Constructive Logic (15-317), Fall 2015
Recitation 14: Exam Review (12/9/15)
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Today's goal is to review for the exam. This review is a selection of the most popular topics from the exam review homework. We will not be able to do all of these problems today, but the remaining problems are good practice.

1 DeMorgan's Law

Show DeMorgan equivalence $\neg(A \vee B) \equiv (\neg A \wedge \neg B)$ under classical logic.

Can DeMorgan equivalence be proved in sequent calculus? Prove or disprove.

2 Restricted Sequent Calculus

Why is it important for the omega context in restricted sequent calculus to be ordered?

3 Soundness and Completeness

Define soundness. Define completeness. Give an example of unsound and complete systems. Take a connective and modify its rules to make it unsound, and then make it sound but incomplete.

Consider the following rules in natural deduction.

$$\dfrac{\begin{array}{cc} \overline{A}^{\,u} & \overline{B}^{\,v} \\ \vdots & \vdots \\ B & A \end{array}}{A \Leftrightarrow B} \Leftrightarrow I^{u,v} \qquad \dfrac{A \Leftrightarrow B \qquad B}{A} \Leftrightarrow E_L \qquad \dfrac{A \Leftrightarrow B \qquad A}{B} \Leftrightarrow E_R$$

Prove they are both complete and sound.

4 Classical Logic

Peirce's Law is $((A \supset B) \supset A) \supset A$. Show that there is no proof of Peirce's Law in sequent calculus. Write a proof of Peirce's Law using classical logic. Do the same for the law of the excluded middle.

5 Linear Logic

Linear logic makes fewer assumptions than sequent calculus does. What assumptions does sequent calculus make that linear logic does not? Claim: making fewer assumptions results in a system that in some ways is more powerful. How can this be? Why are there more connectives in linear logic than in sequent calculus?

What are the benefits of using linear logic v.s. other types of deduction (natural deduction, classical logic, and sequent calculus)? Specifically, for what types of problems would it be better to use linear logic as opposed to sequent calculus? Give an example of such a problem.

Is it possible to add rules to linear logic so that you get something equivalent to sequent calculus? If yes, show two introduction and elimination rules that demonstrate how this transformation occurs. Otherwise, explain why this transformation is impossible.

6 Inversion

Prove that $\oplus L$ is invertible in linear logic.

7 Polarity

Assign polarity to the sequent calculus operators. Do they retain the same sign as their linear logic counterparts? Why or why not?

8 Sequent Calculus

Show admissibility of contraction in sequent calculus.

9 Prolog

This problem is a slight variation of problem 2 from midterm 2 Fall 2013. Consider the following code snippet:

```
replace(X,Y,[],[]).
replace(X,Y,[X|Xs],[Y|Ys]) :- !, replace(X,Y,Xs,Ys).
replace(X,Y,[Z|Xs],[Z|Ys]) :- replace(X,Y,Xs,Ys).
```

The intended meaning of `replace(+X,+Y,+Xs,-Ys)` is that Ys is derived from the list Xs by replacing all occurrences of X by Y.

Task 1. Identify if the cut in the second clause is red or green, and explain why.

Task 2. Write an alternative version of replace without using cut.

10 iff

Consider a new connective that corresponds to $A \vdash B$ and $B \vdash A$.

1. Give its left and right rules.
2. Classify the connective as positive or negative, proving the appropriate inversion properties.