

Midterm I Exam

15-317/657 Constructive Logic
André Platzer

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Name: _____

Andrew ID: _____

Instructions

- Throughout this exam, explain whenever there are notable steps or choices or subtleties and justify the rationale for your particular choice!
- This exam is closed-book with one sheet of notes permitted.
- You have 80 minutes to complete the exam.
- There are 4 problems on 7 pages.
- Read each problem carefully before attempting to solve it.
- Do not spend too much time on any one problem.
- Consider if you might want to skip a problem on a first pass and return to it later.

	Max	Score
Eliminators, Eliminationists, and Eliminated	50	
Rules, the More the Merrier	20	
Conceptual Gadgets	60	
Natural Gadgets	20	
Total:	150	

1 Eliminators, Eliminationists, and Elimination (50 points)

So far, we took the view: “The meaning of a proposition is determined by [...] what counts as a verification of it.” In this question, the eliminationist’s get their say, who believe: “the meaning of a proposition should be determined by what counts as an elimination or a use of it.” They came up with the elimination rules for a new connective \triangleleft :

$$\frac{A \triangleleft B \text{ true}}{B \text{ true}} \triangleleft E_{\gg} \qquad \frac{A \triangleleft B \text{ true} \quad B \text{ true}}{A \text{ true}} \triangleleft E_{\ll}$$

10 Task 1 Give the introduction rule(s) that (harmoniously) fit to $\triangleleft E_{\gg}$ and $\triangleleft E_{\ll}$:

10 Task 2 Prove local soundness for the \triangleleft connective.

10 Task 3 Prove local completeness for the \triangleleft connective.

10 **Task 4** Give rules for verifications and uses of $A \triangleleft B$.

10 **Task 5** Consider this proof term assignment for $\triangleleft E_{\ll}$ and $\triangleleft E_{\gg}$:

$$\frac{M : A \triangleleft B}{\gg(M) : B} \triangleleft E_{\gg} \qquad \frac{M : A \triangleleft B \quad N : B}{\ll(M, N) : A} \triangleleft E_{\ll}$$

Propose a proof term assignment for the introduction rule(s) and write your local reductions using only the proof terms.

2 Rules, the More the Merrier (20 points)

In this question, we consider suggestions for new and improved proof rules that honorable Captain Blackbeard came up with. *Either* show the proof rules to be derived from other natural deduction rules considered in the course. *Or* show that they can be used to prove a formula that we cannot prove soundly and *explain briefly* why that formula should not be proved.

10 Task 1

$$\frac{A \vee B \text{ true} \quad A \text{ true}}{\neg B \text{ true}} \vee E_T$$

10 Task 2

$$\frac{\frac{\perp \text{ true}}{A \text{ true}} \quad \vdots \quad u}{A \text{ true}} PBQ^u$$

3 Conceptual Gadgets (60 points)

Inspector Gadget, has defined the binary connective $!$ as follows:

$$\frac{\frac{\overline{B \text{ true}}^u \quad \vdots}{A \text{ true} \quad \perp \quad \text{true}}}{A!B \text{ true}} !I^u$$

- 10 **Task 1** Define one or more elimination rules that are in harmony with the above rule. You do not need to prove harmony, but points will be deducted if the two are not in harmony.
- 10 **Task 2** Inspector Gadget likes local soundness but doesn't much care for local completeness. According to this view, he has proposed a system for verifying the safety of air traffic control systems. What bad thing might happen when he tries to use his system to verify the safety of a system?
- 10 **Task 3** After reading your answer above, Inspector Gadget has decided to change his ways. He now likes local completeness but, instead, got his system locally unsound. He tries to use this system, again to verify safety of an air traffic control system. What bad thing might happen now?

5 **Task 4** Inspector Gadget has read your answers and now believes in local soundness and local completeness. He has become a strict verificationist and now eats only applesauce and Pop Tarts. When he defines new connectives, he has to write both introduction and elimination rules. Which of those serve to define the *meaning* of each connective?

10 **Task 5** Inspector Gadget is really trying hard. He's come up with a new connective:

$$\frac{A \text{ true} \quad B \supset \perp \text{ true}}{A \heartsuit B \text{ true}} \heartsuit I$$

However, when he presents his work, people are not impressed. Explain (informally, in English) what's wrong with it and why that's bad. Please write a new introduction rule that fixes the problems but has the same meaning.

15 **Task 6** While you're at it, write corresponding elimination rule(s) that are in harmony with the introduction rule you present. Prove that your new introduction rule is equivalent to $\heartsuit I$ by showing that if something can be proved with one rule, it can also be proved by the other from the same evidence.

4 Natural Gadgets (20 points)

Inspector Gadget finally got the hang of natural numbers from natural deductions. He went straight ahead to implement the corresponding reductions in his new proof checker Toughch. Toughch uses “nut” as a better name for the natural numbers

$$\frac{}{0 : \text{nut}} \text{nut}I_0 \qquad \frac{n : \text{nut}}{s n : \text{nut}} \text{nut}I_s$$

Inspector Gadget randomly decided upon assigning the following new and improved proof terms:

$$\frac{\begin{array}{c} \frac{}{x : \text{nut}} \quad \frac{}{u : C(x) \text{ true}}^u \\ \vdots \\ n : \text{nut} \quad N_0 : C(0) \text{ true} \quad N_s : C(x) \text{ true} \end{array}}{G(n, N_0, x. u. N_s) : C(n) \text{ true}} \text{nut}E^{x,u}$$

- 10 **Task 1** Read off the local reductions on proof terms that the proof term assignment from $\text{nut}E^{x,u}$ would induce.

- 10 **Task 2** Toughch uses these reductions to compute with natural numbers. What behavior will Inspector Gadget observe. Explain why.