s) as well as the elimination rule given above are merely derived rules? You needn't prove that this fact, merely state yes or no. However, partial credit may be awarded for partially correct arguments.

**Task 2** (8 points). Consider a connective  $\times$  defined by the following rules:

$$\frac{\overline{A \text{ true}}^u \quad \vdots}{A \times B \text{ true}} \times I^u \quad \frac{A \times B \text{ true}}{B \text{ true}} \times E$$

- a. Is this connective locally sound? If so, provide the local reduction; if not, explain why no such reduction exists, and give (without proving) a replacement for the  $\times E$  rule to make the connective harmonious.
- b. Is this connective locally complete? If so, provide the local expansion; if not, explain why no such expansion exists, and give (without proving) a replacement for the  $\times E$  rule to make the connective harmonious.

## 2 Verifications

Recall the  $*$  connective from last week:

$$\frac{\overline{\top \text{ true}}^u \quad \vdots \quad \frac{A \text{ true}}{*A \text{ true}}{*I^u}}{\frac{*A \text{ true} \quad \top \text{ true}}{A \text{ true}}{*E}}$$

**Task 3** (6 points). Give rules for forming the judgments that  $*A$  has a verification and that  $*A$  can be used.

**Task 4** (5 points). Give a verification for this judgment

$$(\neg A \wedge B) \supset ((A \supset B) \supset (\neg A \supset \neg B)) \supset \perp \text{ true}.$$

**Task 5** (10 points). For each of the following propositions, give a verification and its corresponding proof term. (The proof terms annotations rules for verifications-and-uses follow the same way as the annotation rules from natural-deduction.)

- $\perp \supset \top$
- $\perp \supset \top$  (**Do not use the same verification/proof term as part a. Use a new one.**)
- $(A \supset B) \supset (\neg B \supset \neg A)$
- $(A \supset B) \supset (B \supset C) \supset (A \supset C)$