Constructive Logic (15-317), Fall 2015
Assignment 1: Harmony

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Due Tuesday, September 15, 2015

Welcome to your first assignment that involves Tutch!

The Tutch portion of your work (Section 1) should be submitted electronically using the command

```
$ /afs/andrew/course/15/317/bin/submit -r hw1 <files...>
```

from any Andrew server. You may check the status of your submission by running the command

```
$ /afs/andrew/course/15/317/bin/status hw1
```

If you have trouble running either of these commands, email Anna, Michael, or Vincent.

The written portion of your work (Sections 2 and 3) should be submitted at the beginning of class. If you are familiar with LaTeX, you are encouraged to use this document as a template for typesetting your solutions, but you may alternatively write your solutions *neatly* by hand.

1 Tutch Proofs

Task 1 (10 points)**.** Prove the following theorems using Tutch.

```
Reflexivity: A => A
Distributivity: ((A | B) & C) => (A & C) | (B & C)
Implicandtion: (A => B) => ((A & C) => (B & C))
Implicortion: (A => B) => ((A | C) => (B | C))
Idempotency: ((A => B) & (A => ~B)) => ~A
```

```
Recall that in Tutch ~A is short hand for A => F!
```

On Andrew machines, you can check your progress against the requirements file `/afs/andrew/course/15/317/req/hw1.req` by running the command

```
$ /afs/andrew/course/15/317/bin/tutch -r hw1 <files...>
```

2 The Wheat and the Chaff

Task 2 (10 points)**.** The skill of detecting bogus arguments is critical in both mathematics and politics. The fallacy of *affirming a disjunct* occurs occasionally in everyday bogus arguments. It looks like this:

$$((A \lor B) \land A) \supset \neg B$$

Show that this is bogus in the case where $A \land B$ *true* by proving:

$$(A \land B) \supset ((((A \lor B) \land A) \supset \neg B) \supset \bot)\ \mathit{true}$$

Once again, recall that $\neg B$ is shorthand for $B \supset \bot$.

3 Harmony and Derivability

Task 3 (10 points)**.** Consider a connective defined by the following rules:

$$\dfrac{\begin{array}{cc} \dfrac{}{A\ \mathit{true}}\,u & \dfrac{}{B\ \mathit{true}}\,v \\ \vdots & \vdots \\ B\ \mathit{true} & C\ \mathit{true} \end{array}}{\clubsuit(A,B,C)\ \mathit{true}}\,\clubsuit I^{u,v} \qquad \dfrac{\clubsuit(A,B,C)\ \mathit{true} \quad A\ \mathit{true}}{C\ \mathit{true}}\,\clubsuit E$$

1. Is this connective locally sound? If so, show the reduction; if not, explain (informally) why no such reduction exists.

2. Is this connective locally complete? If so, give an appropriate local expansion; otherwise, explain (informally) why no such expansion exists.

Task 4 (10 points)**.** Suppose that we define another connective with the same introduction rule as for $\clubsuit$, but with two elimination rules, as follows:

$$\dfrac{\begin{array}{cc} \dfrac{}{A\ \mathit{true}}\,u & \dfrac{}{B\ \mathit{true}}\,v \\ \vdots & \vdots \\ B\ \mathit{true} & C\ \mathit{true} \end{array}}{\diamond(A,B,C)\ \mathit{true}}\,\diamond I^{u,v} \qquad \dfrac{\diamond(A,B,C)\ \mathit{true} \quad A\ \mathit{true}}{B\ \mathit{true}}\,\diamond E_1 \qquad \dfrac{\diamond(A,B,C)\ \mathit{true} \quad B\ \mathit{true}}{C\ \mathit{true}}\,\diamond E_2$$

1. Show local soundness and completeness for $\diamond$.

2. Show that $\diamond(A, B, C)$ is definable as $(A \supset B) \wedge (B \supset C)$. You must show two things: (1) if there is a derivation of $\diamond(A, B, C)$ *true*, then there is a derivation of $(A \supset B) \wedge (B \supset C)$ *true*; and (2) the rules for $\diamond$ are derivable in the system without $\diamond$, if we regard $\diamond(A, B, C)$ as an abbreviation for $(A \supset B) \wedge (B \supset C)$.