Operational Semantics of Prolog

Relate Logical Meaning to Operational Meaning

1. Left-to-right subgoal selection
2. First-to-last clause selection with backtrack
3. Unification instantiates schema vars + unknowns
4. ~~Cut commits to particular choice of a clause~~
5. ~~Built-in arithmetic ...~~

Stay on most appropriate, not most accurate level

Prolog

Logic Programming

Definitional Interpreter / John Reynolds '72

Write an interpreter for a language in the language
(alias meta-circular interpreter)
Then replace advanced language features by simpler ones

⓪ Prolog solve(A) :- A. HOLP

logical connectives are data

① Prolog true/0 ,/2 term internalize

```
solve(true).
solve((A,B)) :- solve(A), solve(B).
solve(P) :- clause(P, B), solve(B).
```

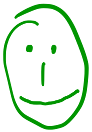
atomic

```
P :- B.
P :- true.
```

Subgoal Order / L2R

solve(A,S) solves goal A with stack S remaining

```
solve(true, true).
solve(true, (A,S)) :- solve(A, S).
solve((A,B), S) :- solve(A, (B,S)).
solve(P, S) :- clause(P, B), solve(B, S).
```



subgoal order

Subgoal Order mae Logically

A/S A under S capturing solve(A,S)

$$\frac{}{T/T}\ \text{done} \qquad \frac{A/S}{T/A\wedge S}\ \text{pop}$$

$$\frac{A/B\wedge S}{A\wedge B/S}\ \text{push}$$

$$\frac{B_1\wedge\ldots\wedge B_n/S}{P/S}\ \text{clause} \qquad \text{rule}\ \frac{B_1\ \text{true}\ \ldots\ B_n\ \text{true}}{P\ \text{true}}$$

```
×   solve(true, true).
    solve(true, (A , S)) :- solve(A, S).
×   solve((A , B), S) :- solve(A, (B , S)).
    solve(P, S) :- clause(P, B), solve(B, S
```

Thm (Sound): If A / S then A true and S true

Proof: By induction on structure of deduction of A/S

- $\mathcal{D} = \dfrac{}{\top / \top}$ done then indeed $\dfrac{}{\top \text{ true}}\top I$

- $\mathcal{D} = \dfrac{\overset{\mathcal{D}_1}{A / S}}{\top / A \wedge S}$ pop

$$A \text{ true by IH } \mathcal{D}_1 \qquad \dfrac{}{\top \text{ true}}\top I$$

$$\dfrac{A \text{ true} \qquad S \text{ true}}{A \wedge S \text{ true}}\wedge I$$

- $\mathcal{D} = \dfrac{\overset{\mathcal{D}_1}{A / B \wedge S}}{A \wedge B / S}$ push

$$A \text{ true by IH } \mathcal{D}_1$$

$$\dfrac{B \wedge S \text{ true}}{S \text{ true}}\wedge E_2 \qquad \dfrac{A \text{ true} \qquad \dfrac{B \wedge S \text{ true IH } \mathcal{D}_1}{B \text{ true}}\wedge E_1}{A \wedge B \text{ true}}\wedge I$$

- $\mathcal{D} = \dfrac{B_1 \wedge \ldots \wedge B_n / S}{P / S}$ clause for rule $\dfrac{B_1 \text{ true} \quad \ldots \quad B_n \text{ true}}{P \text{ true}}R_{99}$

$$\dfrac{\dfrac{B_1 \wedge \ldots \wedge B_n \text{ true}}{B_1 \text{ true}}\wedge E^* \quad \cdots\cdots \quad \dfrac{B_1 \wedge \ldots \wedge B_n \text{ true}}{B_n \text{ true}}\wedge E^*}{P \text{ true}}R_{99}$$

Thm (Complete): If A true and T/S then A/S

Proof:

Thm (Complete): If A true and T/S then A/S

Proof: By induction on structure of deduction of A true

- $\mathcal{D} = \dfrac{}{T\ \text{true}}\,TI$ and T/S then T/S

- $\mathcal{D} = \dfrac{\overset{\mathcal{D}_1}{A_1\ \text{true}} \quad \overset{\mathcal{D}_2}{A_2\ \text{true}}}{A_1 \wedge A_2\ \text{true}}\,\wedge I$ then

$$\dfrac{\overset{\text{IH},\ \mathcal{D}_2}{A_2 / S}}{T / A_2 \wedge S}\,\text{pop}$$

$$\Downarrow\ \mathcal{D}_1\ \text{IH}$$

$$\dfrac{A_1 / A_2 \wedge S}{A_1 \wedge A_2 / S}\,\text{push}$$

- $\mathcal{D} = \dfrac{\overset{\mathcal{D}_1}{B_1\ \text{true}} \ \dots\ \overset{\mathcal{D}_2}{B_2\ \text{true}}}{P\ \text{true}}$ then

$$\dfrac{\overset{\mathcal{D}_2\ \text{IH}}{B_2 / S}}{T / B_2 \wedge S}\,\text{pop}$$

$$\Downarrow\ \mathcal{D}_1\ \text{IH}$$

$$\dfrac{\dfrac{\dfrac{B_1 / B_2 \wedge S}{T / (B_1 \wedge B_2) \wedge S}\,\text{pop}}{B_1 \wedge B_2 / S}\,\text{push}}{P / S}\,\text{clause}$$

Backtracking

$\top \quad \wedge$

make explicit in logic with $\vee \quad \bot$

- normalize clauses to explicit disjunctive form

$$\frac{X \doteq Y}{\text{member}(X, [\cancel{Y} \mid Ys])} \qquad \frac{\text{member}(X, Ys) \quad R5}{\text{member}(X, [Y \mid Ys])}$$

$$\frac{X \doteq Y \;\vee\; (\text{member}(X, Ys) \wedge R5)}{\text{member}(X, [Y \mid Ys])}$$

- Complement with explicit premiss

$$\frac{\bot}{\text{member}(X, [\,])}$$

Logical Backtracking

$(A \wedge S) \vee F$

$A / S / F$ Either A under S or F

$$\frac{}{T / T / F}\ \text{done} \qquad \frac{A / S / F}{T / A \wedge S / F}\ \text{pop} \qquad \frac{A / B \wedge S / F}{A \wedge B / S / F}\ \text{push}$$

$$\frac{B / S / F}{P / S / F}\ \text{clause} \qquad \text{for rule } \frac{B \text{ true}}{P \text{ true}}$$

$$\frac{A / S / B \vee F}{A \vee B / S / F} \qquad \frac{B / T / F}{\bot / S / B \vee F}$$

$$\frac{A / S / (B \wedge S) \vee F}{A \vee B / S / F} \qquad \frac{B / S / F}{\bot / S' / (B \wedge S) \vee F}$$

Thm (Sound): if $A / S / F$ then $(A \wedge S) \vee F$ true

Thm ~~(Complete)~~: ?

$$\frac{div \vee T}{div}$$

div :- div.
div :- true.

Semantics Operational Prolog

subgoal select L2R

clause + backtrack F2L