

1 Concepts

- Why do we use sequent calculus? What are its advantages over natural deduction?
- If I can prove something in sequent calculus can I prove it in natural deduction? What about visa versa?
- What does it mean for sequent calculus to be sound? complete?

Sequent calculus rules

$$\frac{\Gamma, A \Longrightarrow B}{\Gamma \Longrightarrow A \supset B} \supset R \quad \frac{\Gamma, B_1 \supset B_2 \Longrightarrow B_1 \quad \Gamma, B_1 \supset B_2, B_2 \Longrightarrow A}{\Gamma, B_1 \supset B_2 \Longrightarrow A} \supset L$$

$$\frac{\Gamma \Longrightarrow A \quad \Gamma \Longrightarrow B}{\Gamma \Longrightarrow A \wedge B} \wedge R \quad \frac{\Gamma, A \wedge B, A \Longrightarrow C}{\Gamma, A \wedge B \Longrightarrow C} \wedge L_1 \quad \frac{\Gamma, A \wedge B, B \Longrightarrow C}{\Gamma, A \wedge B \Longrightarrow C} \wedge L_2$$

$$\frac{\Gamma \Longrightarrow A}{\Gamma \Longrightarrow A \vee B} \vee R_1 \quad \frac{\Gamma \Longrightarrow B}{\Gamma \Longrightarrow A \vee B} \vee R_2 \quad \frac{\Gamma, A \vee B, A \Longrightarrow C \quad \Gamma, A \vee B, B \Longrightarrow C}{\Gamma, A \vee B \Longrightarrow C} \vee L$$

$$\overline{\Gamma, P \Longrightarrow P} \textit{ init} \quad \overline{\Gamma \Longrightarrow \top} \top R \quad \overline{\Gamma, \perp \Longrightarrow C} \perp L$$

2 Theorems

- **Weakening** For all Γ, A, C , if $\Gamma \Longrightarrow C$, then $\Gamma, A \Longrightarrow C$.
- **Identity** For all Γ, A : $\Gamma, A \Longrightarrow A$.
- **Cut** For all Γ, A, C , if $\Gamma \Longrightarrow A$ and $\Gamma, A \Longrightarrow C$, then $\Gamma \Longrightarrow C$.

3 Sequent Calculus Practice

1. $\Longrightarrow A \wedge B \supset B \wedge A$

2. $\Longrightarrow A \wedge B \supset A$

$$3. \implies A \supset A$$

$$4. \implies (B \vee A) \supset (A \vee B)$$

$$5. \implies (A \supset (B \wedge C)) \supset (A \supset B)$$

$$6. \implies (A \supset B) \supset ((A \wedge C) \supset (B \wedge C))$$

$$7. \implies ((A \vee B) \wedge C) \supset (A \wedge C) \vee (B \wedge C)$$