Constructive Logic (15-317), Fall 2016
Assignment 1: Harmony

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Welcome to your first assignment that involves Tutch!

This assignment is due at the beginning of class on the above date and it must be submitted electronically at autolab. Submit your homework as a `tar` archive containing two files: `hw1.pdf` (your written solutions) and `hw1.tut` (your Tutch solutions).

1 Tutch Proofs

Task 1 (10 points)**.** Prove the following theorems using Tutch.

```
proof absurdity : A & ~A => B;
proof sCombinator : (A => B) => (A => B => C) => (A => C);
proof deMorgin : ~(A | B) => ~A & ~B;
proof deMorgout : ~A & ~B => ~(A | B);
proof covariance : (A => B) => (X => Y | A & Z) => (X => Y | B & Z);
```

Recall that in Tutch `~A` is short hand for `A => F`.

You can check your progress against the provided requirements file `hw1.req` by running the command

```
$ tutch -r ./hw1.req hw1.tut
```

2 The Wheat and the Chaff

Task 2 (10 points)**.** The skill of detecting bogus arguments is critical in both mathematics and politics. The fallacy of *denying the antecedent* occurs occasionally in everyday bogus arguments. It looks like this:

$$(A \supset B) \supset (\neg A \supset \neg B)\ \mathit{true} \tag{DtA}$$

Show that this is bogus in the case where $\neg A \wedge B$ *true* by proving:

$$(\neg A \wedge B) \supset ((A \supset B) \supset (\neg A \supset \neg B)) \supset \bot \ \mathit{true}$$

Once again, recall that $\neg B$ is shorthand for $B \supset \bot$. Be sure to label each inference rule in your proof.

3 Harmony

Task 3 (10 points)**.** Consider a connective $\ltimes$ defined by the following rules:

$$\dfrac{A\ \mathit{true} \quad \begin{array}{c} \overline{A\ \mathit{true}}\ u \\ \vdots \\ B\ \mathit{true} \end{array}}{A \ltimes B\ \mathit{true}}\ {\ltimes}I^u \qquad \dfrac{A \ltimes B\ \mathit{true}}{B\ \mathit{true}}\ {\ltimes}E$$

1. Is this connective locally sound? If so, show the reduction; if not, explain (informally) why no such reduction exists.
2. Is this connective locally complete? If so, give an appropriate local expansion; otherwise, explain (informally) why no such expansion exists.

Task 4 (10 points)**.** Consider a connective $\odot$ with the following elimination rules:

$$\dfrac{A \odot B\ \mathit{true} \quad A\ \mathit{true}}{B\ \mathit{true}}\ {\odot}E_1 \qquad \dfrac{A \odot B\ \mathit{true} \quad B\ \mathit{true}}{A\ \mathit{true}}\ {\odot}E_2$$

(Normally we take the verificationist perspective that introduction rules come first, but this time we'll go in the opposite direction.)

1. Come up with a set of zero or more introduction rules for this connective.
2. Show that the connective is locally sound and complete for your choice of introduction rules.